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ON MINIMAL AND MAXIMAL $\psi g^{\#}$-CLOSED SETS

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ABSTRACT

In this paper, we introduce and study a new class of closed sets called minimal $\psi g^{\#}$-closed, maximal $\psi g^{\#}$-closed sets and their properties. Also we introduce new types of continuous functions called minimal $\psi g^{\#}$-continuous and maximal $\psi g^{\#}$-continuous functions. Finally, we study some properties of these types of continuous functions.

Keywords: *minimal $\psi g^{\#}$-closed, maximal $\psi g^{\#}$-closed, minimal $\psi g^{\#}$-continuous, maximal $\psi g^{\#}$- continuous, minimal $\psi g^{\#}$-irresolute, maximal $\psi g^{\#}$-irresolute.*

1. INTRODUCTION

In 2006, F.Nakaoka and N. Oda introduced the concepts of minimal and maximal closed sets, which plays the significant role in general topology. In 2015, N.Sowmya, M.Elakkiya, N.Balamani introduced the concepts of $\psi g^{\#}$-closed sets in topological spaces. In this paper, we introduce and study a new class of closed sets called minimal $\psi g^{\#}$-closed, maximal $\psi g^{\#}$-closed sets and their properties. Also we introduce a new class of continuous functions called minimal $\psi g^{\#}$-continuous and maximal $\psi g^{\#}$-continuous functions and investigate some of their fundamental properties. Also we define minimal $\psi g^{\#}$-irresolute and maximal $\psi g^{\#}$-irresolute functions in topological spaces.

Throughout this paper, the spaces X, Y and Z always mean topological spaces (X, τ), (Y, σ) and (Z, η) respectively.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X, τ) is called
- (i) Semi-closed [2] if $int(cl(A)) \subseteq A$.
- (ii) α-closed [9] if $cl(int(cl(A))) \subseteq A$.
- (iii) semi-generalized closed [1] (briefly, sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.
- (iv) α-generalized closed [4] (briefly, αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an open set in X.
- (v) ψ-closed [11] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in X.
- (vi) $g^{\#}$-closed [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in X.

Definition 2.2: Let X be a topological space. A nonempty proper closed subset U of X is said to be

- (i) minimal closed [6] if any closed set which is contained in U is ϕ or U.
- (ii) maximal closed [6] if any closed set which is contains U is X or U.

Definition 2.3: Let X and Y be topological spaces. A map $f: X \rightarrow Y$ is called
- (i) minimal continuous if $f^{-1}(A)$ is open in X for every minimal open set A in Y.
- (ii) maximal continuous if $f^{-1}(A)$ is open in X for every maximal open set A in Y.

Definition 2.4: A topological space X is said to be T_{min}(resp. T_{max}) space if every nonempty proper open subset of X is a minimal open set(resp. maximal open).

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Lemma 2.5: Let Y be a T_{min} space. Then f: X → Y is min-continuous if and only if f is max-continuous.

Definition 2.6: A subset A of a topological space (X, τ) is called a $\psi g^{\#}$-closed set if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g^{\#}$-open in X.

A subset A of a topological space (X, τ) is called a $\psi g^{\#}$-open if X \ A is $\psi g^{\#}$-closed in X.

Definition 2.7: A subset A of a topological space X is called
(i) $\psi g^{\#}$-continuous if $f^{-1}(V)$ is $\psi g^{\#}$-closed in X for every closed set V in Y.
(ii) $\psi g^{\#}$-irresolute if $f^{-1}(V)$ is $\psi g^{\#}$-closed in X for every $\psi g^{\#}$-closed set V in Y.

Definition 2.8: A topological space X is called a $T_{\psi g\#}$ space if every $\psi g^{\#}$-closed set is closed in X.

Definition 2.9: For any set A ⊂ X, the $\psi g^{\#}$-closure of A is defined as the intersection of all $\psi g^{\#}$-closed sets contains A and is denoted by $\psi g^{\#}cl(A)$.

For any set A ⊂ X, the $\psi g^{\#}$-interior of A is defined as the union of all $\psi g^{\#}$-open sets contained in A and is denoted by $\psi g^{\#}int(A)$.

Lemma 2.10: For $x \in X$ and a subset A in X, $x \in \psi g^{\#}int(A)$ if and only if $F \cap A \neq \phi$ for every $\psi g^{\#}$-closed set F containing x.

