

NEW AUGMENTED ZAGREB INDICES

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ABSTRACT

We introduce some new augmented Zagreb indices: second, third and fourth augmented Zagreb indices of a molecular graph. A topological index is a numerical parameter mathematically derived from the graph structure. In this paper, we compute augmented zagreb index and Sanskruti index of triangular benzenoid T_m , hexagonal parallelogram nanotube $P(m, n)$ and zigzag-edge coronoid fused with starphene nanotube $ZCS(k, l, m)$ by using the line graphs of the subdivision graphs of these important chemical graphs.

Keywords: augmented Zagreb index, triangular benzenoid, hexagonal parallelogram nanotube, zigzag-edge coronoid fused with starphene nanotube.

Mathematics Subject Classification: 05C05, 05C12, 05C35.

1. INTRODUCTION

Let $G = (V, E)$ be a simple connected graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v . The edge connecting the vertices u and v will be denoted by uv . Let $S_G(v)$ denote the sum of the degrees of all vertices adjacent to a vertex v . We refer to [1] for undefined term and notation.

A molecular graph or a chemical graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. A topological index is a numerical parameter mathematically derived from the graph structure. These topological indices are useful for establishing correlations between the structure of a molecular compound and its physico-chemical properties, see [2].

In [3], Furtula *et al.* introduced the augmented Zagreb index of a graph. The augmented Zagreb index of a graph G is defined as

$$AZI(G) = \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3.$$

This topological index has proved to a valuable predictive index in the study of the heat formation in octanes, and heptanes, whose prediction power is better than atom bond connectivity index, see [3]. This index was also studied, for example, in [4, 5].

Motivated by the previous research in augmented Zagreb index and its wide applications, we now introduce the second, third and fourth augmented Zagreb indices of the molecular graph as follows:

The second augmented Zagreb index of a molecular graph G is defined as

$$AZI_2(G) = \sum_{uv \in E(G)} \left(\frac{n_u n_v}{n_u + n_v - 2} \right)^3$$

where the number n_u of vertices of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G .

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The third augmented Zagreb index of a molecular graph G is defined as

$$AZI_3(G) = \sum_{uv \in E(G)} \left(\frac{m_u m_v}{m_u + m_v - 2} \right)^3$$

where the number m_u of edges of G lying closer to the vertex u than to the vertex v for the edge uv of a graph G .

The fourth augmented Zagreb index of a molecular graph G is defined as

$$AZI_4(G) = \sum_{uv \in E(G)} \left(\frac{\varepsilon(u)\varepsilon(v)}{\varepsilon(u) + \varepsilon(v) - 2} \right)^3$$

where the number $\varepsilon(u)$ is the eccentricity of all vertices adjacent to a vertex u .

In [6], the Sanskruti index of a graph G is introduced by Hosamani and it is defined as

$$S(G) = \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3.$$

In this paper, we compute the augmented Zagreb index and Sanskruti index of line graphs of subdivision graphs of triangular benzenoid T_n , hexagonal parallelogram nanotube $P(m, n)$ and zigzag-edge coronoid fused with starphene nanotube $ZCS(k, l, m)$. For benzenoid structures, see [7].

Recently, some topological indices were studied, for example, in [7, 8, 9, 10, 11, 12, 13, 14].

The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if the corresponding edges of G are adjacent. The subdivision graph $S(G)$ of a graph G is the graph obtained from G by replacing each of its edges by a path of length two.

We need the following results.

Lemma 1 [1]: Let G be a (p, q) graph. Then $L(G)$ has q vertices and $\frac{1}{2} \sum_{i=1}^p d_G(u_i)^2 - q$ edges.

Lemma 2: Let G be a (p, q) graph. Then $S(G)$ has $p+q$ vertices and $2q$ edges.

2. RESULTS FOR TRIANGULAR BENZENOIDS $T_n, n \in \mathbb{N}$

In this section, we consider triangular benzenoids which is a family of benzenoid molecular graphs. We denote the triangular benzenoid molecular graph by T_n in which n is the number of hexagons in the base of a graph, as shown in

Figure 1(a). We see that a triangular benzenoid T_n has $n^2 + 4n + 1$ vertices and $\frac{3}{2}n(n+3)$ edges.

