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RADIO MEAN LABELING OF DOUBLE TRIANGULAR SNAKE GRAPH AND QUADRILATERAL SNAKE GRAPH

Dr. C. DAVID RAJ[1], Dr. A. SUBRAMANIAN[2] AND K. SUNITHA*[3]

[1]Department of Mathematics, Malankara Catholic College, Mariagiri, India.

[2]Department of Mathematics, M. D. T. Hindu College, Tirunelveli, India.

[3]Department of Mathematics, Women's Christian College, Nagercoil, India.

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ABSTRACT

A Radio Mean labeling of a connected graph G is a one to one map h from the vertex set V(G) to the set of natural numbers N such that for any two distinct vertices x and y of G, $d(x, y) + \left\lceil \frac{h(x)+h(y)}{2} \right\rceil \geq 1 + diam(G)$. *The radio mean number of h, rmn(h), is the maximum number assigned to any vertex of G. The radio mean number of G, rmn(G), is the minimum value of rmn(h) taken over all radio mean labelings h of G. In this paper we find the radio mean number of double triangular snake graph and double quadrilateral snake graph.*

Keywords: *Radio mean labeling, Diameter, Double triangular snake graph and Double quadrilateral snake graph.*

1. INTRODUCTION AND DEFINITIONS

Throughout this paper we consider finite, simple, undirected and connected graphs. V(G) and E(G) respectively denote the vertex set and edge set of G. Radio labeling, or multilevel distance labeling, is motivated by the channel assignment problem for radio transmitters [1]. Ponraj *et al*. [3] introduced the notion of radio mean labeling of graphs and investigated radio mean number of some graphs [11]. D.S.T. Ramesh, A. Subramanian and K. Sunitha investigated radio number for some graphs [9, 10] and introduced the radio mean square labeling of some graphs [8]. The span of a labeling h is the maximum integer that h maps to a vertex of G. The radio mean number of G, rmn(G) is the lowest span taken over all radio mean labeling of the graph G. For standard terminology and notations we follow Harary [4] and Gallian [7]. The distance between two vertices x and y of G is denoted by d(x, y) and diam(G) indicate the diameter of G.

Definition 1.1[2]: The distance d(u, v) from a vertex u to a vertex v in a connected graph G is the minimum of the lengths of the u-v paths in G.

Definition 1.2[2]: The eccentricity e(v) of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G.

Definition 1.3[2]: The diameter diam(G) of G is the greatest eccentricity among the vertices of G.

Definition 1.4 [5]: A double triangular snake consists of two triangular snakes that have a common path. That is, a double triangular snake DT_n is a graph obtained from a path $x_1, x_2, \ldots, x_n$ by joining x_i and x_{i+1} to two new vertices y_i and z_i, $1 \leq i \leq n - 1$.

Definition 1.5 [6]: A double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path. That is, a double quadrilateral snake graph DQ_n is a graph obtained from a path $x_1, x_2, \ldots, x_n$ by joining x_i and x_{i+1} to new vertices y_i, y_i' and z_i, z_i' respectively and then joining y_i, z_i and y_i', z_i'.

Corresponding Author: K. Sunitha[3],*
[3]Department of Mathematics, Women's Christian College, Nagercoil, India.

2. MAIN RESULTS

Theorem 2.1: $rmn(DT_n) = 4n - 4,\ n \geq 2$

Proof: Consider a path $x_1, x_2, \ldots, x_n$. Join x_i and x_{i+1} to two new vertices y_i and z_i, $1 \leq i \leq n-1$.

The resultant graph is DT_n whose edge set is $E = \{x_i x_{i+1}, x_i y_i, x_{i+1} y_i, x_i z_i, x_{i+1} z_i \,/\, 1 \leq i \leq n-1\}$ and $diam(DT_n) = n - 1$.

Define a function $h: V(DT_n) \to N$ by

$h(x_i) = 3n+i-4,\ 1 \leq i \leq n$
$h(y_i) = n+2i-3,\ 1 \leq i \leq n-1$
$h(z_i) = n+2i-2,\ 1 \leq i \leq n-1$

Next we check the radio mean condition for h.