3. MINIMAL AND MAXIMAL $\psi g^{\#}$-CLOSED SETS

Definition 3.1: A nonempty proper $\psi g^{\#}$-closed set A of X is said to be minimal $\psi g^{\#}$-closed if any $\psi g^{\#}$-closed set contained in A is ϕ or A.

Definition 3.2: A nonempty proper $\psi g^{\#}$-closed set A of X is said to be maximal $\psi g^{\#}$-closed if any $\psi g^{\#}$-closed set contains A is X or A.

Example 3.3: Let X = {1, 2, 3} with the topology τ = {ϕ, {1}, {2}, {1, 2}, {1, 3}, X}.

Then $\psi g^{\#}$-closed sets are ϕ, {2}, {3}, {1, 3}, {2, 3}, X.

Here the minimal $\psi g^{\#}$-closed sets are {2}, {3} and the maximal $\psi g^{\#}$-closed sets are {1, 3}, {2, 3}.

Theorem 3.4: Let X be a topological space and $F \subseteq X$. Then F is minimal $\psi g^{\#}$-closed if and only if X \ F is maximal $\psi g^{\#}$-open.

Proof: Let F be a minimal $\psi g^{\#}$-closed set in X. Let V be a $\psi g^{\#}$-open set such that $X \setminus F \subseteq V$. Since X \V is a $\psi g^{\#}$-closed set contained in a minimal $\psi g^{\#}$-closed set F, then X \ V = ϕ or X \ V = F. That is V = X or V = X \ F. Therefore X \ F is a maximal $\psi g^{\#}$-open set.

Conversely, let X \ F be maximal $\psi g^{\#}$-open. Let U be a $\psi g^{\#}$-closed such that $U \subseteq F$. But X \ U is $\psi g^{\#}$-open and X \ F is maximal $\psi g^{\#}$-open, then X \ U = X \ F or X \ U = X. That is, U = F or U = ϕ. Therefore F is a minimal $\psi g^{\#}$-closed set.

Remark 3.5: Maximal closed and Maximal $\psi g^{\#}$-closed sets are independent to each other.

Example 3.6: Let X = {a, b, c} with the topology τ = {ϕ, {a, b}, X} and τ^c = {ϕ, {c}, X}.
Then $\psi g^{\#}$-closed sets are ϕ,{c}, {b, c}, {a, c}, X.

Here {c} is a maximal closed set but not maximal $\psi g^{\#}$-closed and {b, c}, {a, c} are maximal $\psi g^{\#}$-closed but not a maximal closed set.

Theorem 3.7: Let (X, τ) be a topological space.
(a) If F is a minimal $\psi g^{\#}$-closed set and W is a $\psi g^{\#}$-closed set such that $F \cap W \neq \phi$, then $F \subseteq W$.
(b) If F and W are minimal $\psi g^{\#}$-closed sets such that $F \cap W \neq \phi$, then F = W.

Proof:
(a) If $F \cap W \neq \phi$, then $F \cap W \subseteq F$. But $F \cap W$ is a nonempty $\psi g^{\#}$-closed set contained in a minimal $\psi g^{\#}$-closed set F, so $F \cap W = F$. Thus $F \subseteq W$.
(b) If $F \cap W \neq \phi$, then by (a), $F \subseteq W$ and $W \subseteq F$. Thus F = W.

Theorem 3.8: Let F be a minimal $\psi g^{\#}$-closed set. If x is an element of F, then $F \subseteq W$ for any $\psi g^{\#}$-closed set W containing x.

Proof: Let $x \in F$ and W is a $\psi g^{\#}$-closed set containing x. Then $F \cap W \neq \phi$.

So, by Theorem 3.7, $F \subseteq W$.

Corollary 3.9: Let F be a minimal $\psi g^{\#}$-closed set and $x \in F$. Then $F = \cap$ {W: W is a $\psi g^{\#}$-closed set containing x}.

Theorem 3.10: Let W be a nonempty finite $\psi g^{\#}$-closed set in a topological space X. Then there exists atleast one (finite) minimal $\psi g^{\#}$-closed set F such that $F \subseteq W$.

