

ON NANO (τ_1, τ_2) GENERALIZED β CLOSED SETS
 AND NANO (τ_1, τ_2) GENERALIZED β OPEN SETS IN NANO BITOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce Nano (τ_1, τ_2) generalized β closed sets and Nano (τ_1, τ_2) generalized β open sets in Nano bitopological spaces. Also the characteristics and properties of Nano (τ_1, τ_2) β closed sets and Nano (τ_1, τ_2) β open sets are studied respectively.

Keywords: Nano (τ_1, τ_2) β interior, Nano (τ_1, τ_2) β closure, Nano (τ_1, τ_2) generalized β closed sets, Nano (τ_1, τ_2) generalized β open sets.

1. INTRODUCTION

The notion of Nano topology was introduced by Lellis Thivagar [6] which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and he also defined Nano closed set, Nano interior and Nano closure. Levine [7] introduced generalized closed sets as a generalization of closed sets in topological spaces. Abd El Monsef *et al.* [1] introduced the notion of β -open set in topology, further investigation of Nano β open sets was given by Gnanambal [4]. Shalini *et al.* [8] have introduced Nano generalized β closed sets in Nano topology. Kelly [5] introduced the concept of bitopological space in and Fukutake [3] introduced the generalized closed sets in bitopological space. Bhuvaneswari *et al.* [2] introduced the Nano bitopological space. In this paper we introduce Nano (τ_1, τ_2) generalized β closed sets and Nano (τ_1, τ_2) generalized β open sets and some of its properties are investigated.

2. PRELIMINARIES

Definition 2.1[6]: Let U be the universe, R be an equivalence relation on U and where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

- U and $\phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. hence $\tau_R(X)$ is called the Nano topology on U with respect to X , $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U . Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

Definition 2.2[6]: If $(U, \tau_R(X))$ is a Nano topological space where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $N \text{ int}(A)$. $N \text{ int}(A)$ is the largest Nano open subset of A .
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A .

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Definition 2.3 [7]: A subset A of (X, τ) is called generalized closed set (briefly g closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

Definition 2.4 [2]: A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition 2.5 [8]: A subset A of Nano topological space $(U, \tau_R(X))$ is called Nano generalized β closed set (briefly Ng β closed) if $N\beta cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition 2.6 [3]: A subset A of (X, τ_1, τ_2) is called (τ_1, τ_2) generalized closed set (briefly (τ_1, τ_2) g closed) if $\tau_2 cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1 open in (X, τ_1, τ_2) .

3. NANO (τ_1, τ_2) GENERALIZED β CLOSED SETS

Definition 3.1: Let U be the universe, R_1 and R_2 are equivalence relations on U and X_1 and X_2 are subsets of U . Then $\tau_{R_1}(X_1)$ and $\tau_{R_2}(X_2)$ satisfies the following axioms:

- U and $\emptyset \in \tau_{R_1}(X_1)$ and $\tau_{R_2}(X_2)$.
- The union of the elements of any sub collection of $\tau_{R_1}(X_1)$ is in $\tau_{R_1}(X_1)$ and $\tau_{R_2}(X_2)$ is in $\tau_{R_2}(X_2)$.
- The intersection of the elements of any finite sub collection of $\tau_{R_1}(X_1)$ is in $\tau_{R_1}(X_1)$ and $\tau_{R_2}(X_2)$ is in $\tau_{R_2}(X_2)$.

Hence $\tau_{R_1}(X_1)$ and $\tau_{R_2}(X_2)$ is called the Nano bitopology on U with respect to X_1 and X_2 , $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano (τ_1, τ_2) open sets in U and elements of $[\tau_{R_1}(X_1)]^C$ and $[\tau_{R_2}(X_2)]^C$ are called Nano (τ_1, τ_2) closed sets.