Case-a: Consider the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i)+h(x_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n+i+j-8}{2} \right\rceil \geq n = 1 + diam(DT_n)$$

Case-b: Consider the pair (x_i, y_j), $1 \leq i \leq n$, $1 \leq j \leq n-1$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i)+h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+i+2j-7}{2} \right\rceil \geq n$$

Case-c: Consider the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n-1$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i)+h(y_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2n+2i+2j-6}{2} \right\rceil \geq n,\ n \geq 3$$

Case-d: Consider the pair (x_i, z_j), $1 \leq i \leq n$, $1 \leq j \leq n-1$

$$d(x_i, z_j) + \left\lceil \frac{h(x_i)+h(z_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+i+2j-6}{2} \right\rceil \geq n$$

Case-e: Consider the pair (z_i, z_j), $i \neq j$, $1 \leq i, j \leq n-1$

$$d(z_i, z_j) + \left\lceil \frac{h(z_i)+h(z_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2n+2i+2j-4}{2} \right\rceil \geq n$$

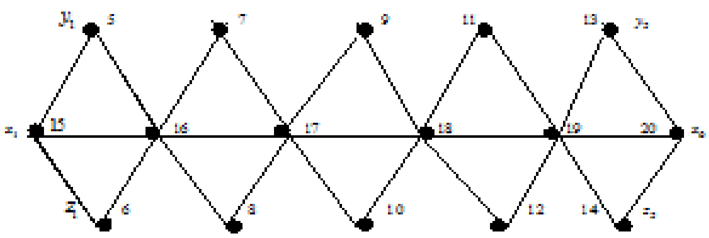
Case-f: Consider the pair (y_i, z_j), $1 \leq i, j \leq n-1$

$$d(y_i, z_j) + \left\lceil \frac{h(y_i)+h(z_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{2n+2i+2j-5}{2} \right\rceil \geq n$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of DT_n.
Therefore $rmn(DT_n) \leq rmn(h) = 4n - 4$

Since h is injective, $rmn(DT_n) \geq 4n - 4$ for all radio mean labelings h and hence $rmn(DT_n) = 4n - 4$, $n \geq 2$.

Example 2.1:



$rmn(DT_6) = 20$
Figure 1

Theorem 2.2: $rmn(DQ_n) = 6(n - 1),\ n \geq 2$

Proof: Let $x_1, x_2, ..., x_n$ be a path. For $1 \leq i \leq n - 1$, add vertices y_i and z_i and join them with x_i. For $1 \leq i \leq n - 1$, add vertices y_i' and z_i' and join them with x_{i+1}. Finally join y_i and y_i' and join z_i and z_i'. The resultant graph is DQ_n whose edge set is $E(DQ_n) = \{x_i x_{i+1}, x_{i+1} xy_i', x_{i+1} z_i', x_iy_i, y_i y_i', x_iz_i, z_iz_i' / 1 \leq i \leq n - 1\}$ and $diam(DQ_n) = n – 1$. Define a function $h: V(DQ_n) \rightarrow N$ by

$h(x_i) = 5n\text{-}6+i,\ 1 \leq i \leq n$
$h(y_i) = n+2i\text{-}3,\ 1 \leq i \leq n - 1$
$h(y_i') = 3n+2i\text{-}4,\ 1 \leq i \leq n - 1$
$h(z_i) = 3n+2i\text{-}5,\ 1 \leq i \leq n - 1$
$h(z_i') = n+2i\text{-}2,\ 1 \leq i \leq n - 1$

Now we check the radio mean condition for h.

Case-a: Consider the pair (y_i, y_j'), $1 \leq i, j \leq n - 1$

$$d(y_i, y_i') + \left\lceil \frac{h(y_i) + h(y_i')}{2} \right\rceil \geq 1 + \left\lceil \frac{4n + 2i + 2j - 7}{2} \right\rceil \geq n = 1 + diam(DQ_n)$$

Case-b: Consider the pair (y_i, y_j), $i \neq j$, $1 \leq i, j \leq n - 1$

$$d(y_i, y_j) + \left\lceil \frac{h(y_i) + h(y_j)}{2} \right\rceil \geq 3 + \left\lceil \frac{2n + 2i + 2j - 6}{2} \right\rceil \geq n$$

Case-c: Consider the pair (y_i', y_j'), $i \neq j$ and $1 \leq i, j \leq n - 1$

$$d(y_i', y_j') + \left\lceil \frac{h(y_i') + h(y_j')}{2} \right\rceil \geq 3 + \left\lceil \frac{6n + 2i + 2j - 8}{2} \right\rceil \geq n,\ n \geq 3$$

Case-d: Consider the pair (x_i, x_j), $i \neq j$, $1 \leq i, j \leq n$

$$d(x_i, x_j) + \left\lceil \frac{h(x_i) + h(x_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{10n + i + j - 12}{2} \right\rceil \geq n$$

Case-e: Consider the pair (x_i, y_j), $1 \leq i \leq n$ and $1 \leq j \leq n - 1$

$$d(x_i, y_j) + \left\lceil \frac{h(x_i) + h(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n + i + 2j - 9}{2} \right\rceil \geq n$$

