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INDEPENDENT MAJORITY NEIGHBORHOOD POLYNOMIAL OF A GRAPH

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ABSTRACT

In this article we have introduced new polynomial Independent Majority Neighborhood polynomial of a graph G is defined as $N_{iM}(G,x) = \sum_{i=n_{iM}(G)}^{p} n_{iM}(G,i)x^i$, where $n_{iM}(G,i)$ is the number of independent majority neighborhood sets of size i and $n_{iM}(G)$ is the Independent majority neighborhood number of a graph. Also we have determined this new polynomial structure for some classes of graphs.

Key words: *Independent Majority neighborhood number, Independent Majority neighborhood polynomial.*

1. INTRODUCTION

Let G = (V, E) be a simple graph with p vertices and q edges for any vertex $v \in V$, the open neighborhood of v is the set of $N(v) = \{u \in V / uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$ [8]. A set of points S in a graph G is a neighborhood set of G if $G = \cup_{v \in S} \langle N[v] \rangle$ where $\langle N[v] \rangle$ is the sub graph of G induced by v and all points adjacent to v minimum cardinality of neighborhood set of G is the neighborhood number of a graph G [8]. A neighborhood set S of G is an independent neighborhood set if no two vertices in S are adjacent [7]. Let $G = (V, E)$ be a graph. A set $S \subseteq V(G)$ is called a majority neighborhood set if $G_M = \bigcup_{v \in S} \langle N[v] \rangle$ contains at least $\left\lceil \frac{p}{2} \right\rceil$ vertices and at least $\left\lceil \frac{q}{2} \right\rceil$ edges. [5] A majority neighborhood set S is called a minimal majority neighborhood set if no proper subset of S is a majority neighborhood set. The minimum cardinality of a minimal majority neighborhood set is called the majority neighborhood number of G and is denoted by $n_M(G)$. This parameter has been studied by Swaminathan.V and Joseline Manora. J [4]. Neighborhood polynomial $N(G,x)$ of a graph G has been introduced by J. Josline Manora and I. Paulraj Jayasimmman [6].

2. INDEPENDENT MAJORITY NEIGHBORHOOD POLYNOMIAL OF A GRAPH

Definition 2.1: Let $G = (V, E)$ be a graph of order p with the independent majority neighborhood number $n_{iM}(G)$. Then the independent majority neighborhood polynomial of G is defined as $N_{iM}(G,x) = \sum_{i=n_{iM}(G)}^{p} n_{iM}(G,i)x^i$, where $n_{iM}(G,i)$ is the number of independent majority neighborhood sets of size i.

The following example illustrates the new definition.

Let $G = P_7$ be a path of length 7 with, $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and $q = 6$ then $n_{iM}(G) = 2$. Therefore, the independent majority neighborhood sets of size 2, 3 and 4 are the following, $n_{iM}(G,2)x^2 = |N_{iM}(G,2)|x^2 = 14x^2$, $n_{iM}(G,3)x^3 = |N_{iM}(G,3)| = 10x^3$. Then exists no other independent majority neighborhood sets of sizes i=4, 5, 6 and 7. Hence, $N_{iM}(P_7,x) = 14x^2 + 10x^3$.

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Proposition 2.2: Let $G = \overline{K_p}$ be the totally disconnected graph with order $p \geq 2$. Then the independent majority neighborhood polynomial of G is $N_{iM}(G,x) = \sum_{i=\lceil \frac{p}{2} \rceil}^{p} \binom{p}{i} x^i$.

Proof: Since $n_{iM}(G) = \left\lceil \frac{p}{2} \right\rceil$, then the independent majority neighborhood sets are

$$n_{iM}\left(G, \left\lceil \frac{p}{2} \right\rceil\right) = \binom{p}{\lceil \frac{p}{2} \rceil}, n_{iM}\left(G, \left\lceil \frac{p}{2} \right\rceil + 1\right) = \binom{p}{\lceil \frac{p}{2} \rceil + 1}, n_{iM}\left(G, \left\lceil \frac{p}{2} \right\rceil + 2\right) = \binom{p}{\lceil \frac{p}{2} \rceil + 2}, \ldots n_{iM}(G,p) = \binom{p}{p}.$$

Therefore the independent majority neighborhood polynomial is $N_{iM}(G,x) = \sum_{i=\lceil \frac{p}{2} \rceil}^{p} \binom{p}{i} x^i$.