Definition 3.2: If $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is a Nano bitopological space where X_1 and X_2 are subsets of U and if $A \subseteq U$, then

- The Nano (τ_1, τ_2) interior of a set A is defined as the union of all Nano (τ_1, τ_2) open subsets contained in A and is denoted by $N\tau_1\tau_2 \text{ int}(A)$. $N\tau_1\tau_2 \text{ int}(A)$ is the largest Nano (τ_1, τ_2) open subset of A .
- The Nano (τ_1, τ_2) closure of a set A is defined as the intersection of all Nano (τ_1, τ_2) closed sets containing A and is denoted by $N\tau_1\tau_2 cl(A)$. $N\tau_1\tau_2 cl(A)$ is the smallest Nano (τ_1, τ_2) closed set containing A .

Definition 3.3: Let $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

- Nano (τ_1, τ_2) semi open if $A \subseteq N\tau_2 cl(N\tau_1 \text{ int}(A))$
- Nano (τ_1, τ_2) pre open if $A \subseteq N\tau_2 \text{ int}(N\tau_1 cl(A))$
- Nano (τ_1, τ_2) α open if $A \subseteq N\tau_1 \text{ int}[N\tau_2 cl(N\tau_1 \text{ int}(A))]$
- Nano (τ_1, τ_2) regular open if $A = N\tau_2 \text{ int}(N\tau_1 cl(A))$
- Nano (τ_1, τ_2) β open (Nano (1,2) semi-pre open) if $A \subseteq N\tau_1 cl[N\tau_2 \text{ int}(N\tau_1 cl(A))]$

The family of Nano (τ_1, τ_2) semi open (resp. Nano (τ_1, τ_2) pre open, Nano (τ_1, τ_2) α open, Nano (τ_1, τ_2) regular open, Nano (τ_1, τ_2) β open) sets in U is denoted by $N(\tau_1, \tau_2) SO(U, X)$ (resp. $N(\tau_1, \tau_2) PO(U, X)$, $N(\tau_1, \tau_2) \alpha O(U, X)$, $N(\tau_1, \tau_2) RO(U, X)$ and $N(\tau_1, \tau_2) \beta O(U, X)$).

The complement of Nano (τ_1, τ_2) semi open (resp. Nano (τ_1, τ_2) pre open, Nano (τ_1, τ_2) α open, Nano (τ_1, τ_2) regular open, Nano (τ_1, τ_2) β open) sets in U is Nano (τ_1, τ_2) semi closed (resp. Nano (τ_1, τ_2) pre closed, Nano (τ_1, τ_2) α closed, Nano (τ_1, τ_2) regular closed, Nano (τ_1, τ_2) β closed).

Definition 3.4: A subset A of $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is called Nano (τ_1, τ_2) generalized closed set (briefly $N(\tau_1, \tau_2)$ g closed) if $N\tau_2 cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano τ_1 open in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Definition 3.5: If $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is a Nano bitopological space where X_1 and X_2 are subsets of U and if $A \subseteq U$ then

- The Nano (τ_1, τ_2) β interior of a set A is defined as the union of all Nano (τ_1, τ_2) β open subsets contained in A and is denoted by $N\tau_1\tau_2\beta\text{int}(A)$. $N\tau_1\tau_2\beta\text{int}(A)$ is the largest Nano (τ_1, τ_2) β open subset of A .
- The Nano (τ_1, τ_2) β closure of a set A is defined as the intersection of all Nano (τ_1, τ_2) β closed sets containing A and is denoted by $N\tau_1\tau_2\beta cl(A)$. $N\tau_1\tau_2\beta cl(A)$ is the smallest Nano (τ_1, τ_2) β closed set containing A .

Definition 3.6: A subset A of Nano bitopological space $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is called Nano (τ_1, τ_2) generalized β closed set (briefly $N(\tau_1, \tau_2)$ g β closed) if $N\tau_2\beta cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano τ_1 open in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Theorem 3.8: If A is Nano τ_2 closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ then it is Nano (τ_1, τ_2) g β closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ but not conversely.