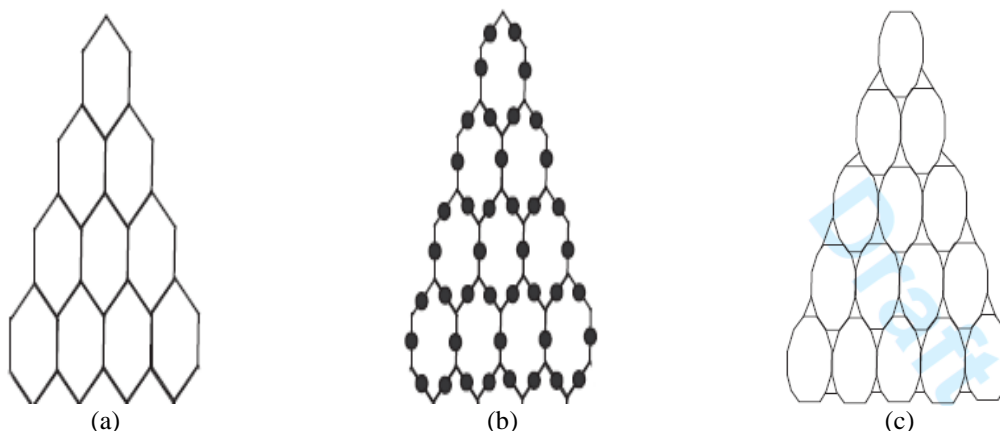


Figure-1: (a) triangular benzenoid T_4 , (b) subdivision graph of T_4 , (c) Line graph of the subdivision graph of T_4 .

The line graph of the subdivision graph of triangular benzenoid T_4 is shown in Figure 1(c).

We compute augmented Zagreb index of the line graph of the subdivision graph of a triangular benzenoid.

Theorem 2.1: Let G be the line graph of the subdivision graph of a triangular benzenoid T_n , $n \in \mathbb{N}$. Then $AZI(G) = 51.2578125 n^2 + 89.0859375n - 44.34375$.

Proof: The graph of a triangular benzenoid T_n has n^2+4n+1 vertices and $\frac{3}{2}n(n+3)$ edges. By Lemma 2 and

Lemma 1, the line graph of the subdivision graph G has $3n(n+3)$ vertices and $\frac{3}{2}(3n^2+7n-2)$ edges. Further, the edge partition of G based on degree of end vertices of each edge is given in Table 1.

$d_G(u), d_G(v) \setminus uv \in E(G)$	(2, 2)	(2, 3)	(3,3)
Number of edges	$3(n+3)$	$6(n-1)$	$\frac{3}{2}(3n^2+n-4)$

Table-1: Edge partition of G

To compute $AZI(G)$, we see that

$$\begin{aligned}
 AZI(G) &= \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u)+d_G(v)-2} \right)^3 \\
 &= 3(n+3) \left(\frac{2 \times 2}{2+2-2} \right)^3 + 6(n-1) \left(\frac{2 \times 3}{2+3-2} \right)^3 + \frac{3}{2}(3n^2+n-4) \left(\frac{3 \times 3}{3+3-2} \right)^3 \\
 &= 51.2578125n^2 + 89.0859375n - 44.34375.
 \end{aligned}$$

In the following theorem, we compute Sanskruti index of the line graph of the subdivision graph of a triangular benzenoid.

Theorem 2.2: Let G be the line graph of the subdivision graph of a triangular benzenoid T_n , $n \in \mathbb{N}$. Then

$$\begin{aligned}
 S(G) &= \frac{9}{2} \left(\frac{81}{16} \right)^3 n^2 + \left[3 \left(\frac{25}{8} \right)^3 + 6 \left(\frac{40}{11} \right)^3 + 3 \left(\frac{32}{7} \right)^3 + 6 \left(\frac{72}{15} \right)^3 - \frac{15}{2} \left(\frac{81}{16} \right)^3 \right] n \\
 &\quad + 9 \left(\frac{8}{3} \right)^3 + 6 \left(\frac{20}{7} \right)^3 - 6 \left(\frac{25}{8} \right)^3 - 6 \left(\frac{40}{11} \right)^3 - 3 \left(\frac{32}{7} \right)^3 + 3 \left(\frac{81}{16} \right)^3, \text{ if } n \neq 1, \\
 &= \frac{256}{3}, \text{ if } n = 1,
 \end{aligned}$$