Proof: If W is a minimal $\psi g^{\#}$-closed set, we set F = W. If W is not a minimal $\psi g^{\#}$-closed set, then there exists a finite $\psi g^{\#}$-closed set W_1 such that $\phi \neq W_1 \subset W$. If W_1 is a minimal $\psi g^{\#}$-closed set, we set $F = W_1$. If W_1 is not minimal $\psi g^{\#}$-closed set, then there exists a finite $\psi g^{\#}$-closed set W_2 such that $\phi \neq W_2 \subset W_1 \subset W$. Continuing this process, we have a sequence of $\psi g^{\#}$-closed sets $W \supset W_1 \supset W_2 \supset \ldots \supset W_k \ldots$. Since W is a finite set, this process repeats only finitely many. Then finally, we get a minimal $\psi g^{\#}$-closed set $F = W_n$ for some positive integer n.

Corollary 3.11: If W is a finite minimal closed set, then there exists atleast one minimal $\psi g^{\#}$-closed set F such that $F \subseteq W$.

Theorem 3.12: Let W be a proper nonempty cofinite $\psi g^{\#}$-closed subset of a topological space X. Then there exists at least one (cofinite) maximal $\psi g^{\#}$-closed set F such that $W \subseteq F$.

Proof: If W is a maximal $\psi g^{\#}$-closed set, we set F = W. If W is not a maximal $\psi g^{\#}$-closed set, then there exists a cofinite $\psi g^{\#}$-closed set W_1 such that $W \subset W_1 \neq X$. If W_1 is maximal $\psi g^{\#}$-closed, we set $F = W_1$. If W_1 is not maximal $\psi g^{\#}$-closed, then there exists a cofinite $\psi g^{\#}$-closed set W_2 such that $W \subset W_1 \subset W_2 \neq X$. Continuing this process, we have a sequence of $\psi g^{\#}$-closed sets $W \subset W_1 \subset W_2 \subset \ldots \subset W_k \ldots$. Since W is a cofinite set, this process repeats only finitely many. Then finally, we get a maximal $\psi g^{\#}$-closed set $F = W_n$ for some positive integer n.

Theorem 3.13: Let A be a nonempty $\psi g^{\#}$-closed set. Then the following three conditions are equivalent :

(i) A is a minimal $\psi g^{\#}$-closed set.
(ii) $A \subset \psi g^{\#}int(U)$, for any nonempty subset U of A.
(iii) $\psi g^{\#}int(A) = \psi g^{\#}int(U)$, for any nonempty subset U of A.

Proof: To prove **(i) ⇒ (ii):** Let $x \in A$ and U be a nonempty subset of A. Then there is a $\psi g^{\#}$-closed set B containing x such that $A \subset B$. Now, $U = U \cap A \subset U \cap B$. Since U is nonempty, we have $U \cap B \neq \phi$. Again since B is any $\psi g^{\#}$-closed set containing x and by Lemma 2.10, $x \in \psi g^{\#}int(U)$. Hence $A \subset \psi g^{\#}int(U)$, for any nonempty subset U of A.

To prove **(ii) ⇒ (iii):** Let U be a nonempty subset of A and $A \subset \psi g^{\#}int(U)$. Then $\psi g^{\#}int(U) \subset \psi g^{\#}int(A)$ and $\psi g^{\#}int(A) \subset \psi g^{\#}int(U)$. Hence $\psi g^{\#}int(A) = \psi g^{\#}int(U)$, for any nonempty subset U of A.

To prove **(iii) ⇒ (i):** Let $\psi g^{\#}int(A) = \psi g^{\#}int(U)$, for any nonempty subset U of A. Suppose A is not a minimal $\psi g^{\#}$-closed set. Then there exists a nonempty $\psi g^{\#}$-closed set B such that $B \subset A$ and $B \neq A$. Now there exist an element $x \in A$ such that $x \notin B$. That is, $\psi g^{\#}int(\{x\}) \subset \psi g^{\#}int(X \setminus B) = X \setminus B$, since $X \setminus B$ is $\psi g^{\#}$-open set in X. It follows that $\psi g^{\#}int(\{x\}) \neq \psi g^{\#}int(A)$. This contradiction to the fact that $\psi g^{\#}int(\{x\}) = \psi g^{\#}int(A)$, for any nonempty subset $\{x\}$ of A. Thus A is a minimal $\psi g^{\#}$-closed set.