Proof: Since every Nano closed set is Nano g β closed set, the proof follows.

Example 3.9: Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{c\}, \{b, d\}\}$, $X_1 = \{a, b\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$, $U/R_2 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$, $X_2 = \{a, d\}$, $\tau_{R_2}(X_2) = \{U, \phi, \{a, d\}\}$. Here the set $\{a, c, d\}$ is Nano (τ_1, τ_2) g β closed but not Nano τ_2 closed in U .

Theorem 3.10: Every Nano τ_2 pre closed set is Nano (τ_1, τ_2) g β closed set but not conversely.

Proof: Let A be Nano τ_2 pre closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ and let G be a Nano τ_1 open set such that $A \subseteq G$. Since every Nano pre closed is Nano g β closed, we have $N\tau_2\beta cl(A) \subseteq G$. Hence A is Nano $(1, 2)$ g β closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Example 3.11: Let $U = \{a, b, c, d\}$ with $U/R_2 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$, $X_1 = \{a, d\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{a, d\}\}$, $U/R_2 = \{\{a\}, \{c\}, \{b, d\}\}$, $X_2 = \{a, b\}$, $\tau_{R_2}(X_2) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Here the set $\{a, b\}$ is Nano (τ_1, τ_2) g β closed but not Nano τ_2 pre closed in U .

Theorem 3.12: Every Nano τ_2 regular closed set is Nano (τ_1, τ_2) g β closed set but not conversely.

Proof: Since every Nano regular closed set is Nano g β closed set, the proof follows.

Example 3.13: Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$, $X_1 = \{a, d\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{a, d\}\}$, $U/R_2 = \{\{a\}, \{c\}, \{b, d\}\}$, $X_2 = \{a, b\}$, $\tau_{R_2}(X_2) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Here the set $\{c, d\}$ is Nano $(1, 2)$ g β closed but not Nano τ_2 pre closed in U .

Remark 3.14: The union of two Nano (τ_1, τ_2) g β closed sets need not be Nano (τ_1, τ_2) g β closed set which can be seen from the following example.

Example 3.15: Let $U = \{a, b, c, d\}$ with $U/R_1 = \{\{a\}, \{c\}, \{b, d\}\}$, $X_1 = \{a, b\}$, $\tau_{R_1}(X_1) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$, $U/R_2 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$, $X_2 = \{a, d\}$, $\tau_{R_2}(X_2) = \{U, \phi, \{a, d\}\}$. Here the sets $\{a\}$ and $\{b, d\}$ are Nano (1,2) g β closed sets but $\{a\} \cup \{b, d\} = \{a, b, d\}$ is not Nano (τ_1, τ_2) g β closed set in U .

Theorem 3.16: If a set A is Nano (τ_1, τ_2) g β closed set in a Nano bitopological space $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$, then $N\tau_2\beta cl(A) - A$ contains no non-empty Nano τ_1 closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Proof: Suppose that F is a Nano τ_1 closed set such that $F \subseteq N\tau_2\beta cl(A) - A$. Now $F \subseteq N\tau_2\beta cl(A)$ and $F \subseteq A^c$ then $A \subseteq U - F$, $U - F$ is Nano τ_1 open set and A is Nano (τ_1, τ_2) g β closed. Therefore $N\tau_2\beta cl(A) \subseteq U - F$. That is $F \subseteq U - (N\tau_2\beta cl(A))$. Hence $F \subseteq N\tau_2\beta cl(A) \cap (U - (N\tau_2\beta cl(A))) = \phi$, $F = \phi$. Therefore $N\tau_2\beta cl(A) - A$ contains no non-empty Nano τ_1 closed set in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Remark 3.17: If a set A in a Nano bitopological space $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is Nano $\tau_2\beta$ closed then $N\tau_2\beta cl(A) - A = \phi$.

