International Journal of Mathematical Archive-8(8), 2017, 134-141
Available online through www.ijma.info ISSN 2229 – 5046

# ON R# CLOSED SETS IN TOPOLOGICAL SPACES

BASAVARAJ M. ITTANAGI, RAGHAVENDRA K*

Department of Mathematics,
Siddaganga Institute of Technology, Tumakuru-03, Affiliated to VTU, Karnataka state, India.

*Department of Mathematics,
ACS College of Engineering, Bangalore-74, Affiliated to VTU, Karnataka state, India.

*(Received On: 27-07-17; Revised & Accepted On: 17-08-17)*

***ABSTRACT***

*In this paper, a new class of closed sets called $R^{\#}$ closed sets in topological spaces are introduced and studied. A subset A of a topological space $(X,\tau)$ is called $R^{\#}$-closed if U contains generalized closure of A whenever U contains A and U is R* open set in $(X,\tau)$. This new class of Closed sets lies between the g-closed sets and rg-closed sets in topological spaces. Also some of their properties have been investigated.*

***Keywords:*** *R* -closed sets, w-closed sets, rg-closed sets and $R^{\#}$-closed sets.*

***2010 Mathematics Subject Classification:*** *54A05, 54A10.*

## 1. INTRODUCTION

In the year 1970, Levine [58] introduced the concept of generalized closed sets in topological spaces, Also N.Palaniapan *et al.* [38], P.sundaram *et al.* [44], S.S.Benchalli *et al.* [7], C.Janaki *et al.* [19] introduced and studied regular generalized closed sets, W-closed sets, RW-closed sets, R*-closed sets in topological spaces respectively. In this paper an attempt is made to study a new class of closed sets called $R^{\#}$-closed sets in topological spaces.

Throughout this paper $(X,\tau)$ represent non-empty topological spaces. For a subset A of a topological space $(X,\tau)$, cl(A), int(A), scl(A), $\alpha$cl(A) and spcl(A)) denote the closure of A, the interior of A, the semi-closure of A, the $\alpha$-closure of A and the semi pre closure of A in a topological space X respectively. We recall the following definitions, which are prerequisites for present study.

## 2. PRELIMINARIES

**Definition 2.1:** A subset A of a topological space $(X,\tau)$ is called a

1. Generalized closed set (g-closed) [58] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X, \tau)$.
2. R*-closed set [19] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi -open in $(X, \tau)$.
3. RW-closed set [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi -open in $(X, \tau)$.
4. Generalized pre regular closed set (gpr-closed) [15] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in $(X, \tau)$.
5. Generalized semi pre closed set (gsp-closed) [14] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X, \tau)$.
6. Regular generalized closed set (rg-closed) [38] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular-open in $(X, \tau)$.
7. Regular weak generalized closed set (rwg-closed) [30] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in $(X, \tau)$.
8. w-closed set[44] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi -open in $(X, \tau)$.
9. gspr-closed set[35] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular -open in $(X, \tau)$.
10. r^g-closed set[43] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular -open in $(X, \tau)$.

*Corresponding Author: Raghavendra K*, *Department of Mathematics,*
*ACS College of Engineering, Bangalore-74, Affiliated to VTU, Karnataka state, India.*

## 3. BASIC PROPERTIES OF R#-CLOSED SETS IN TOPOLOGICAL SPACES

**Definition 3.1:** A subset A of a space (X,τ) is called R#-closed if gcl(A) ⊆ U whenever A ⊆ U and U is R*open in (X, τ), we use the notation R#-C(X) to denote set of all R#-closed sets in (X, τ).

**Example 3.2:**

i) Let X = {a, b, c d}, τ = **{**ϕ, X, {a}, {b}, {a, b}, {a, b, c}**}.** Then
Closed sets in (X, τ) are X, ϕ, {d},{c, d},{a, c},{a, c, d}, {b, c, d}
R#-Closed sets in (X, τ) are X, ϕ, {d},{a, c}, {a, d},{b, c}, {b, d},{c, d},{a, b, c}{a, b, d},{a, c, d},{b, c, d}
R#-Open sets in (X, τ) are X, ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}{b, c}, {b, d}{a, b, c}

ii) Let X = {a, b, c, d}, τ = **{**ϕ, X, {a}, {a, b},{a, b, c}**}.** Then
Closed sets in (X, τ) are X, ϕ, {d},{c, d}, {b, c, d}
R#-Closed sets in (X, τ) are X, ϕ, {d}, {a, d}, {b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
R#-Open sets in (X, τ) are X, ϕ, {a}, {b}, {c}, {a, b}, {a, c},{b, c},{a, b, c}

**Theorem 3.3:** Every g-closed set in X is R#-closed set but not conversely.

**Proof:** Let A be a g-closed set in topological space X. Let U be any R*-open set in X such that A ⊂ U. Since A is g-closed, we have gcl (A) = A ⊆ U. Therefore gcl(A)=A⊆U .Hence A is R#-closed set in X.