Proof: The edge partition based on the degree sum of neighbor vertices of each edge of G is obtained, as given in Table 2.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4,4)	(4,5)	(5,5)	(5, 8)	(8, 8)	(8, 9)	(9, 9)
Number of edges	9	6	$3(n-2)$	$6(n-1)$	$3(n-1)$	$6(n-1)$	$\frac{3}{2}(3n^2-5n+2)$

Table-2: Edge partition of G

Case-1: Suppose $n \neq 1$.

To compute $S(G)$, we see that

$$\begin{aligned}
 S(G) &= \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)-2} \right)^3 \\
 &= 9 \left(\frac{4 \times 4}{4+4-2} \right)^3 + 6 \left(\frac{4 \times 5}{4+5-2} \right)^3 + 3(n-2) \left(\frac{5 \times 5}{5+5-2} \right)^3 + 6(n-1) \left(\frac{5 \times 8}{5+8-2} \right)^3 \\
 &\quad + 3(n-1) \left(\frac{8 \times 8}{8+8-2} \right)^3 + 6(n-1) \left(\frac{8 \times 9}{8+9-2} \right)^3 + \frac{3}{2}(3n^2-5n+2) \left(\frac{9 \times 9}{9+9-2} \right)^3 \\
 &= \frac{9}{2} \left(\frac{81}{16} \right)^3 n^2 + \left[3 \left(\frac{25}{8} \right)^3 + 6 \left(\frac{40}{11} \right)^3 + 3 \left(\frac{32}{7} \right)^3 + 6 \left(\frac{72}{15} \right)^3 - \frac{15}{2} \left(\frac{81}{16} \right)^3 \right] n \\
 &\quad + 9 \left(\frac{8}{3} \right)^3 + 6 \left(\frac{20}{7} \right)^3 - 6 \left(\frac{25}{8} \right)^3 - 6 \left(\frac{40}{11} \right)^3 - 3 \left(\frac{32}{7} \right)^3 + 3 \left(\frac{81}{16} \right)^3.
 \end{aligned}$$

Case-2: Suppose $n = 1$.

To compute $S(G)$, we see that

$$S(G) = \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3 = 9 \left(\frac{4 \times 4}{4 + 4 - 2} \right)^3 = \frac{256}{3}.$$

3. RESULTS FOR HEXAGONAL PARALLELOGRAM $P(m, n)$ FOR ANY $m, n \in \mathbb{N}$ NANOTUBES

In this section, we consider hexagonal parallelogram nanotubes. These nanotubes usually symbolized as $P(m, n)$ for any $m, n \in \mathbb{N}$ in which m is the number of hexagons in any row and n is the number hexagons in any column, see Figure 2(a). A hexagonal parallelogram $P(m, n)$ has $2(m+n+mn)$ vertices and $3mn + 2m + 2n - 1$ edges.

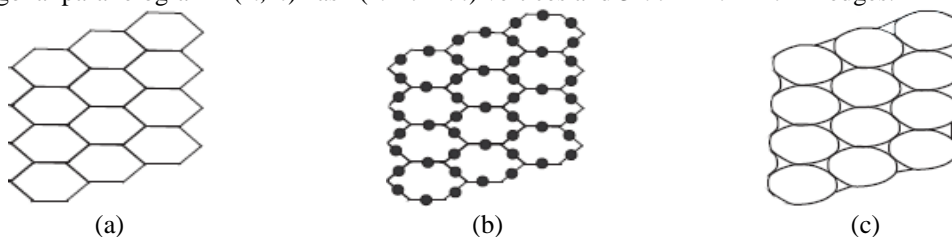


Figure-2: (a) hexagonal parallelogram $P(3, 4)$ (b) subdivision graph of $P(3, 4)$ (c) line graph of the subdivision graph of $P(3,4)$

The line graph of the subdivision graph of hexagonal parallelogram $P(3,4)$ is depicted in Figure 2(c).

In the following theorem, we compute augmented Zagreb index of the line graph of the subdivision graph of a hexagonal parallelogram.