Proposition 2.3: Let $G = K_{1,p}$ be a star graph of order $p \geq 2$. Then the independent majority neighborhood polynomial of G is $N_{iM}(G,x) = x + \sum_{i=\lceil \frac{p}{2} \rceil}^{p} \binom{p}{i} x^i$.

Proposition 2.4: For a complete graph $G = K_p$ with ≥ 3 . Then the independent majority neighborhood polynomial of G is $N_{iM}(G,x) = px$.

Theorem 2.5: Let $G = D_{r,s}$ be the double star with $r, s \geq 2$ Then the independent majority neighborhood polynomial of G is

$$N_{iM}(G,x) = \begin{cases} 2x + \left(((1+x)^r - 1) + ((1+x)^s - 1)\right)x^{i+1} + \sum_{i=\lceil \frac{q}{2} \rceil}^{r+s} \binom{r+s}{i} x^i \ , \ if\ r < s \\ 2x\left(1 + ((1+x)^{2r} - 1)\right) + \sum_{i=\lceil \frac{q}{2} \rceil}^{2r} \binom{2r}{i} x^i \ , \qquad if\ r = s \end{cases}$$

Proof: Let $V(G) = \{u, v, u_1, u_2, \ldots, u_r, v_1, v_2, \ldots, v_s\}$ with $p = r + s + 2$. Let $X = \{u, v\}$ be the centre vertex set of G, $r = X_1 = N[u] = \{u_1, u_2, u_3, \ldots, u_r\}$ with $|X_1| = r$ and $X_2 = N[v] = \{v_1, v_2, v_3, \ldots, v_5\}$ with $|X_2| = s$. Since $n_{iM}(G) = 1$.

Case-(i): When $r < s$. Without loss of generality let $r < s$. Then independent majority neighborhood set of G of the size *i =1* $n_{iM}(G) = 1$ are $N_{iM}(G,1) = \{\{u\},\{v\}\}$, $|N_{iM}(G,1)| = n_{iM}(G,1) = 2$. This gives $N_{iM}(G,1)x = 2x$. Choose the independent majority neighborhood set with cardinality $i = 2$ then $N_{iM}(G,2) = \begin{Bmatrix} \{\{u\} \cup \{v_i\} / \ u \in X, v_i \in X_2\}, \\ \{\{v\} \cup \{u_i\} / \ v \in X, u_i \in X_1\} \end{Bmatrix}$

$\Rightarrow \ |N_{iM}(G,2)| = n_{iM}(G,2) = \binom{r}{1} + \binom{s}{1} = r + s.$ $\quad n_{iM}(G,2) = \left(\binom{r}{1} + \binom{s}{1}\right)x^2$. Next choose the independent majority neighborhood set of size $i = 3$ is $N_{iM}(G,2) = \begin{Bmatrix} \left\{\{u\} \cup \{v_i v_j\} / \ i \neq j \ , u \in X, v_i v_j \in X_2\right\}, \\ \left\{\{v\} \cup \{u_i u_j\} / \ i \neq j, v \in X, u_i u_j \in X_1\right\} \end{Bmatrix}$. Therefore, $n_{iM}(G,3) = \left(\binom{r}{2} + \binom{s}{2}\right)x^3$. The independent majority neighborhood set of the size is $i = \left\lceil \frac{q}{2} \right\rceil$ then the independent majority neighborhood sets are $N_{iM}\left(G, \left\lceil \frac{q}{2} \right\rceil\right) = \begin{Bmatrix} \{u_1, u_2, u_3, \ldots, u_i\} \cup \{v_1, v_2, v_3, \ldots, v_j\} u_i \in X_1, v_j \in X_2, \\ |X_1 + X_2| = \left\lceil \frac{q}{2} \right\rceil \end{Bmatrix}$

