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EVEN VERTEX ODD MEAN LABELING OF H-GRAPH

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ABSTRACT

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function f:V(G)→{0, 2, 4, ... 2q-2,2q} such that the induced map f: E(G)→ {1, 3, 5, ... 2q-1} defined by* $f^*(uv)=\frac{f(u)+f(v)}{2}$ *is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we study the even vertex odd mean behaviour of H-graph.*

Keywords: *Even vertex odd mean labeling, even vertex odd mean graph.*

AMS subject classification (2010): *05C78.*

1. INTRODUCTION

Throughout this paper we restrict our attention to finite, simple and undirected graphs. The set of vertices and the set of edges of a graph G will be denoted by V(G) and E(G) respectively and let $p = |V(G)|$, $q = |E(G)|$.For general graph theorectic notations we follow F.Harary[6].A graph labeling is a mapping that carries a set of elements(usually vertices and /or edges) into a set of numbers .Many kinds of labeling have been studied an excellent survey of graph labeling can be found in[2].Most of the graph labeling techniques found their origin with graceful labeling which was introduced by Rosa.A(1967) . Let G(V, E) be a graph with p vertices and q edges.

The concept of mean labeling was introduced and studied by Somasundaram and Ponraj [9]. Further some more results on mean graphs are discussed in [4, 5]. A graph G is said to be a mean graph if there exists an injective function f:V(G) → {0, 1, 2, …q} such that the induced map
f*:E(G)→{1, 2, …q} defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

Manickam and Marudai [8] have introduced the concept of odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective map f:V(G)→{0, 1, …2q–1} defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gopi [3]. A graph G is said to be even mean if there exists an injective function f: V(G)→{0,1,…2q} such that the induced map f*:E(G)→{2,4,…2q} defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

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A graph G is said to have an even vertex odd mean labeling if there exists an injective function $f:V(G)\to\{0, 2, \ldots 2q-2, 2q\}$ such that the induced map $f^*:E(G)\to\{1, 3, \ldots 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called even vertex odd mean graph [1, 10].

In this paper, we proved that the even vertex odd meanness of H-graphs.

2. MAIN RESULTS

Definition 2.1: The H-graph of a path P_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots v_n$ and $u_1, u_2, \ldots u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ by an edge if n is odd and the vertices $V_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even.

Theorem 2.2: The H-graph of a path P_n in $(n \ge 3)$ is a even vertex odd mean graph.

Proof: Let $\{v_i, 1\le i \le n, u_i, 1\le i \le n\}$ be the vertices and $\{e, e_i, 1 \le i \le n-1, e'_i, 1\le i \le n-1\}$ be the edges which are denoted as in figure 1.1

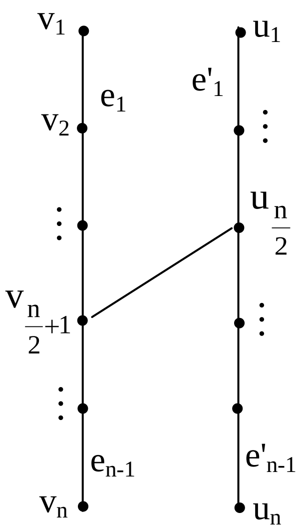

Figure-1.1: Ordinary labeling of H-graph of path P_n

First we label the vertices as follows

Define $f : v\to\{0, 2, \ldots 2q\}$ by

For $1 \le i \le n$

$f(v_i) = 2(i-1)$

$f(u_i) = 2n + 2(i-1)$

Then the induced edge labels are :

for $1 \le i \le n-1$

$f^*(e_i) = 2i-1$

$f^*(e'_i) = 2n+2i-1$

$f^*(e) = 2n-1$

Therefore $f^*(E) = \{1, 3, 5, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the H-graph of a path P_n $(n\ge3)$ is a even vertex odd mean graph.

H-graph of P_6 is shown in figure 1.2

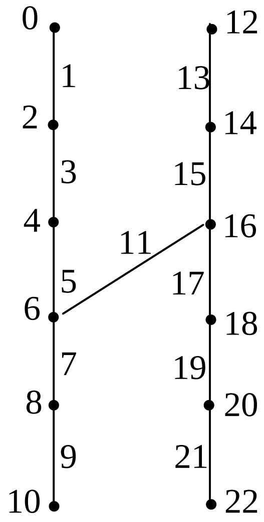

Figure-1.2: Even Vertex Odd Mean Labelling of H graph of a path P_6

Definition 2.3: The graph $H\odot mk_1$ is a graph obtained from the H-graph by attaching i pendant vertices at each i^{th} vertex on the two paths on n vertices for $1 \leq i \leq n$.