Theorem 3.1: Let G be the line graph of the subdivision graph of a hexagonal parallelogram $P(m, n)$ for any $m, n \in \mathbb{N}$. Then

$$AZI(G) = 102.515625 mn + 25.21875m + 25.21875n - 56.953125.$$

Proof: The graph of a hexagonal parallelogram $P(m, n)$ has $2(m+n+mn)$ vertices and $3mn + 2m + 2n - 1$ edges. By Lemma 2 and Lemma 1, the line graph of the subdivision graph G has $2(3mn+2m+2n-1)$ vertices and $9mn + 4m + 4n - 5$ edges. Further, the edge partition of G based on degree of end vertices of each edge is given in Table 3.

$d_G(u), d_G(v) \setminus e = uv \in E(G)$	(2, 2)	(2, 3)	(3,3)
Number of edges	$2(m+n+4)$	$4(m+n-2)$	$9mn-2m-2n-5$

Table-3: Edge degree partition of G

To compute $AZI(G)$, we see that

$$\begin{aligned} AZI(G) &= \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3 \\ &= 2(m+n+4) \left(\frac{2 \times 2}{2+2-2} \right)^3 + 4(m+n-2) \left(\frac{2 \times 3}{2+3-2} \right)^3 + (9mn-2m-2n-5) \left(\frac{3 \times 3}{3+3-2} \right)^3 \\ &= 102.515625mn + 25.21875m + 25.21875n - 56.953125. \end{aligned}$$

In the following theorem, we compute Sanskruti index of the line graph of the subdivision graph of a hexagonal parallelogram.

Theorem 3.2: Let G be the line graph of the subdivision graph of a hexagonal parallelogram $P(m, n)$ for any $m, n \in \mathbb{N}$. Then

$$\begin{aligned} S(G) &= \left[2 \left(\frac{25}{8} \right)^3 + 4 \left(\frac{40}{11} \right)^3 + 2 \left(\frac{32}{7} \right)^3 + 4 \left(\frac{72}{15} \right)^3 + \left(\frac{81}{16} \right)^3 \right] n \\ &\quad + 10 \left(\frac{8}{3} \right)^3 + 4 \left(\frac{20}{7} \right)^3 - 4 \left(\frac{25}{8} \right)^3 - 4 \left(\frac{40}{11} \right)^3 - 2 \left(\frac{32}{7} \right)^3 - 4 \left(\frac{72}{15} \right)^3 - \left(\frac{81}{16} \right)^3, \quad \text{if } n \neq 1, m = 1, \end{aligned}$$

$$\begin{aligned}
&= 9\left(\frac{81}{16}\right)^3 mn + \left[2\left(\frac{25}{8}\right)^3 + 4\left(\frac{40}{11}\right)^3 + 2\left(\frac{32}{7}\right)^3 + 4\left(\frac{72}{15}\right)^3 - 8\left(\frac{81}{16}\right)^3 \right] (m+n) \\
&+ 8\left(\frac{8}{3}\right)^3 + 8\left(\frac{20}{7}\right)^3 - 8\left(\frac{25}{8}\right)^3 - 8\left(\frac{40}{11}\right)^3 - 4\left(\frac{32}{7}\right)^3 - 8\left(\frac{72}{15}\right)^3 + 7\left(\frac{81}{16}\right)^3, \quad \text{if } n \neq 1, m > 1.
\end{aligned}$$

Proof: Case 1. The edge partition based on the degree sum of neighbor vertices of each edge of G is obtained, as given Table 4.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4,4)	(4,5)	(5,5)	(5, 8)	(8, 8)	(8, 9)	(9,9)
Number of edges	10	4	$2(n-2)$	$4(n-1)$	$2(n-1)$	$4(n-1)$	$n-1$

Table-4: Edge partition of G

Suppose $n \neq 1$ and $m = 1$.