**Example 3.4:** Let X= {a, b, c, d} and $\tau$={X, ∅, {a},{b},{a, b},{a, b, c}} then the set A={a, c} is R#-closed set but not g-closed in X.

**Corollary 3.5:**

i. Every closed set is R#-closed set in X.
ii. Every regular closed sets is R#-closed set in X.
iii. Every w-closed set in R#-closed set in X.
iv. Every $\hat{g}$-closed set is R#-closed set in X

**Proof:**

i. Every closed set is g-closed [58] and follows from theorem 3.3.
ii. Every regular closed set is closed, from stone [57] and then follows corollary 3.5.i).
iii. Every w-closed set is g-closed [44] follows from theorem 3.3.
iv. Every $\hat{g}$-closed set is g-closed [51] follows from theorem 3.3.

**Remark 3.6:** The converse of the above Corollary 3.5 need not be true as seen from the following example.

**Example 3.7:** Let X = {a, b, c, d} $\tau$={X, ∅, {a},{b},{a, b},{a, b, c}}, A={a, b, d} is R#-closed set but not a closed (respectively r-closed, w-closed, $\hat{g}$-closed) in X.

**Theorem 3.8:** Every R#-closed is rg-closed set in X but not conversely.

**Proof:** Let A be a R#-closed set in X. Let U be any open set in X such that A⊆ U. Since every open set is R* open set and A is R#-closed set, we have gcl(A)⊆U. Thus gcl(A)⊆U, U is open in X. Therefore A is rg-closed in X.

**Example 3.9:** Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b},{a, b}, {a, b, c}}. Then the set A={c} is rg-closed set but not R#-closed set in X.

**Corollary 3.10:** For a topological space (X, τ) the following are hold

i) Every R#-closed set is rgb-closed set
ii) Every R#-closed set is wgr$\alpha$-closed set
iii) Every R#-closed set is gpr-closed, gspr-closed, and rg$\beta$-closed
iv) Every R#-closed set is r^g-closed and rwg-closed

**Proof:**

i) Let A be a R#-closed set in X. Let U be any regular open in X such that A⊆U. Since every regular open set is R*-open set in X and A is R#-closed set in X it follows that gcl(A)⊆U. Therefore gcl(A)⊆U, U is open in X. Hence A is rgb-closed set in X.
ii) Let A be a R#-closed set in X. Let U be any r$\alpha$-open in X such that A⊆U. Since every r$\alpha$- open set is R*-open set in X and A is R#-closed set in X it follows that gcl(A)⊆U. Therefore gcl(A)⊆U, U is regular open in X. Hence A is wgr$\alpha$ -closed set in X.

iii) Every rg closed and gpr-closed is gspr closed [35], and also gpr-closed is rg$\beta$-closed [43] and follows from Theorem 3.8.
iv) Every rg is r^g-closed [43], rwg –closed [30] and follows from Theorem 3.9(iii)

**Remark 3.11:** The following examples are shows that the $R^{\#}$-closed sets are independent with some existing closed sets in topological spaces p-closed [32], s-closed [23], α-closed sets [59], semi pre-closed sets [5], b-closed sets [4], rs-closed sets [11], gs-closed sets [1], gα-closed sets [25], αg-closed sets [24], gsp-closed sets [14], gp-closed sets [26], g*-closed [49], swg-closed sets [35], wg-closed sets [31], rgα-closed sets [47], g*p-closed sets [50], wα-closed sets [8], gwα-closed sets [8], R*[19], rgw-closed sets [28], pgpr-closed sets [6], rps-closed [45] sets, gprw-closed sets [29], αrw-closed sets [55], $\alpha$gp-closed sets [36], $\beta wg^{*}$-closed sets [13],$**_{g\alpha}$-closed sets [54], gαb-closed sets [56], sgb-closed sets [18], rg*b-closed sets [17], ps-closed sets [48], αgs-closed sets [41], g#s-closed sets [53], α**g-closed sets [25],g**-closed sets [16], gb-closed sets [2], swg*-closed sets [31], gr-closed sets [9], rps-closed sets [45], $\beta wg\ **$-closed sets [46], g#α-closed sets [37], g*s-closed sets [14], #gα-closed sets [12], **s**#gα-closed sets [22], g*p-closed sets [50], gps-closed sets [42], g#p#-closed sets [3].