Theorem 2.4: The $H\odot mk_1$ is a even vertex odd sum graph.

Proof: Let $\{v_i, u_i, 1 \leq i \leq n$ and $v_{ij}, u_{ij}, 1 \leq i \leq n, 1 \leq j \leq m\}$ be the vertices and $\{a_i, b_i, 1 \leq i \leq n\text{–}1$ and $a_{ij}, b_{ij}, 1 \leq i \leq n, 1 \leq j \leq m, b\}$ be the edges which are denoted as in figure 1.3

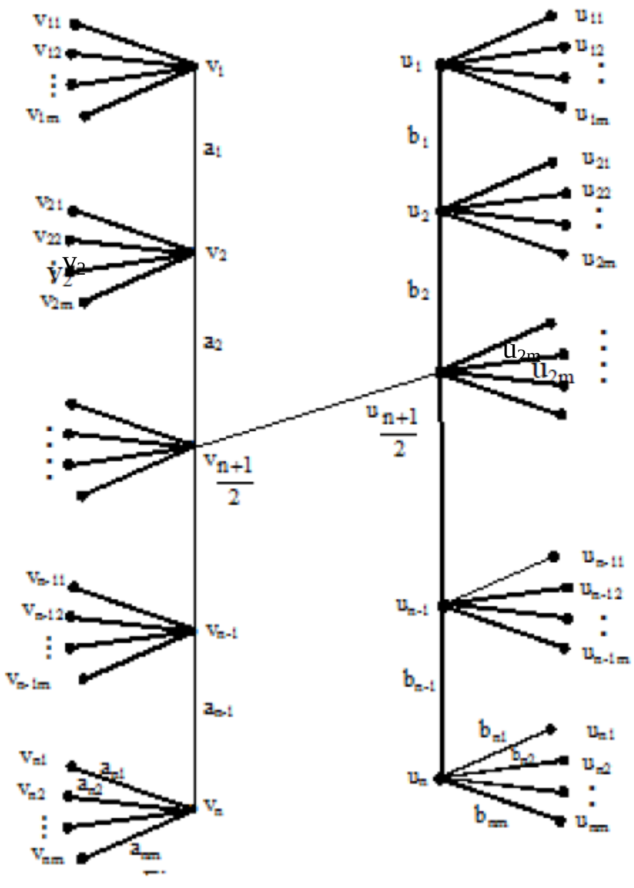

Figure 1.3: Ordinary labeling of $H\odot mk_1$

Case-(i): n is odd

First we label the vertices as follows:

Define $f: V \rightarrow \{0, 2, \ldots. 2q\}$

For $1 \le i \le n$

$$f(v_i) = \begin{cases} 2(i-1)(m+1) & i \text{ is odd} \\ 4m+2+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} 2(n-1)(m+1)+4m+2+2(i-1)(m+1) & i \text{ is odd} \\ 2(n-1)(m+1)+4(m+1)+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

$1 \le i \le n$, $1 \le j \le m$

$$f(v_{ij}) = \begin{cases} 4j-2+2(i-1)(m+1) & i \text{ is odd} \\ 4j+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

$$f(u_{ij}) = \begin{cases} 2(n-1)(m+1)+4j+2(i-1)(m+1) & i \text{ is odd} \\ 2(n-1)(m+1)+4m+4j+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

Case-(ii): n is even

For $1 \le i \le n$

$$f(v_i) = \begin{cases} 2(i-1)(m+1) & i \text{ is odd} \\ 4m+2+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

$$f(u_i) = \begin{cases} 4m+2(n-2)(m+1)+4+2(m+1)(i-1) & i \text{ is odd} \\ 8m+2(n-2)(m+1)+6+2(m+1)(i-2) & i \text{ is even} \end{cases}$$