To compute $S(G)$, we see that

$$\begin{aligned}
S(G) &= \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3 \\
&= 10\left(\frac{4 \times 4}{4+4-2}\right)^3 + 4\left(\frac{4 \times 5}{4+5-2}\right)^3 + 2(n-2)\left(\frac{5 \times 5}{5+5-2}\right)^3 + 4(n-1)\left(\frac{5 \times 8}{5+8-2}\right)^3 \\
&+ 2(n-1)\left(\frac{8 \times 8}{8+8-2}\right)^3 + 4(n-1)\left(\frac{8 \times 9}{8+9-2}\right)^3 + (n-1)\left(\frac{9 \times 9}{9+9-2}\right)^3 \\
&= \left[2\left(\frac{25}{8}\right)^3 + 4\left(\frac{40}{11}\right)^3 + 2\left(\frac{32}{7}\right)^3 + 4\left(\frac{72}{15}\right)^3 + \left(\frac{81}{16}\right)^3 \right] n \\
&+ 10\left(\frac{8}{3}\right)^3 + 4\left(\frac{20}{7}\right)^3 - 4\left(\frac{25}{8}\right)^3 - 4\left(\frac{40}{11}\right)^3 - 2\left(\frac{32}{7}\right)^3 - 4\left(\frac{72}{15}\right)^3 - \left(\frac{81}{16}\right)^3.
\end{aligned}$$

Case-2: The edge partition based on the degree sum of neighbor vertices of each edge of G is obtained, as given in Table 5.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4,4)	(4,5)	(5,5)	(5, 8)	(8, 8)	(8, 9)	(9,9)
Number of edges	8	8	$2(m+n-4)$	$4(m+n-2)$	$2(m+n-2)$	$4(m+n-2)$	$9mn-8(m+n)+7$

Table-5: Edge partition of G

Suppose $n \neq 1$, and $m > 1$.

To compute $S(G)$, we see that

$$\begin{aligned}
S(G) &= \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3 \\
&= 8\left(\frac{4 \times 4}{4+4-2}\right)^3 + 8\left(\frac{4 \times 5}{4+5-2}\right)^3 + 2(m+n-4)\left(\frac{5 \times 5}{5+5-2}\right)^3 + 4(m+n-2)\left(\frac{5 \times 8}{5+8-2}\right)^3 \\
&+ 2(m+n-2)\left(\frac{8 \times 8}{8+8-2}\right)^3 + 4(m+n-2)\left(\frac{8 \times 9}{8+9-2}\right)^3 + (9mn-8(m+n)+7)\left(\frac{9 \times 9}{9+9-2}\right)^3 \\
&= 9\left(\frac{81}{16}\right)^3 mn + \left[2\left(\frac{25}{8}\right)^3 + 4\left(\frac{40}{11}\right)^3 + 2\left(\frac{32}{7}\right)^3 + 4\left(\frac{72}{15}\right)^3 - 8\left(\frac{81}{16}\right)^3 \right] (m+n) \\
&+ 8\left(\frac{8}{3}\right)^3 + 8\left(\frac{20}{7}\right)^3 - 8\left(\frac{25}{8}\right)^3 - 8\left(\frac{40}{11}\right)^3 - 4\left(\frac{32}{7}\right)^3 - 8\left(\frac{72}{15}\right)^3 + 7\left(\frac{81}{16}\right)^3.
\end{aligned}$$

4. RESULTS FOR ZIGZAG-EDGE CORONOID FUSED WITH STARPHENE NANOTUBES $ZCS(k, l, m)$

In this section, we consider the system which is a composite benzenoid obtained by a zigzag-edge coronoid $ZC(k, l, m)$ with a starphene $St(k, l, m)$. This system is called a zigzag-edge coronoid fused with starphene nanotubes, denoted by $ZCS(k, l, m)$, see Figure 3(a). We see that a zigzag edge coronoid fused with starphene nanotube $ZCS(k, l, m)$ has $36k - 54$ vertices and $15(k+l+m) - 63$ edges.