**Example 3.12:** Let X = {a, b, c d}, τ = **{**ϕ, X, {a}, {b},{a, b} {a, b, c}**}.** Then

i. α-closed sets in (X,τ)are X,ϕ,{c},{d},{c, d}, {a, c, d},{b, c, d}
ii. rs- closed sets in (X, $\tau$) are X,ϕ,{a},{b},{a, c},{a, d},{b, c},{b, d},{a, c, d},{b, c, d}
iii. gs- closed sets in (X, $\tau$) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
iv. gα- closed sets in (X,$\tau$) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
v. αg- closed sets in (X,$\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
vi. gsp- closed sets in (X,$\tau$) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
vii. gp- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
viii. g*- closed sets in (X, $\tau$) are X, ϕ,{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
ix. wg- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
x. rgα- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}
xi. g*p- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xii. wα- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xiii. gwα- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xiv. R* closed sets in (X, $\tau$) are X,ϕ{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}
xv. rgw- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xvi. pgpr- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xvii. rps- closed sets in (X, $\tau$) are X,ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}
xviii. gprw- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, b},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xix. αrw- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xx. αgp- closed sets in (X, $\tau$) are X,ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxi. ps- closed sets in (X, $\tau$) are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c}, {a, b, d}, {a, c, d},{b, c, d}
xxii. p- closed sets in (X, $\tau$) are X, ϕ, {c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d}, {b, c, d}
xxiii. αgs- closed sets in (X, $\tau$) are X, ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxiv. g#s- closed sets in (X,$\tau$) are X, ϕ, {a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, c, d}, {b, c, d}}
xxv. α**g- closed sets in (X, $\tau$) are X, ϕ,{a, b},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d}, {b, c, d}
xxvi. gb- closed sets in (X,$\tau$) are X, ϕ,{a},{b},{c},{d},{a, c},{a, d},{b, c},{b, d},{c, d},{a, b, d}, {a, c, d}, {b, c, d}}
xxvii. $\beta$wg**-closed set in (X,$\tau$) are X, ϕ,{c},{d},{a, b},{a, c},{a, d},{b, c},{b, d}, {c, d},{a, b, d},{a, c, d}, {b, c, d}{a, b, c}
xxviii. g#α- closed sets in (X,$\tau$) are X, ϕ,{c},{d},{c, d},{a, c, d},{b, c, d}
xxix. og$^{\#}$-p- closed sets in (X,$\tau$) are X, ϕ,{c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d},{b, c, d}
xxx. g*p- closed sets in (X,$\tau$) are X, ϕ, {c},{d},{a, d},{b, d},{c, d},{a, b, d},{a, c, d}, {b , c, d}
xxxi. g#p#- closed sets in (X, $\tau$) are X, ϕ,{c}, {d}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d},{a, c, d}, {b, c, d}{a, b, c},{a, b, d}}