Case-f: Consider the pair (x_i, y_j'), $1 \leq i \leq n$ and $1 \leq j \leq n - 1$

$$d(x_i, y_j') + \left\lceil \frac{h(x_i) + h(y_j')}{2} \right\rceil \geq 2 + \left\lceil \frac{8n + i + 2j - 10}{2} \right\rceil \geq n$$

Case-g: Consider the pair (x_i, z_j), $1 \leq i \leq n$ and $1 \leq j \leq n - 1$

$$d(x_i, z_j) + \left\lceil \frac{h(x_i) + h(z_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n + i + 2j - 11}{2} \right\rceil \geq n$$

Case-h: Consider the pair (x_i, z_j'), $1 \leq i \leq n$ and $1 \leq j \leq n - 1$

$$d(x_i, z_j') + \left\lceil \frac{h(x_i) + h(z_j')}{2} \right\rceil \geq 2 + \left\lceil \frac{6n + i + 2j - 8}{2} \right\rceil \geq n$$

Case-i: Consider the pair (y_i, z_j), $1 \leq i, j \leq n - 1$

$$d(y_i, z_j) + \left\lceil \frac{h(y_i) + h(z_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n + 2i + 2j - 8}{2} \right\rceil \geq n$$

Case-j: Consider the pair (y_i', z_j'), $1 \le i, j \le n - 1$

$$d(y_i', z_j') + \left\lceil \frac{h(y_i') + h(z_j')}{2} \right\rceil \ge 2 + \left\lceil \frac{4n + 2i + 2j - 6}{2} \right\rceil \ge n$$

Case-k: Consider the pair (y_i, z_j'), $1 \le i, j \le n - 1$

$$d(y_i, z_j') + \left\lceil \frac{h(y_i) + h(z_j')}{2} \right\rceil \ge 3 + \left\lceil \frac{2n + 2i + 2j - 5}{2} \right\rceil \ge n$$

Case-l: Consider the pair (y_i', z_j), $1 \le i, j \le n - 1$

$$d(y_i', z_j) + \left\lceil \frac{h(y_i') + h(z_j)}{2} \right\rceil \ge 2 + \left\lceil \frac{6n + 2i + 2j - 9}{2} \right\rceil \ge n$$

Case-m: Consider the pair (z_i, z_j), $i \ne j$, $1 \le i, j \le n - 1$

$$d(z_i, z_j) + \left\lceil \frac{h(z_i) + h(z_j)}{2} \right\rceil \ge 3 + \left\lceil \frac{6n + 2i + 2j - 10}{2} \right\rceil \ge n, \; n \ge 3$$

Case-n: Consider the pair (z_i', z_j'), $i \ne j$ and $1 \le i, j \le n - 1$

$$d(z_i', z_j') + \left\lceil \frac{h(z_i') + h(z_j')}{2} \right\rceil \ge 3 + \left\lceil \frac{2n + 2i + 2j - 4}{2} \right\rceil \ge n, \; n \ge 3$$

Case-o: Consider the pair (z_i, z_j'), $1 \le i, j \le n - 1$

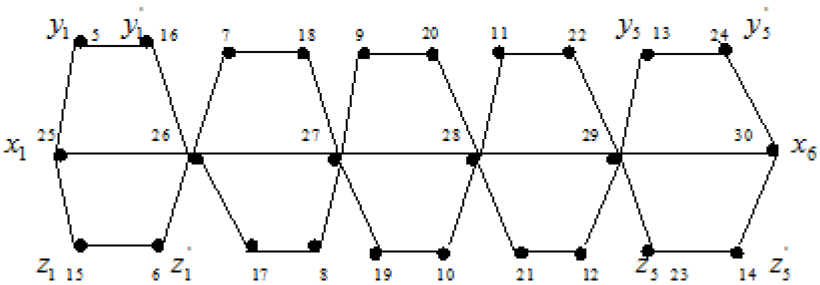
$$d(z_i, z_j') + \left\lceil \frac{h(z_i) + h(z_j')}{2} \right\rceil \ge 1 + \left\lceil \frac{4n + 2i + 2j - 7}{2} \right\rceil \ge n$$

Thus, the radio mean condition is satisfied for all pairs of vertices. Hence h is a valid radio mean labeling of DQ_n.

Therefore $rmn(DQ_n) \le rmn(h) = 6(n - 1)$

Since h is injective, $rmn(DQ_n) \ge 6(n - 1)$ for all radio mean labelings h and hence $rmn(DQ_n) = 6(n - 1)$, $n \ge 2$

Example 2.2:



$rmn(DQ_6) = 30$

Figure-2

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