$$\Rightarrow \left|N_{iM}\left(G, \left\lceil \frac{q}{2} \right\rceil\right)\right| = n_{iM}\left(G, \left\lceil \frac{q}{2} \right\rceil\right) = \binom{r+s}{\lceil \frac{q}{2} \rceil}, \text{ this gives } N_{iM}\left(G, \left\lceil \frac{q}{2} \right\rceil\right) x^{\lceil \frac{q}{2} \rceil} = \binom{r+s}{\lceil \frac{q}{2} \rceil} x^{\lceil \frac{q}{2} \rceil}.$$

Hence,

$$N_{iM}(G,x) = \begin{Bmatrix} 2x + \left(\binom{r}{1} + \binom{s}{1}\right)x^2 + \left(\binom{r}{2} + \binom{s}{2}\right) x^3 + \cdots + \binom{r}{r} x^r + \binom{s}{2} x^s \\ + \binom{r+s}{\lceil \frac{q}{2} \rceil} x^{\lceil \frac{q}{2} \rceil} + \binom{r+s}{\lceil \frac{q}{2} \rceil + 1} x^{\lceil \frac{q}{2} \rceil + 1} + \cdots + \binom{r+s}{r+s} x^{r+s}, \ r < s \end{Bmatrix}$$

$$= 2x + \sum_{i=1}^{r} \binom{r}{i} x^{i+1} + \sum_{i=1}^{s} \binom{s}{i} x^{i+1} + \sum_{i=\lceil \frac{q}{2} \rceil}^{r+s} \binom{r+s}{i} x^i$$

Hence,

$$N_{iM}(G,x) = 2x + \left(((1+x)^r - 1) + ((1+x)^s - 1)\right) + \sum_{i=\lceil \frac{q}{2} \rceil}^{r+s} \binom{r+s}{i} x^i \ , if\ r < s$$

Case-(ii): If $r = s$ then the independent majority neighborhood sets of the size $n_{iM}(G) = 1$ are $N_{iM}(G,1) = \{\{u\},\{v\}/u,v \in X\} \Rightarrow |N_{iM}(G,1)| = n_{iM}(G,1) = 2$. This gives $N_{iM}(G,1)x = 2x$.

Next independent majority neighborhood sets of the size of $i = 2$ is $|N_{iM}(G,2)| = n_{iM}(G,2) = \left(\binom{r}{1} + \binom{s}{1}\right) = \left(\binom{r}{1} + \binom{r}{1}\right) = 2\binom{r}{1}$. Therefore $N_{iM}(G,2)x^2 = 2\binom{r}{2}$. For the size $i = 3$ is $2\binom{r}{3}x^3$. Hence $N_{iM}(G,i)x^i = 2\binom{r}{i}x^i$. The independent majority neighborhood sets of the size $\left\lceil\frac{q}{2}\right\rceil$ is $N_{iM}\left(G,\left\lceil\frac{q}{2}\right\rceil\right)x^{\left\lceil\frac{q}{2}\right\rceil} = \binom{r+s}{\left\lceil\frac{q}{2}\right\rceil}x^{\left\lceil\frac{q}{2}\right\rceil} = \binom{2r}{\left\lceil\frac{q}{2}\right\rceil}x^{\left\lceil\frac{q}{2}\right\rceil}$. Thus,

$$N_{iM}(G,x) = 2x + \left(2\binom{r}{1}x^2 + 2\binom{r}{2}x^3 + \cdots + 2\binom{r}{r}x^i\right) + \binom{2r}{\left\lceil\frac{q}{2}\right\rceil}x^{\left\lceil\frac{q}{2}\right\rceil} + \binom{2r}{\left\lceil\frac{q}{2}\right\rceil+1}x^{\left\lceil\frac{q}{2}\right\rceil+1} + \cdots + \binom{2r}{2r}x^{2r}.$$

$$N_{iM}(G,x) = \left\{2\left(x + \sum_{i=1}^{2r}\binom{2r}{i}x^{i+1}\right) + \sum_{i=\left\lceil\frac{q}{2}\right\rceil}^{2r}\binom{2r}{i}x^i\right.$$