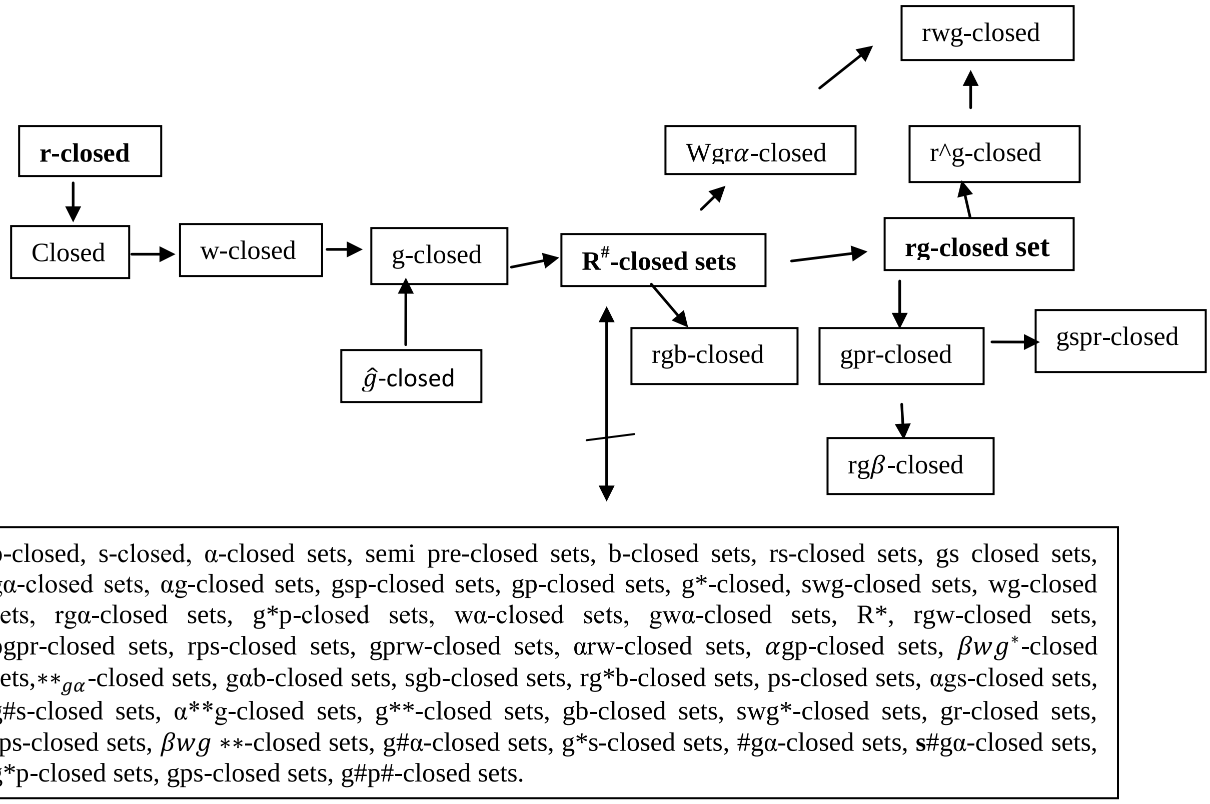
**Conclusion:** From the above example we proved that the following closed sets are independent to $R^{\#}$-closed sets: $\alpha$-closed set, rs- closed set, gs- closed set, g$\alpha$- closed set,$\alpha$g- closed set, gsp- closed set, gp- closed set, g*- closed set, wg-closed set, rg$\alpha$- closed set, g*p- closed set, w$\alpha$- closed set, gw_- closed set, R*closed set, rgw- closed set, pgpr-closed set, rps- closed set, gprw- closed set,$\alpha$rwclosed set, gp- closed set, ps- closed set, p- closed set, gs- closed set, $g^{\#}$s- closed set, $\alpha$**g-closed set, gb- closed set, $\beta$wg**-closed set, $g^{\#}\alpha$- closed set, og#p- closed set, g*p- closed set, $g^{\#}p^{\#}$- closed set.

**Example 3.13:** Let X = {a, b, c, d}, τ = **{**ϕ, X, {a}, {a, b} {a, b, c}**}.** Then

i. closed sets in (X,$\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
ii. sp- closed sets in (X,$\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
iii. b- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
iv. swg- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
v. gw$\alpha$- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
vi. rgw- closed sets in (X, $\tau$) are X, ϕ, {c}, {d}, {a, b}, {c, d}, {a, b, c}, {a, c, d}{a, b, d} {b, c, d}
vii. pgpr- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
viii. rps- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
ix. $\beta wg *$- closed sets in (X, $\tau$) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {b, c, d}
x. $** g\alpha$- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xi. $g\alpha b$- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xii. $sgb$- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xiii. rg*b- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xiv. gb- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}
xv. swg*- closed sets in (X, $\tau$) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}{a, c, d} {a, b, d}, {b, c, d}
xvi. gr- closed sets in (X, $\tau$) are X, ϕ, {d}, {a, d}, {b, d}, {c, d} {a, c, d} {a, b, d}, {b, c, d}
xvii. rps- closed sets in (X, $\tau$) are X, ϕ, {d}, {a, d}, {b, d}, {c, d}{a, c, d}{a, b, d}, {b, c, d}}
xviii. g#$\alpha$- closed sets in (X, $\tau$) are X, ϕ,
xix. {b}, {c}, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {a, b, d}, {b, c, d}
xx. g*s- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}
xxi. #g$\alpha$- closed sets in (X, $\tau$) are X, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {b, c, d}

**Conclusion:** From the above example we proved that the following closed sets are independent to $R^{\#}$-closed sets: s-closed set, sp- closed set, b- closed set, swg-closed set, gw$\alpha$-closed set,rgw- closed set, pgpr- closed set, rps- closed set, $\beta$wg*-closed set,**g$\alpha$- closed set, g$\alpha$b- closed set, sgb-closed set, rg*b- closed set, gb-closed set, swg*- closed set, gr- closed set, rps- closed set, g#$\alpha$ - closed set, g*s-closed set, #g$\alpha$- closed set.