For $1 \le i \le n$, $1 \le j \le m$

$$f(v_{ij}) = \begin{cases} 4j-2+2(i-1)(m+1) & i \text{ is odd} \\ 4j+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

$$f(u_{ij}) = \begin{cases} 4m+2+2(n-2)(m+1)+4j+2(i-1)(m+1) & i \text{ is odd} \\ 4m+2(n-2)(m+1)+4j+4+2(i-2)(m+1) & i \text{ is even} \end{cases}$$

Then the induced edge labels are:

For $1 \le i \le n-1$
$f^*(a_i) = 2m+1+2(i-1)(m+1)$
$f^*(b_i) = 4m+3+2(m+1)(n-1)+2(i-1)(m+1)$

For $1 \le i \le n$, $1 \le j \le m$
$f^*(a_{ij}) = 2j-1+2(m+1)(i-1)$
$f^*(b_{ij}) = 2m+1+2(m+1)(n-1)+2j+2(i-1)(m+1)$
$f^*(b) = 2m+1+2(m+1)(n-1)$

Therefore, $f^*(E) = \{1, 3, \ldots 2q-1\}$. So, f is a even vertex odd mean labeling and hence, the $H \odot mk_1$ is a even vertex odd mean graph $H \odot 3k_1$ and $H \odot 4k_1$ is shows in figures 1.4 and 1.5 respectively

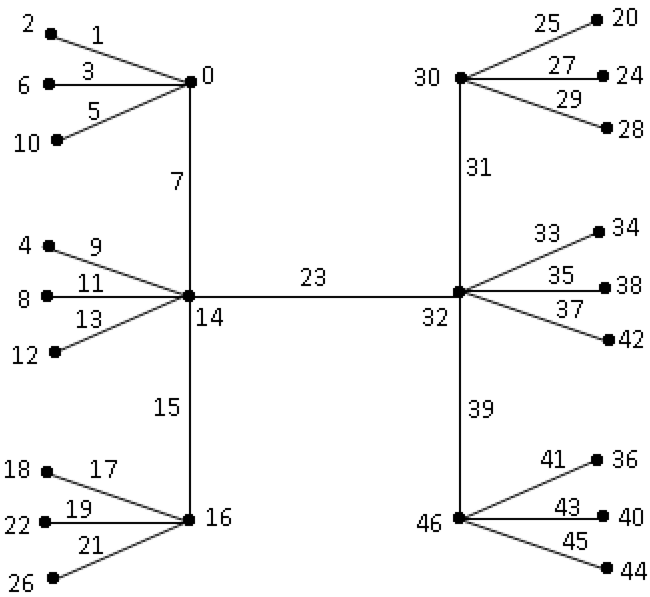

Figure-1.4: H⊙ $3k_1$

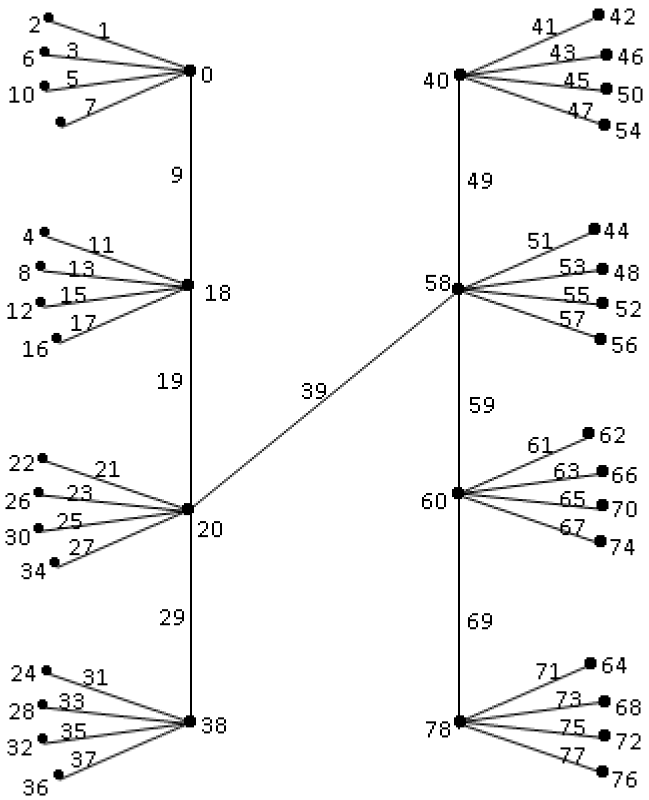

Figure-1.5: H⊙ $4k_1$

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