Hence,

$$N_{iM}(G,x) = \left\{2x\left(1 + ((1+x)^{2r} - 1)\right) + \sum_{i=\left\lceil\frac{q}{2}\right\rceil}^{2r}\binom{2r}{i}x^i, if\ r = s\right.$$

From the above two cases,

$$N_{iM}(G,x) = \begin{cases} 2x + \left(((1+x)^r - 1) + ((1+x)^s - 1)\right) + \sum_{i=\left\lceil\frac{q}{2}\right\rceil}^{r+s}\binom{r+s}{i}x^i & ,if\ r < s \\ 2x\left(1 + ((1+x)^{2r} - 1)\right) + \sum_{i=\left\lceil\frac{q}{2}\right\rceil}^{2r}\binom{2r}{i}x^i & ,\ if\ r = s \end{cases}$$

Theorem 2.6: For $G = K_{m,n}$ be a complete bipartite graph with then independent majority neighborhood polynomial of G is

$$N_{iM}(G,x) = \begin{cases} \sum_{i=\left\lceil\frac{m}{2}\right\rceil}^{m}\binom{m}{i}x^i + \sum_{i=\left\lceil\frac{n}{2}\right\rceil}^{n}\binom{n}{i}x^i, & if\ m < n \\ 2\sum_{i=\left\lceil\frac{m}{2}\right\rceil}^{m}\binom{m}{i}x^i, & if\ m = n \end{cases}$$

Proof: Let $G = K_{m,n}$ be a complete bipartite graph $m,n \geq 2$ with the partition $V_1(G) = \{v_1,v_2,v_3,\ldots,v_m\}$ and $V_2(G) = \{v_1,v_2,v_3,\ldots,v_n\}$

Case-(i): If $m < n$ then $n_{iM}(G) = \left\lceil\frac{m}{2}\right\rceil$. Since $V_1(G)$ and $V_2(G)$ are the independent set and $\left(V_1(G)\right) = V_2(G)$ $N\left(V_2(G)\right) = V_1(G)$. Therefore combination of vertices of $V_1(G)$ with $V_2(G)$ are not an independent set. Then the independent majority neighborhood sets are the combination of vertices of $V_1(G)$ with the size $n_{iM}(G) = \left\lceil\frac{m}{2}\right\rceil, \left\lceil\frac{m}{2}\right\rceil + 1, \left\lceil\frac{m}{2}\right\rceil + 2, \ldots, m$ are $\binom{m}{\left\lceil\frac{m}{2}\right\rceil}, \binom{m}{\left\lceil\frac{m}{2}\right\rceil+1}, \binom{m}{\left\lceil\frac{m}{2}\right\rceil+2}, \binom{m}{\left\lceil\frac{m}{2}\right\rceil+3}, \ldots, \binom{m}{m}$ respectively and independent majority neighborhood sets with the combinations of vertices of $V_2(G)$ with the size $i = \left\lceil\frac{n}{2}\right\rceil, \left\lceil\frac{n}{2}\right\rceil + 1, \left\lceil\frac{n}{2}\right\rceil + 2, \ldots, n$ are $\binom{n}{\left\lceil\frac{n}{2}\right\rceil}, \binom{n}{\left\lceil\frac{n}{2}\right\rceil+1}, \binom{n}{\left\lceil\frac{n}{2}\right\rceil+2}, , \ldots, \binom{n}{n}$ respectively.