**Remark 3.14:** From the above results discussion and known results we have the following implications




**Notations:**
A $\longrightarrow$ B means the set A implies the set B but not conversely
A $\not\leftrightarrow$ B means the set A and the set B are independent of each other.

**Theorem 3.15:** The union of any two $R^{\#}$-closed sets of X is $R^{\#}$-closed s

**Proof:** Let A and B are the $R^{\#}$-closed sets in topological space (X, $\tau$). Let U be R*-open set in X such that A∪B⊆U, then A⊆U and B⊆U, since A and B are the $R^{\#}$-closed sets, gcl(A)⊆U, gcl(B)⊆U, and we know that gcl(A)∪ gcl(B)= gcl(A∪B)⊆U. Therefore A∪B is $R^{\#}$-closed set in X.

**Remark 3.16:** The intersection of two $R^{\#}$-closed sets of topological space (X, $\tau$) is generally not a $R^{\#}$-closed set in X.

**Example 3.17:** Let X = {a, b, c, d} and $\tau$={X, $\phi$, {a}, {b}, {a, b}, {a, b, c}} then the set A={a, c} and B={a, d} are $R^{\#}$-closed sets in X, but A∩B={a} is not $R^{\#}$-closed set in X.

**Theorem 3.18:** If a subset A of topological space (X, $\tau$) is a $R^{\#}$-closed set in X then gcl(A)-A. does not contain any non empty R*-closed set in X.

**Proof:** Let A be a $R^{\#}$-closed set in X and suppose F be a non empty R*-closed subset of gcl(A)-A F⊆gcl(A)-A⟹ F⊆gcl(A)⋂(X-A) ⟹ F⊆gcl(A) —— (1) & F⊆X-A
⟹ A⊆X-F and X-F is R*-open set and A is an $R^{\#}$-cl~~osed set~~, gcl(A) ⊆X-F
⟹ F⊆X-gcl(A) —— (2) from equations (1) and (2) we gwt F⊆gcl(A)∩(X-gcl(A))=$\phi$
⟹ F=Φ thus gcl(A)-A does not contain any non empty R*-closed set in X.

**Remark 3.19:** The converse of the above Theorem 3.17 need not be true as seen from the following example 3.20.

**Example 3.20:** Let X = {a, b, c, d} τ = **{** X, ϕ, {a}, {a, b}, {a, b, c}**}** then the set A={a}, gcl{a} = {ac}, gcl{A}-A={a}-{a, c}={c} does not contain any non empty $R^{*}$ closed set in X but A is not $R^{\#}$-closed set.

**Theorem 3.21:** If A is an $R^{\#}$-closed set in (X, τ) and A⊆B⊆gcl(A) then B is also $R^{\#}$-closed set in X.

**Proof:** If it is given that A is $R^{\#}$-closed set in X then we have to prove that B is also $R^{\#}$-closed set in X. Let U be an R*-open set of X such that B⊆U.Since A⊆B and A is $R^{\#}$-closed set, gcl(A) ⊆U & A⊆U Now B⊆gcl(A) ⇒gcl(B) ⊆gcl(gcl(A))=gcl(A) ⊆U. Therefore gcl(B) ⊆U Hence B is $R^{\#}$-closed set in X.

**Remark 3.22:** The converse of the above theorem 3.20 need not to be true as seen from the following example 3.23

**Example 3.23:** Let X={a, b, c} and $\tau$={x, $\phi$, {a}, {b}, {a, b}, {a, b, c}} then the set A={c, d} B={a, c, d} such that A and B are $R^{\#}$-closed sets in X, but A⊆B and B is not a subset of gcl(A) ,because gcl(A) ={c, d}.

**Theorem 3.24:** Let A be a $R^{\#}$-closed in X. Then A is g-closed if and only if gcl(A)-A is R*-closed.

**Proof: Necessity:** suppose A be a g-closed set in X then gcl(A)=A that is gcl(A)-A=$\phi$ which is R*-closed.

**Sufficiency:** Suppose A is $R^{\#}$-closed in X and gcl(A)-A is R*-closed from theorem 3.18 then gcl(A)-A=$\phi$ ⇒gcl(A) =A Therefore A is g-closed.

**Theorem 3.25:** Let$A \subseteq Y \subseteq X$, (X, τ) and (Y, σ) are topological spaces. If A is a $R^{\#}$-closed set in (X,τ). Then A is $R^{\#}$-closed relative to (Y, σ).