Theorem 3.14: Let A be a nonempty subset of X and F a $\psi g^{\#}$-closed set in A. If F is a minimal $\psi g^{\#}$-closed in A, then F is a minimal $\psi g^{\#}$-closed set in X.

Proof: Let F be a nonempty proper $\psi g^{\#}$-closed set in X. Let U be a nonempty subset of F. Since F is minimal $\psi g^{\#}$-closed in A and by Theorem 3.13, we have $F \subseteq \psi g^{\#}int_A(U) = \psi g^{\#}int_X(U) \cap A \subseteq \psi g^{\#}int_X(U)$. Thus by Theorem 3.13, F is a minimal $\psi g^{\#}$-closed set in X.

Theorem 3.15: Let A, B, C be minimal $\psi g^{\#}$-closed sets such that $A \neq B$. If $C \subset A \cup B$, then either A = C or B = C.

Proof: If A = C, then there is nothing to prove. If $A \neq C$, by Theorem 3.7, we have $B \cup C = B \cup (C \cup \phi) = B \cup (C \cup (A \cap B)) = B \cup ((C \cup A) \cap (C \cup B)) = (B \cup C \cup A) \cap (B \cup C \cup B) = (A \cup B) \cap (C \cup B) = (A \cap C) \cup B = \phi \cup B = B$. This implies that $C \subseteq B$. Since B and C are minimal $\psi g^{\#}$-closed sets, we have B = C.

Theorem 3.16: If A, B, C be minimal $\psi g^{\#}$-closed sets which are different from each other. Then $(A \cup B) \not\subset (A \cup C)$.

Proof: Suppose $(A \cup B) \subset (A \cup C)$.
Then $(A \cup B) \cap (C \cup B) \subset (A \cup C) \cap (C \cup B)$. Hence $(A \cap C) \cup B \subset C \cup (A \cap B)$.

By Theorem 3.7, $A \cap C = \phi$, $A \cap B = \phi$. Then $\phi \cup B \subset C \cup \phi \Rightarrow B \subset C$. By Definition 3.1, we have B = C, which is contradiction to A, B and C are different from each other. Therefore $(A \cup B) \not\subset (A \cup C)$.

Theorem 3.17: Let A and $\{A_i / i \in \Lambda\}$ be minimal $\psi g^{\#}$-closed sets. If $A \subset \bigcup_{i\in\Lambda} A_i$, then there exists an element $i \in \Lambda$ such that $A = A_i$.

Proof: Let $A \subset \bigcup_{i\in\Lambda} A_i$. Then $A \cap \bigcup_{i\in\Lambda} A_i = A$. Since $A \cap A_i \neq \phi$ and by Theorem 3.7, we have $A = A_i$ for any $i \in \Lambda$. Hence there exists an element $i \in \Lambda$ such that $A = A_i$.

Theorem 3.18: Let A and $\{A_i / i \in \Lambda\}$ be minimal $\psi g^{\#}$-closed sets. If $A \neq A_i$ for any $i \in \Lambda$, then $(\bigcup_{i\in\Lambda} A_i) \cap A = \phi$.

Proof: Suppose $(\bigcup_{i\in\Lambda} A_i) \cap A \neq \phi$. That is, $\bigcup_{i\in\Lambda} (A_i \cap A) \neq \phi$. Then there exists an element $i \in \Lambda$ such that $A \cap A_i \neq \phi$. By Theorem 3.7, we have $A = A_i$, which is contradiction to $A \neq A_i$ for any $i \in \Lambda$. Hence $(\bigcup_{i\in\Lambda} A_i) \cap A = \phi$.