Proof: Assume that A is Nano $\tau_2\beta$ closed. Since $N\tau_2\beta cl(A) = A$, $N\tau_2\beta cl(A) - A = \phi$.

Theorem 3.18: For each point x of U , a singleton $\{x\}$ is Nano τ_1 closed or $\{x\}^c$ is Nano (τ_1, τ_2) g β closed.

Proof: Suppose $\{x\}$ is not Nano τ_1 closed. Since $\{x\}^c$ is not Nano τ_1 open, a Nano τ_1 open containing $\{x\}^c$ is only U . Then $N\tau_2\beta cl(\{x\}^c) \subseteq U$ and $\{x\}^c$ is Nano (1,2) g β closed.

Theorem 3.19: If A is Nano (τ_1, τ_2) g β closed then $N\tau_2\beta cl(x) \cap A \neq \phi$ for some $x \in N\tau_2\beta cl(A)$ but not conversely.

Proof: If $N\tau_2\beta cl(x) \cap A = \phi$ for $x \in N\tau_2\beta cl(A)$, then $A \subseteq (N\tau_2\beta cl(x))^c$. Since A is Nano (τ_1, τ_2) g β closed set, we have $N\tau_2\beta cl(A) \subseteq (N\tau_2\beta cl(x))^c$. This implies $x \notin N\tau_2\beta cl(A)$ which is a contradiction.

Theorem 3.20: If A is Nano (τ_1, τ_2) g β closed in a Nano bitopological space $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ and $A \subseteq B \subseteq N\tau_2\beta cl(A)$, then B is also Nano (1,2) g β closed in $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$.

Proof: Let $B \subseteq G$ where G is Nano τ_1 open set in U . Then $A \subseteq B$ implies $A \subseteq G$. As A is Nano (1, 2) g β closed, we have $N\tau_2\beta cl(A) \subseteq G$. Now $B \subseteq N\tau_2\beta cl(A)$, implies

$N\tau_2\beta cl(B) \subseteq N\tau_2\beta cl(N\tau_2\beta cl(A)) = N\tau_2\beta cl(A) \subseteq G$. Thus $N\tau_2\beta cl(B) \subseteq G$. Therefore B is Nano (1,2) g β closed set in U .

4. NANO (τ_1, τ_2) GENERALIZED β OPEN SETS

Definition 4.1: A subset A of a Nano bitopological space $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ is called Nano (τ_1, τ_2) generalized β open (briefly Nano (τ_1, τ_2) g β open), if its complement A^c is Nano (τ_1, τ_2) g β closed.

The collection of all Nano (τ_1, τ_2) g β open subsets of U is denoted by $N(\tau_1, \tau_2) G \beta O(U, X)$.

Theorem 4.2: Every Nano τ_2 open set in $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ is Nano $(\tau_1, \tau_2)g\beta$ open set in $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ but not conversely.

Proof: Since every Nano open set is Nano $g\beta$ open, the proof follows.

Example 4.3: Let $U = \{a, b, c, d, e\}$ with $U/R1 = \{\{a\}, \{b, c, d\}, \{e\}\}$, $X_1 = \{a, b\}$, $\tau_{R1}(X_1) = \{U, \phi, \{a\}, \{b, c, d\}, \{a, b, c, d\}\}$, $U/R2 = \{\{a, b\}, \{c, e\}, \{d\}\}$, $X_2 = \{a, d\}$, $\tau_{R2}(X_2) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Here the set $\{a, c, e\}$ is Nano $(\tau_1, \tau_2)g\beta$ open but not Nano τ_2 open in U .

Remark 4.4: The intersection of two Nano $(\tau_1, \tau_2)g\beta$ open sets need not be Nano $(\tau_1, \tau_2)g\beta$ open set which can be seen from the following example.