**Proof:** Let $A \subseteq Y \bigcap G$ , where G is R*-open since A is $R^{\#}$-closed set in X .Then A⊆X and gcl(A)⊆G this implies that Y∩gcl(A)⊆Y∩G where Y∩gcl(A) is g-closed set of A in Y. Thus A is $R^{\#}$-closed set relative to Y.

**Theorem 3.26:**

i) If A is regular open and rg-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
ii) If A is g-open and rg-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
iii) If A is a regular-open and rwg-closed set in (X, τ) then A is $R^{\#}$-closed in (X, τ).
iv) If A is a regular-open and gpr-closed set in (X, τ) then A is $R^{\#}$-closed in (X, τ).
v) If A is regular open and r^g-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).
vi) If A is regular open and $\beta wg^{**}$-closed set in (X, τ) then A is $R^{\#}$-closed set in (X, τ).

**Proof:**

i. Let A be a regular open and rg-closed set in X. Let U be any R*-open set in X such that A⊆U. Since A is regular open and rg-closed in X by definition gcl(A)⊆(A)⊆U. Hence A is $R^{\#}$-closed set in X.
ii. Let A be a g-open and rg-closed set in X. Let U be any R*-open in X such that A⊆U since A is g-open and rg-closed in X by definition gcl(A)⊆A then gcl(A)⊆A⊆U. Hence A is $R^{\#}$-closed set in X.

iii. Let A be a regular open and rwg-closed set in X. Let U be any R*-open set in X such that A ⊆ U since A is regular open and rwg-closed in X, by definition $cl(\text{int}(A)) \subseteq A$ we know that
$cl(\text{int}(A)) \subseteq cl(\text{int}(cl(A))) \subseteq A \subseteq U \quad Then \quad gcl(A) \subseteq U$. Hence A is $R^{\#}$-closed set in X.

iv. Let A be a regular open and gpr-closed set in X. Let U be any R*-open set in X such that A⊆U.Since A is regular open and gpr-closed in X, by definition pcl(A))⊆A then we know that pcl(A)⊆gcl(A)⊆A hence gcl(A)⊆U. Hence A is $R^{\#}$-closed set in X.

v. Let A be a regular open and r$\hat{g}$-closed set in X. Let U be any R*-open set in X such that A⊆U.Since A is regular open and r^g-closed in X by definition gcl(A)⊆U then A is $R^{\#}$-closed set in X.

vi. Let A is regular open and $\beta wg^{**}$-closed set in X. Let U be any R*-open set in X such that A⊆U since A is regular open and $\beta wg^{**}$-closed in X by definition gcl(A)⊆U then gcl(A)⊆A⊆U. Hence A is $R^{\#}$-closed set in X.

**Remark 3.27:** If A is both semi-open and $R^{\#}$-closed set in (X, τ), then A need not be $\hat{g}$-closed in general as seen from the following example 3.28.

**Example 3.28:** Consider X={a, b, c, d} $\tau$={x, $\phi$, {a}, {a, b}, {a, b, c}}, A={a, c, d} is both semi-open and $R^{\#}$-closed but it is not $\hat{g}$- closed set.

**Theorem 3.29:** If A is both open and g-closed set in X then A is $R^{\#}$ closed set in X.

**Proof:** Let A be open and g-closed in X. Let U be any R*-open set in X such that A⊆U, by definition cl(A))⊆A ⊆U and gcl(A)=A .This implies that cl(A)⊆gcl(A)⊆A⊆U. Hence gcl(A)⊆U. Therefore A is $R^{\#}$-closed set in X.

**Theorem 3.30:** If a subset A of a topological space X is both regular open and $R^{\#}$-closed set in X then it is g-closed set in X.

**Proof:** Suppose a subset A of a topological space X is regular open and $R^{\#}$-closed as every regular open is R*-open now A⊆A then definition of $R^{\#}$-closed , gcl(A)⊆A and also A⊆gcl(A) then gcl(A)=A. Hence A is g-closed in X.