4. MINIMAL $\psi g^{\#}$-CONTINUOUS FUNCTION

Definition 4.1: Let X and Y be topological spaces. A map $f: X \to Y$ is called

(i) minimal $\psi g^{\#}$-continuous (briefly, min-$\psi g^{\#}$-continuous) if $f^{-1}(M)$ is a $\psi g^{\#}$-open set in X for every minimal $\psi g^{\#}$-open set M in Y.
(ii) maximal $\psi g^{\#}$-continuous (briefly, max-$\psi g^{\#}$-continuous) if $f^{-1}(M)$ is a $\psi g^{\#}$-open set in X for every maximal $\psi g^{\#}$-open set M in Y.
(iii) minimal $\psi g^{\#}$-irresolute (briefly, min-$\psi g^{\#}$-irresolute) if $f^{-1}(M)$ is a minimal $\psi g^{\#}$-open set in X for every minimal $\psi g^{\#}$-open set M in Y.
(iv) maximal $\psi g^{\#}$-irresolute (briefly, max-$\psi g^{\#}$-irresolute) if $f^{-1}(M)$ is a maximal $\psi g^{\#}$-open set in X for every maximal $\psi g^{\#}$-open set M in Y.
(v) minimal-maximal $\psi g^{\#}$-continuous (briefly, min-max $\psi g^{\#}$-continuous) if $f^{-1}(M)$ is a maximal $\psi g^{\#}$-open set in X for every minimal $\psi g^{\#}$-open set M in Y.
(vi) maximal-minimal $\psi g^{\#}$-continuous (briefly, max-min $\psi g^{\#}$-continuous) if $f^{-1}(M)$ is a minimal $\psi g^{\#}$-open set in X for every maximal $\psi g^{\#}$-open set M in Y.

Theorem 4.2: Let X and Y be the topological spaces. A map $f : X \to Y$ is minimal(resp. maximal) $\psi g^{\#}$-continuous if and only if the inverse image of each maximal(resp. minimal) $\psi g^{\#}$-closed set in Y is a $\psi g^{\#}$-closed set in X.

Proof: Obvious.

Theorem 4.3: Let X and Y be the topological spaces and A be a nonempty subset of X. If $f: X \to Y$ is minimal (resp. maximal) $\psi g^{\#}$-continuous then the restriction map $f_A : A \to Y$ is a minimal(resp. maximal) $\psi g^{\#}$-continuous.

Proof: Let $f: X \to Y$ be minimal ψg#-continuous. Let B be any minimal $\psi g^{\#}$-open set in Y. Since f is minimal $\psi g^{\#}$-continuous, $f^{-1}(B)$ is a $\psi g^{\#}$-open set in X. But $f_A^{-1}(B) = A \cap f^{-1}(B)$ and $A \cap f^{-1}(B)$ is a $\psi g^{\#}$-open set in A. Therefore f_A is minimal $\psi g^{\#}$-continuous.

Remark 4.4: The composition of two minimal $\psi g^{\#}$-continuous function need not be a minimal $\psi g^{\#}$-continuous.

Example 4.5:. Let X = Y = Z = {a, b, c} with the topology $\tau_X = \{\phi, \{a, b\}, X\}$, $\tau_Y = \{\phi, \{a\}, \{b\}, \{a, b\}, \{b,c\}, Y\}$ and $\tau_Z = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, Z\}$.

Define $f : X \to Y$ by f(a) = c, f(b) = b, f(c) = a and $g : Y \to Z$ by g(a) = a, g(b) = b, g(c) = a.

Then f and g are minimal $\psi g^{\#}$-continuous. But $g\circ f$ is not a minimal $\psi g^{\#}$-continuous, since the set {a} is a minimal $\psi g^{\#}$-closed set in Z while $(g\circ f)^{-1}(\{a\}) = \{c\}$ which is not a $\psi g^{\#}$-closed set in Y.

Theorem 4.6: If f: X → Y is $\psi g^{\#}$-continuous, Y is $T_{\psi g\#}$ space and g: Y→ Z is minimal (resp. maximal) $\psi g^{\#}$-continuous. Then g ○ f : X → Z is minimal(resp. maximal) $\psi g^{\#}$-continuous.

Proof: Let A be any minimal $\psi g^{\#}$-open set in Z. Since g is minimal $\psi g^{\#}$-continuous, $g^{-1}(A)$ is $\psi g^{\#}$-open in Y. But Y is $T_{\psi g\#}$ space, $g^{-1}(A)$ is open in Y. Again since f is $\psi g^{\#}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a $\psi g^{\#}$-open set in X. Hence g ○ f is minimal $\psi g^{\#}$-continuous.

Theorem 4.7: Every minimal (resp. maximal) $\psi g^{\#}$-irresolute map is a minimal (resp. maximal) $\psi g^{\#}$-continuous map.

Proof: Let f: X → Y be a minimal $\psi g^{\#}$-irresolute map. Let A be any minimal $\psi g^{\#}$-open set in Y. Since f is minimal $\psi g^{\#}$-irresolute, $f^{-1}(A)$ is a minimal $\psi g^{\#}$-open set in X. That is $f^{-1}(A)$ is a $\psi g^{\#}$-open set in X. Hence f is minimal $\psi g^{\#}$-continuous.