Example 4.5: Let $U = \{a, b, c, d, e\}$ with $U/R1 = \{\{a\}, \{b, c, d\}, \{e\}\}$, $X_1 = \{a, b\}$, $\tau_{R1}(X_1) = \{U, \phi, \{a\}, \{b, c, d\}, \{a, b, c, d\}\}$, $U/R2 = \{\{a, b\}, \{c, e\}, \{d\}\}$, $X_2 = \{a, d\}$, $\tau_{R2}(X_2) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$. Here the sets $\{a, e\}$ and $\{d, e\}$ are Nano $(\tau_1, \tau_2)g\beta$ open sets but $\{a, e\} \cap \{d, e\} = \{e\}$ is not Nano $(\tau_1, \tau_2)g\beta$ open set in U .

Theorem 4.6: A subset A in a Nano bitopological space $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ is Nano $(\tau_1, \tau_2)g\beta$ open if and only if $F \subseteq N\tau_2\beta \text{int}(A)$ whenever F is Nano τ_1 closed and $F \subseteq A$.

Proof: Assume that A is Nano $(\tau_1, \tau_2)g\beta$ open set in $(U, \tau_R(X))$. Let F be Nano τ_1 closed and $F \subseteq A$, then $A^c \subseteq F^c$ implies F^c is Nano τ_1 open. Since A^c is Nano $(\tau_1, \tau_2)g\beta$ closed set $N\tau_2\beta \text{cl}(A^c) \subseteq F^c$. Since $(N\tau_2\beta \text{int}(A))^c = N\tau_2\beta \text{cl}(A^c)$, $(N\tau_2\beta \text{int}(A))^c \subseteq F^c$. Therefore $F \subseteq N\tau_2\beta \text{int}(A)$.

Conversely assume that $F \subseteq N\tau_2\beta \text{int}(A)$ whenever F is Nano τ_1 closed set and $F \subseteq A$. Then $(N\tau_2\beta \text{int}(A))^c \subseteq F^c$. Thus $N\tau_2\beta \text{cl}(A^c) \subseteq F^c$. Hence A^c is Nano $(\tau_1, \tau_2)g\beta$ closed set and A is Nano $(\tau_1, \tau_2)g\beta$ open set in U .

Theorem 4.7: If A is a Nano $(\tau_1, \tau_2)g\beta$ open set in a Nano bitopological space $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$ and $N\tau_2\beta \text{int}(A) \subseteq B \subseteq A$, then B is also Nano $(\tau_1, \tau_2)g\beta$ open.

Proof: Let A is Nano $(\tau_1, \tau_2)g\beta$ open set and $N\tau_2\beta \text{int}(A) \subseteq B \subseteq A$. Then $A^c \subseteq B^c \subseteq (N\tau_2\beta \text{int}(A))^c$ implies $A^c \subseteq B^c \subseteq N\beta \text{cl}(A^c)$. Since A^c is Nano $(\tau_1, \tau_2)g\beta$ closed, B^c is also Nano $(\tau_1, \tau_2)g\beta$ closed. Therefore B is Nano $(\tau_1, \tau_2)g\beta$ open.

Theorem 4.8: If a set A is Nano $(\tau_1, \tau_2)g\beta$ open in a Nanobitopological space $(U, \tau_{R1}(X_1), \tau_{R2}(X_2))$, then $G = U$ whenever G is Nano τ_1 open and $N\tau_2\beta \text{int}(A) \cup A^c \subseteq G$.

Proof: Let A be Nano $(\tau_1, \tau_2)g\beta$ open, G be Nano τ_1 open set and $N\tau_2\beta \text{int}(A) \cup A^c \subseteq G$. Then $G^c \subseteq (N\tau_2\beta \text{int}(A) \cup A^c)^c = N\tau_2\beta \text{cl}(A^c) - A^c$, since G^c is Nano τ_1 closed and A^c is Nano $(\tau_1, \tau_2)g\beta$ closed. By theorem 3.13[8] we have $G^c = \phi$. Therefore $G = U$.

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