## REFERENCES


1. S.P. Arya & T.M. Nour, characterizations of s-normal spaces, Indian J.pure Application. Math 21 (1990), 717-719.
2. Ahmad Al-Omari and, Mohd. Salmi and Md. Noorani, On Generalized b-Closed Sets, Bull. Malays.Math. Sci. Soc. (2) 32(1) (2009), 19–30
3. K. Alli, A. Subramanian and S. Pious Missier, $g^{\#}p^{\#}$-closed sets in topological spaces, International Journal of Mathematics and Soft Computing Vol.3, No.3. (2013), 55- 60.
4. D. Andrijevi´c, On b-open sets, Mat. Vesnik 48 (1996), no. 1–2, 59–64.
5. D.Andrijevic, Semi-preopen sets, Mat. Vesnik, 38(1) (1986), 24-32.
6. M.Anitha and P.Thangavelu, On Pre-Generalized pre-regular closed sets, Acta Ciencia Indica, 31M (4) (2005),1035 – 1040
7. S.S. Benchalli and R.S. Wali, On RW-closed sets in topological spaces, Malays. Math. sci. Soc. 30 (2007), 99-110
8. S. S. Benchalli, P. G. Patil and T. D. Rayanagaudar, ωα-Closed sets is Topological Spaces, The Global. J. Appl. Math. and Math. Sci 2, 53-63
9. Sharmistha Bhattacharya, On Generalized Regular Closed Sets, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 3, 145 – 152
10. P.Bhattacharyya and B.K.Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29 (1987),376-382
11. D.E.Cameron**,** Properties of s-closed spaces Proc. Amer. Math. Soc., 72 (1978), 581-586.
12. R. Devi, H. Maki, V. Kokilavani, The group structure of #gα-closed sets in topological spaces, International Journal of General Topology, 2 (2009), 21-30.
13. C. Dhanapakyam, J. Subashini and K. Indirani, ON - βwg* set and continuity in topological spaces, IJMA-5(5), 2014, 222-233.
14. J.Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser.A. Math., 16(1995), 35-48.
15. Y.Gnanambal, On generalized pre-regular closed sets in topological spaces, Indian J. Pure Appl.Math., 28 (1997), 351-360
16. Pauline Mary Helen. M, Veronica Vijayan, Ponnuthai Selvarani, g**-closed sets in topological spaces, IJMA 3(5), (2012), 1-15
17. K Indirani and G Sindhu, On Regular Generalized Star B closed sets, IJMA. International Journal of general topology, 2(1)(2009), 21- 30
18. D.Iyappan and N.Nagaveni, On semi generalized b-closed set, Int. Journal of Math. Analysis, Vol.6, 2012, no. 26, 1251 – 1264.