Theorem 4.8: Let X and Y be the topological spaces. A map f: X → Y is minimal (resp. maximal) $\psi g^{\#}$-irresolute if and only if the inverse image of each maximal (resp. minimal) $\psi g^{\#}$-closed set in Y is a maximal (resp. minimal) $\psi g^{\#}$-closed set in X.

Proof: Obvious.

Theorem 4.9: If f: X → Y and g: Y → Z are minimal(resp. maximal) $\psi g^{\#}$-irresolute, then g ○ f : X→ Z is a minimal(resp. maximal) $\psi g^{\#}$-irresolute map.

Proof: Let A be any minimal $\psi g^{\#}$-open set in Z. Since g is minimal $\psi g^{\#}$-irresolute, $g^{-1}(A)$ is a minimal $\psi g^{\#}$-open set in Y. Again Since f is minimal $\psi g^{\#}$-irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is minimal $\psi g^{\#}$-open in X. Therefore g ○ f is minimal $\psi g^{\#}$-irresolute.

Theorem 4.10: Every minimal-maximal (resp. maximal-minimal) $\psi g^{\#}$-continuous map is minimal (resp. maximal) $\psi g^{\#}$-continuous.

Proof: Let f: X → Y be a minimal-maximal $\psi g^{\#}$-continuous map. Let A be a minimal $\psi g^{\#}$-open set in Y. Since f is minimal-maximal $\psi g^{\#}$-continuous, $f^{-1}(A)$ is a maximal $\psi g^{\#}$-open set in X. Since every maximal $\psi g^{\#}$-open set is a $\psi g^{\#}$-open set, $f^{-1}(A)$ is $\psi g^{\#}$-open in X. Hence f is minimal $\psi g^{\#}$-continuous.

Theorem 4.11: Let X and Y be the topological spaces. A map f : X→ Y is minimal-maximal (resp. maximal-minimal) $\psi g^{\#}$-continuous if and only if the inverse image of each maximal(resp. minimal) $\psi g^{\#}$-closed in Y is a minimal(resp. maximal) $\psi g^{\#}$-closed set in X.

Proof: Obvious.

Theorem 4.12: If f: X→ Y is maximal (resp. minimal) $\psi g^{\#}$-irresolute and g: Y→ Z is minimal -maximal (resp. maximal-minimal) $\psi g^{\#}$-continuous, then g○f : X → Z is minimal-maximal(resp. maximal-minimal) $\psi g^{\#}$-continuous.

Proof: Let A be a minimal $\psi g^{\#}$-open set in Z. Since g is minimal-maximal $\psi g^{\#}$-continuous, $g^{-1}(A)$ is a maximal $\psi g^{\#}$-open set in Y. Again since f is maximal $\psi g^{\#}$-irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a maximal $\psi g^{\#}$-open set in X. Hence g ○ f is minimal-maximal $\psi g^{\#}$-continuous.

Theorem 4.13: If f: X → Y is maximal (resp. minimal) $\psi g^{\#}$-continuous and g: Y → Z is minimal -maximal (resp. maximal-minimal) $\psi g^{\#}$-continuous, then g ○ f : X → Z is a minimal(resp. maximal) $\psi g^{\#}$-continuous.

Proof: Obvious.

Theorem 4.14: If f : X → Y is $\psi g^{\#}$-continuous and Y is $T_{\psi g\#}$ space and g : Y → Z is minimal-maximal (resp. maximal-minimal) $\psi g^{\#}$-continuous, then g ○ f : X → Z is minimal(resp. maximal) $\psi g^{\#}$-continuous.

Proof: Obvious.

Definition 4.15: A topological space (X, τ) is called a
(i) $T_{min-\psi g\#}$ space if every $\psi g^{\#}$-open set is a minimal $\psi g^{\#}$-open set.
(ii) $T_{max-\psi g\#}$ space if every $\psi g^{\#}$-open set is a maximal $\psi g^{\#}$-open set.

Theorem 4.16: Let f: X → Y be a minimal (resp. maximal) $\psi g^{\#}$-continuous and Y be a $T_{min-\psi g\#}$(resp. $T_{max-\psi g\#}$) space. Then f is $\psi g^{\#}$-continuous.