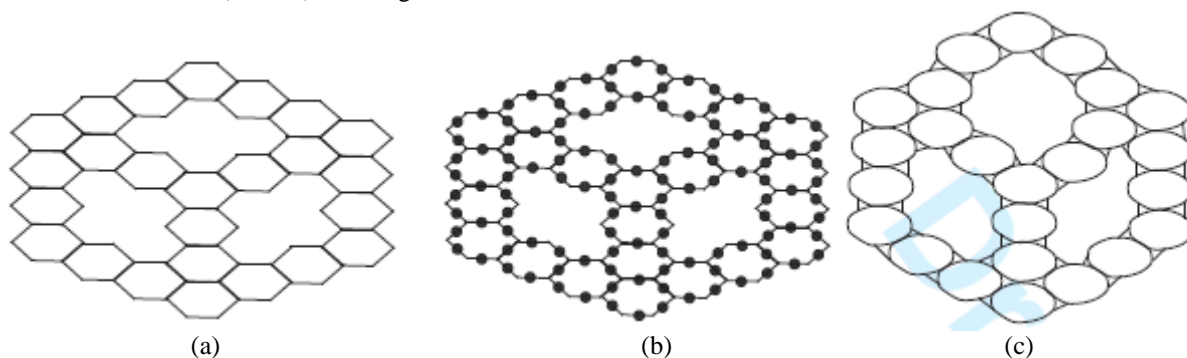


Figure-3: (a) Zigzag-edge coronoid fused with starphene nanotube $ZCS(4, 4, 4)$, (b) Subdivision graph of $ZCS(4, 4, 4)$, (c) Line graph of subdivision graph of $ZCS(4, 4, 4)$

The line graph of the subdivision graph of zigzag-edge coronoid fused with starphene nanotube $ZCS(4, 4, 4)$ is shown in Figure 3(c).

In the following theorem, we compute augmented Zagreb index of the line graph of the subdivision graph of a zigzag coronoid fused with starphene nanotube.

Theorem 4.1: Let G be the line graph of the subdivision graph of a zigzag-edge coronoid fused with starphene nanotube $ZCS(k, l, m)$ for every $k = l = m \geq 4$. Then

$$AZI(G) = 383.203125 (k + l + m) - 1356.234375.$$

Proof: The graph of a zigzag edge coronoid fused with starphene nanotube $ZCS(k, l, m)$ has $36k - 54$ vertices and $15(k + l + m) - 63$ edges. By Lemma 2 and Lemma 1, the line graph of the subdivision graph G has $30(k + l + m) - 126$ vertices and $39(k + l + m) - 153$ edges. Further, the edge partition of G based on degree of end vertices of each edge is given in Table 6.

$d_G(u), d_G(v) \setminus e = uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	$6(k + l + m) - 30$	$12(k + l + m) - 84$	$21(k + l + m) - 39$

Table-6: Edge partition of G

To compute $AZI(G)$, we see that

$$\begin{aligned}
 AZI(G) &= \sum_{uv \in E(G)} \left(\frac{d_G(u)d_G(v)}{d_G(u) + d_G(v) - 2} \right)^3 \\
 &= [6(k + l + m) - 30] \left(\frac{2 \times 2}{2 + 2 - 2} \right)^3 + [12(k + l + m) - 84] \left(\frac{2 \times 3}{2 + 3 - 2} \right)^3 + [21(k + l + m) - 39] \left(\frac{3 \times 3}{3 + 3 - 2} \right)^3 \\
 &= 383.203125 (k + l + m) - 1356.234375.
 \end{aligned}$$

In the next theorem, we compute Sankruti index of the line graph of the subdivision graph of $ZCS(k, l, m)$.

Theorem 4.2: Let G be the line graph of the subdivision graph of a zigzag-edge coronoid fused with starphene nanotube $ZCS(k, l, m)$ for every $k = l = m \geq 4$. Then

$$\begin{aligned}
 S(G) &= \left[6 \left(\frac{25}{8} \right)^3 + 12 \left(\frac{40}{11} \right)^3 + 6 \left(\frac{32}{7} \right)^3 + 12 \left(\frac{72}{15} \right)^3 + 3 \left(\frac{81}{16} \right)^3 \right] (k + l + m) \\
 &\quad + 6 \left(\frac{8}{3} \right)^3 + 12 \left(\frac{20}{7} \right)^3 - 8 \left(\frac{25}{8} \right)^3 - 84 \left(\frac{40}{11} \right)^3 - 54 \left(\frac{32}{7} \right)^3 - 60 \left(\frac{72}{15} \right)^3 + 75 \left(\frac{81}{16} \right)^3.
 \end{aligned}$$

Proof: The edge partition based on the degree sum of neighbor vertices of each edge of G is obtained, as given in Table 7.