19. C. Janaki and Renu Thomas, On R* closed sets in topological space s, IJMA-3(8), 2012, 3067-3074.
20. C.Janaki, Studies on лgα-closed sets in Topology, Asian Journal of Current Engg and Maths Volume 1, (5), (2012), 241-246.
21. A. Jayalakshmi & C. Janaki, On wgrα-Closed Sets in Topological Spaces, International Journal of Mathematical Archive-3(6), 2012, 2386-2392.
22. V. Kokilavani and M. Vivek Prabu, Semi # Generalized α Closed Sets and Semi #Generalized α-Homeomorp - hism in Topological Spaces, Proceedings of National Conference on Recent Advances in Mathematical Analysis and Applications – NCRAMAA-2013, 153-161.
23. N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
24. H.Maki, R.Devi and K.Balachandran, Associated topologies of generalized α-closed sets and α generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 15 (1994), 51-63.
25. H.Maki, R.Devi and K.Balachandran, generalized α-closed sets in topology, Bull.Fukuoka Univ. Ed. Part III, 42(1993), 13- 21.
26. H.Maki, J.umehra and T Noiri, Every topological space is pre-T1/2 mem.Fac.Sci.Kochi univ.Ser.A.math, 17 (1996) 33-42.
27. K. Mariappa, S. Sekar, On Regular Generalized b Closed Set, Int. Journal of Math. Analysis, Vol. 7, 2013, no. 13, 613 – 624. Math. Soc., 41 (1937), 374-481.Mathematical Analysis and Applications NCRAMAA-(2013), 153-161
28. Sanjay Mishra, Nitin Bhardwaj and Varun Joshi, On Regular Generalized weakly (rgw)- Closed Sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 6, 2012, no. 39, 1939 – 1952
29. Sanjay Mishra, Nitin Bhardwaj and Varun Joshi, On Generalized Pre Regular Weakly (gprw)-Closed Sets in Topological Spaces, International Mathematical Forum, Vol. 7, 2012, no. 40,1981-1992
30. P.Rajeswari and L.Senthil kumar, Regular weakely generalized closed sets, Int.J.Computer Applications (0995-8887), Vol 43-no 18, April 2012.
31. C. Mukundhan and N. Nagaveni, Weaker Form of a Generalized Closed Set, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 20, 949 - 961
32. A.S. Mashour M.E Abd EI-Mansef and S.N.EI-Deeb, on pre-continuous and weak pre-continuous mappings, proc-math. soc.Egypt 53(1982), 47-53.
33. A. Narmadha, N. Nagaveni and T. Noiri, On regular b-Open Sets in Topological Spaces Int. Journal of Math. Analysis, Vol. 7, 2013, no. 19, 937 – 948.
34. N.Nagaveni studies on generalization of homeomorphism in topological spaces PhD, thesis Bharthier University, Coimbatore (1999).
35. G. B. Navalagi and Chandrashekarappa, On gspr closed sets in topological spaces, Vol. 2, Nos 1-2, (2010), pp. 51-58.
36. G.B. Navalagi, S.V. Gurushantanavar, and Chandrashekarappa A.S, on α generalized preclosed sets in topological spaces, International Journal of Mathematics and Computing Applications, Vol. 2, Nos 1-2, January-December 2010, pp. 35-44.
37. K.Nono, R.Devi, M.Devipriya, K. Muthukumaraswamy and H. Maki, On g #α-closed sets and the Digital Plane, Bull. Fukuoka Univ. Ed. Part III, 53(2004), 15-24.
38. N.Palaniappan and K.C.Rao, regular generalized closed sets, Kyungpook, Math. J., 33 (1993), 211-219.
39. Y.Palaniappan, On Regular Generalized β-Closed Sets, International Journal of Scientific & Engineering Research, Volume 4, Issue 4, April-2013 1410.
40. J.H. Park, J.K.Park mildly generalized closed sets, almost normal and mildly normal spaces, chaos, solutions and Fractals 20(2004), 1103-1111.
41. Rajamani M., Viswanathan K, On αgs-closed sets in topological spaces, Acta Ciencia Indica, Vol. 1, 2005, pp, 293-303.
42. Sampoornam, Gnanambal Ilango and N. Selvanayaki, Generalized Pre-Semi Closed Sets in Topological Spaces, International Journal of Mathematical Archive-4(6), 2013, 151-154
43. Savithiri D and Janaki C, .On regular^ generalized Closed Sets in Topological Spaces, International Journal of Mathematical Archive-4(4), 2013, 162-169.
44. P.Sunadaram and M.Sheik John, On w-closed sets in Topology, Acta Ciencia Indica, 4 (2000), 389-392.
45. T. Shyla Isac Mary and P. Thangavelu, On Regular Pre-Semiclosed sets in topological spaces, International Journal of General Topology Vol. 4, Nos. 1-2 January-December 2011, pp. 17–26.
46. Subashini jesu rajan, On βwg** set and Continuity in Topological Spaces, Research Gate, Article July 2014. The Global. J. Appl. Math. and Math. Sci. 2, 2009, 53-63.
47. A. Vadivel and K. Vairamanickam, rgα-Closed Sets and rgα-Open Sets in Topological Spaces, Int. Journal of Math. Analysis, Vol. 3, 2009, no. 37, 1803 – 1819.
48. M.K.R.S. Veera kumar, g#-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser.A. Math 24 (2003), 1-13.
49. M.K.R.S. Veera kumar, between closed and g-closed sets.Mem.Fac.Sci.Kochi univ, Math.21 (2000) 1-19.
50. M.K.R.S. Veera Kumar, Pre semi closed sets, Indian, J. Math.44(2) 2002, 165-181.
51. M.K.R.S. Veera Kumar, On g^ closed sets in topological spaces, Bulletin Allahabad Math. Soc.18 (2003), 99-112.

52. M. K. R. S. Veerakumar, g*-preclosed sets Acta Ciencia Indica (Mathematics) Meerut, Vol. 1, 2002, 51 – 60.
53. M.K.R.S Veerakumar, g# semi-closed sets in topological spaces. International Journal of Scientific & Research Publications, Vol 2,Issue 6, June 2012. ISSN 2250-3153.
54. N.V.Velicko, H-closed topological spaces. Amer. Math. Soc. Transl. 78 (1968), 103-118.
55. M. Vigneshwaran and A. Singaravelan, Applications of **gα-closed sets in Topologica Spaces, IJMA-5(10), 2014, 139-150
56. L.Vinayagamoorthi, N.Nagaveni, On generalized-αb closed sets, Proceeding ICMD Allahabad, Puspha Publication Vol.1. 2010-11.Vol.3, No.3, (2013), 55-60
57. M. Stone, Application of the theory of Boolean rings to general topology, Trans.Amer.Math.Soc., 41(1937), 374-481.
58. N.levine, Generalized closed sets in Topology, Rend.Circ.Mat.Palermo, 19(2) (1970), 89-96.
59. O.Njastad on some classes of nearly open sets,paific J.math 15(1965), 961-970.

**Source of support: Nil, Conflict of interest: None Declared.**