Proof: Let f be a minimal $\psi g^{\#}$-continuous. Let A be a open set in Y. Since every open set is $\psi g^{\#}$-open, A is $\psi g^{\#}$-open in Y. By hypothesis Y is $T_{min-\psi g\#}$ space, we have A is a minimal $\psi g^{\#}$-open set in Y. Since f is minimal $\psi g^{\#}$-continuous, $f^{-1}(A)$ is $\psi g^{\#}$-open in X. Therefore f is $\psi g^{\#}$-continuous.

Theorem 4.17: Let f : X → Y and g : Y →Z be minimal $\psi g^{\#}$-continuous and Y is $T_{min\text{-}\psi g\#}$ space, then g ○ f : X → Z is minimal $\psi g^{\#}$-continuous.

Proof: Obvious.

Theorem 4.18: Let f: X → Y be a minimal(resp. maximal) $\psi g^{\#}$-irresolute and let Y be a $T_{min-\psi g\#}$(resp. $T_{max-\psi g\#}$) space. Then f is $\psi g^{\#}$-continuous.

Proof: Obvious.

Theorem 4.19: Let f: X → Y be a minimal -maximal(resp. maximal-minimal) $\psi g^{\#}$-continuous and let Y be a $T_{min-\psi g\#}$(resp. $T_{max-\psi g\#}$) space. Then f is $\psi g^{\#}$-continuous.

Proof: Obvious.

Theorem 4.20: If f: X → Y and g: Y → Z are minimal-maximal(resp. maximal-minimal) $\psi g^{\#}$-continuous maps and if Y is a $T_{min-\psi g\#}$(resp. $T_{max-\psi g\#}$) space, then g ○ f : X → Z is minimal-maximal(resp. maximal-minimal) $\psi g^{\#}$-continuous.

Proof: Let A be any minimal $\psi g^{\#}$-open set in Z. Since g is minimal-maximal $\psi g^{\#}$-continuous, $g^{-1}(A)$ is a maximal $\psi g^{\#}$-open set in Y. It follows that $g^{-1}(A)$ is $\psi g^{\#}$-open in Y. Since Y is $T_{min-\psi g\#}$ space, $g^{-1}(A)$ is a minimal $\psi g^{\#}$-open set in Y. Again since f is minimal-maximal $\psi g^{\#}$-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is a maximal $\psi g^{\#}$-open set in X.

Hence g ○ f is minimal-maximal $\psi g^{\#}$-continuous.

REFERENCES

1. P. Bhattacharyya and B.K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29, (1987), 376-382.
2. N. Levine, Semi-open sets and Semi-Continuity in Topological Spaces, Amer. Math. Monthly, 70, (1963), 36-41.
3. N. Levine, Generalized Closed Sets in Topology, Rend. Circ. Mat. Palermo, 19(2), (1970), 89-96.
4. H. Maki, R. Devi, and K. Balachandran, Associate Topologies of Generalized α-Closed Sets and α-Generalized Closed Sets, Mem. Fac. Kochi Univ. Ser. A. Math., 15, (1994), 51-63.
5. A.Mukharjee, On Maximal, Minimal Open and Closed Sets, Com-mun.Korean.Math.Soc. 30, (2015), 277-282.
6. F.Nakaoka and N.Oda, Some applications of Minimal Closed sets and maximal closed sets, Int. J. Math. Sci., (2006), Article ID 18647, 1-8.
7. F.Nakaoka and N.Oda , Some application of minimal open sets, Int.J.Math.Sci., 278,(2001), 471-478.
8. F.Nakaoka and N.Oda, Some properties of maximal open sets. Int. J. Math. Sci., 21, (2003), 1331-1340.
9. O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15, (1965), 961-970.
10. N.Sowmya, M.Elakkiya, N.Balamani, $\psi g^{\#}$-closed sets in topological spaces, IJAFRC 2015, ISSN 2348-4853.
11. M.K.R.S. Veera kumar, Between semi-closed sets and semi-preclosed sets, Rend. Istit. Mat. Univ. Trieste, (ITALY) XXXXII, (2000), 25-41.
12. M.K.R.S. Veera kumar, $g^{\#}$-closed sets in topological spaces, Mem. Fac. Sci. Kochi Univ. Ser. A. Math, 24, (2003), 1-13.

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