$S_G(u), S_G(v) \setminus uv \in E(G)$	(4,4)	(4,5)	(5,5)	(5, 8)	(8, 8)	(8, 9)	(9,9)
Number of edges	6	12	$6(k+l+m-8)$	$12(k+l+m-7)$	$6(k+l+m-9)$	$12(k+l+m-5)$	$3(k+l+m+25)$

Table-7: Edge partition of G

To compute $S(G)$, we see that

$$\begin{aligned}
 S(G) &= \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u)+S_G(v)-2} \right)^3 \\
 &= 6 \left(\frac{4 \times 4}{4+4-2} \right)^3 + 12 \left(\frac{4 \times 5}{4+5-2} \right)^3 + 6(k+l+m-8) \left(\frac{5 \times 5}{5+5-2} \right)^3 + 12(k+l+m-7) \left(\frac{5 \times 8}{5+8-2} \right)^3 \\
 &\quad + 6(k+l+m-9) \left(\frac{8 \times 8}{8+8-2} \right)^3 + 12(k+l+m-5) \left(\frac{8 \times 9}{8+9-2} \right)^3 + 3(k+l+m+25) \left(\frac{9 \times 9}{9+9-2} \right)^3 \\
 &= \left[6 \left(\frac{25}{8} \right)^3 + 12 \left(\frac{40}{11} \right)^3 + 6 \left(\frac{32}{7} \right)^3 + 12 \left(\frac{72}{15} \right)^3 + 3 \left(\frac{81}{16} \right)^3 \right] (k+l+m) \\
 &\quad + 6 \left(\frac{8}{3} \right)^3 + 12 \left(\frac{20}{7} \right)^3 - 8 \left(\frac{25}{8} \right)^3 - 84 \left(\frac{40}{11} \right)^3 - 54 \left(\frac{32}{7} \right)^3 - 60 \left(\frac{72}{15} \right)^3 + 75 \left(\frac{81}{16} \right)^3.
 \end{aligned}$$

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. I. Gutman and N. Trinajstić, Graph theory and molecular orbitals. Total π -electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* 17 (1972) 535-538.
3. B. Furtula, A. Graovac and D. Vukićević, Augmented Zagreb index, *J. Math. Chem.* 48(2010) 370-380.
4. V.R.Kulli, Some topological indices of certain nanotubes, *Journal of Computer and Mathematical Sciences*, 8(1) (2017) 1-7.
5. V.R.Kulli, Computation of some topological indices of certain networks, *International Journal of Mathematical Archive*, 8(2) (2017) 99-106.
6. S.M. Hosamani, Computing Sanskruti index of certain nanostructures, *J. Appl. Math. Comput.* (2016) DOI: 10.1007/s12190-016-1016-9.
7. S.Akhter and M. Imran, On molecular topological properties of benzenoid structures, *Canadian J. Chem.* (2016) 1-27.
8. V.R. Kulli, On K edge index of some nanostructures, *Journal of Computer and Mathematical Sciences*, 7(7) (2016) 373-378.
9. V.R. Kulli, Computation of general topological indices for titania nanotubes, *International Journal of Mathematical Archive*, 7(12) (2016) 33-38.
10. V.R.Kulli, F -index and reformulated Zagreb index of certain nanostructures, *International Research Journal of Pure Algebra*, 7(1) (2017) 489-495.
11. V.R.Kulli, Multiplicative connectivity indices of nanostructures, *Journal of Ultra Scientist of Physical Sciences*, A 29(1) (2017) 1-10. DOI: <http://dx.doi.org/10.22147/jusps-A/290101>.
12. V.R.Kulli, Two new multiplicative atom bond connectivity indices, *Annals of Pure and Applied Mathematics*, 13(1) (2017) 1-7. DOI: <http://dx.doi.org/10.22457/apam.v13n1a1>.
13. V.R.Kulli, Some new multiplicative geometric-arithmetic indices, *Journal of Ultra Scientist of Physical Sciences*, A, 29(2) (2017) 52-57. DOI: <http://dx.doi.org/10.22147/jusps-A/290201>.
14. V.R.Kulli, New K Banhatti topological indices, *International Journal of Fuzzy Mathematical Archive*, 12(1) (2017) 29-37. DOI: <http://dx.doi.org/10.22457/ijfma.v12n1a4>.

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