Hence,
$$N_{iM}(G,x) = \begin{cases} \binom{m}{\left\lceil\frac{m}{2}\right\rceil}x^{\left\lceil\frac{m}{2}\right\rceil} + \binom{m}{\left\lceil\frac{m}{2}\right\rceil+1}x^{\left\lceil\frac{m}{2}\right\rceil+1} + \binom{m}{\left\lceil\frac{m}{2}\right\rceil+2}x^{\left\lceil\frac{m}{2}\right\rceil+2} + \binom{m}{\left\lceil\frac{m}{2}\right\rceil+3}x^{\left\lceil\frac{m}{2}\right\rceil+3} \\ +..+\binom{m}{m}x^m + \binom{n}{\left\lceil\frac{n}{2}\right\rceil}x^{\left\lceil\frac{n}{2}\right\rceil} + \binom{n}{\left\lceil\frac{n}{2}\right\rceil+1}x^{\left\lceil\frac{n}{2}\right\rceil+1} + \binom{n}{\left\lceil\frac{n}{2}\right\rceil+2}x^{\left\lceil\frac{n}{2}\right\rceil+2}+.,+\binom{n}{n}x^n \\ \text{if m} < n \end{cases}$$

$$N_{iM}(G,x) = \left\{\sum_{i=\left\lceil\frac{m}{2}\right\rceil}^{m}\binom{m}{i}x^i + \sum_{i=\left\lceil\frac{n}{2}\right\rceil}^{n}\binom{n}{i}x^i,\quad if\ m < n\right.$$

Case-(ii): Let $m = n$. Then $V_1(G) = V_2(G)$.. Therefore $n_{iM}(G)$ - sets are the combination of vertices of $V_1(G)$ and $V_2(G)$ with the size the size $n_{iM}(G) = \left\lceil\frac{m}{2}\right\rceil, \left\lceil\frac{m}{2}\right\rceil + 1, \left\lceil\frac{m}{2}\right\rceil + 2, \ldots, m$ are $2\left(\binom{m}{\left\lceil\frac{m}{2}\right\rceil}, \binom{m}{\left\lceil\frac{m}{2}\right\rceil+1}, \binom{m}{\left\lceil\frac{m}{2}\right\rceil+2}, \ldots, \binom{m}{m}\right)$ respectively.

Therefore,

$$N_{iM}(G,x) = \left\{\sum_{i=\left\lceil\frac{m}{2}\right\rceil}^{m}2\binom{m}{i}x^i,\ if\ m = n\right.$$

Hence from the two cases,

$$N_{iM}(G,x) = \begin{cases} \sum_{i=\lceil \frac{m}{2} \rceil}^{m} \binom{m}{i} x^i + \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \binom{n}{i} x^i, & if\ m < n \\ 2\sum_{i=\lceil \frac{m}{2} \rceil}^{m} \binom{m}{i} x^i, & if\ m = n \end{cases}$$

REFERENCES

1. Alikhani S., Y.H. Peng, - Dominating sets and domination polynomial of cycles, Global Journal of Pure and Applied Mathematics 4 (2008) 2, 151–162.
2. Alikhani, S. and Y.H. Peng, - Dominating sets and Domination Polynomials of Paths, International Journal of Mathematics and Mathematical Science, Vol. 2009, Article ID 542040.
3. Alikhani.S and Y.H. Peng, - Introduction to domination Polynomial of a graph, arXiv: 095 225 IvI (2009).
4. JoselineManora. J and Swaminathan. V, Majority Neighbourhood Number of Graph, Scientia Magna, Northwest University, P.R. China, Vol. (6), No. 2 (2010), 20-25.
5. JoselineManora. J, Swaminathan. V, Results on majority dominating set, Science Magna, North – West University, X'tian, P. R. China, Vol. 7, No. 3(2011), 53 – 58.
6. JoselineManora. J and Paulraj Jayasimman. I. - Neighborhood set Polynomial of a graph, International Journal Applied Mathematical Sciences, ISSN 0973-0176 Volume 6, Number 1 (2013), pp. 91-97.
7. Sampathkumar.E and Prabha S. Neeralagi - Independent, Perfect, Connected Neighborhood number or a graph**,** Journal of Combinatorics Information & Sciences, Vol.19, 139-145 (1994).
8. Sampathkumar.E and Prabha S. Neeralagi - Neighborhood number of a graph, Indian J. Pure.Appl.Math., 16(2) 126-132.Feb.1985.

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