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ON A MANEEAL'S PEDAL CONIC OF ORDER ONE

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ABSTRACT

In this article we present a conic which passes through the six notable points, they are the feet of perpendiculars drawn from the internal and external Maneeal's point of order one and also we study the related properties of the conic.

Keywords: *Maneeal's conic, Maneeal's pedal conic of order one, Internal & External Maneeal's pedal conic of order one, A,B,C- Partial Maneeal's pedal conic of order one, Maneeals, carnot's theorem.*

1. INTRODUCTION

Given an arbitrary triangle ABC, n be an integer, consider the points D_n, E_n, F_n on the sides BC, CA, AB respectively satisfying $\left|\frac{AC}{AB}\right|^n = \left|\frac{CD_n}{BD_n}\right|$, $\left|\frac{AB}{BC}\right|^n = \left|\frac{AE_n}{CE_n}\right|$, $\left|\frac{BC}{AC}\right|^n = \left|\frac{BF_n}{AF_n}\right|$, then we call the cevians AD_n , BE_n, CF_n as the order n *Maneeals* of the triangle ABC, and their point of concurrence M_n is called as *Maneeal's point of order of n*, For given M_n we can choose points P_n, Q_n, R_n on the sides *BC, CA, AB* respectively, In such a manner that line segments M_nP_n, M_nQ_n, M_nR_n are perpendicular to the correspoding sides of the triangle ABC, the triangle with vertices P_n, Q_n, R_n is called as *Maneeal's Pedal triangle of order n*. It easy to check, That Angular Bisectors are examples of the Maneeals with integer n=1 and Incenter of the triangle is example of Maneesl's point of order 1. The elaborate study about Maneeals and its properties, applications can be found in [1], [2].

It is always interesting when several notable points lie on some sort of familiar curve, It is well known that five coplanar non collinear points describes a unique conic. If it passes through atleast one more point then it gives a siginificant conic. The purpose of this note is to demonstrate that there is a conic can be recognized as ***Maneeal's pedal conic of order one***, which passes through the six points three intouch, nine extouch points (it is clear that intouch and extouch points are the points of contact of incircle, excircle with the sides of the reference triangle, intouch and extouch points are the foot of perpendicular drawn from internal and external Maneeal's Point of order one, where as internal and external Maneeal's Point of order one are incenter and excenter).

In this article we discuss about three types of conics which passes through the six points out of these 12 points (three intouch and nine extouch points).

2. NOTATION AND BACKGROUND

Let ABC be a non equilateral triangle. We denote its side-lengths by a, b, c, perimeter by 2s, its area by Δ and its circumradius by R, its inradius by r and exradii by r_1, r_2, r_3 respectively. Let ${}_AI$, ${}_BI$, and ${}_CI$ be the intouch points in which the incircle meets the sides *BC, CA* and *AB*, let ${}_AI_1$, ${}_BI_1$, and ${}_CI_1$ be the extouch points in which the *A*-excircle meets the sides *BC, CA* and *AB*, let ${}_AI_2$, ${}_BI_2$, and ${}_CI_2$ be the extouch points in which the *B*-excircle meets the sides *BC, CA* and *AB* , let ${}_AI_3$, ${}_BI_3$, and ${}_CI_3$ be the extouch points in which the *C*-excircle meets the sides *BC, CA* and *AB.*

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The following metric relations are well known

The lengths of segments divided by incircle at the points where it touches the sides BC, CA and AB is given by

$$B_AI = s-b,\ C_AI = s-c,\ C_BI = s-c,\ A_BI = s-a,\ A_CI = s-a,\ B_cI = s-b$$

And

The lengths of segments divided by *A*-excircle at the points where it touches the sides BC, CA and AB is given by

$$B_AI_1 = s - c = B_CI_1,\ C_AI_1 = s - b = C_BI_1,\ A_CI_1 = s = A_BI_1$$

The lengths of segments divided by *B*-excircle at the points where it touches the sides BC, CA and AB is given by

$$C_BI_2 = s - a = C_AI_2,\ A_BI_2 = s - c = A_CI_2,\ B_AI_2 = s = B_CI_2$$

The lengths of segments divided by *C*-excircle at the points where it touches the sides BC, CA and AB is given by

$$A_CI_3 = s - b = A_BI_3,\ B_CI_3 = s - a = B_AI_3,\ C_BI_3 = s = C_AI_3$$

We shall make use of the following basic results on conics associated with a triangle. brief discussion on this by respected Paul Yiu can be found in [3].

Consider a conic with barycentric equation $Ax^2 + By^2 + Cz^2 + 2Hxy + 2Gxz + 2Fyz = 0$

Since this equation can be expressed in the form $\begin{pmatrix} x & y & z \end{pmatrix}\begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

Let us call $M = \begin{pmatrix} A & H & G \\ H & B & F \\ G & F & C \end{pmatrix}$ the matrix of the conic.

The adjoint matrix of M,

Namely $M^{\#} = \begin{pmatrix} BC-F^2 & FG-CH & HF-BG \\ FG-CH & AC-G^2 & GH-AF \\ HF-BG & GH-AF & AB-H^2 \end{pmatrix}$ (I)

Let $G = (1:1:1)$ homogeneous barycentric coordinates of centroid of triangle ABC.

The characteristic of M is the number is given by

$$\begin{aligned} characterstic(M) &= \lambda(M) = GM^{\#}G^T \\ &= 2(AF+BG+CH) + (G^2+H^2+F^2-2GH-2HF-2FG) - (AB+BC+CA) \\ &= \text{The Negative of the sum of the entries of } M^{\#} \end{aligned}$$

And it is clear that the given conic is an ellipse , a parabola or a hyperabola according as the $\lambda(M)$ is positive, zero or negative.

Let $C(M)$ is the center of conic, has coordinates formed by the columns sum of $M^{\#}$
That is

$$C(M) = GM^{\#} = \begin{pmatrix} [F(G+H-F)+BC-BG-CH]:[G(F+H-G)+AC-AF-CH]: \\ [H(G+F-H)+AB-AF-BG] \end{pmatrix}$$

The following table gives the barycentric coordinates of intouch and extouch points on the sides

${}_AI$, ${}_BI$, ${}_CI$	(0 : s-c : s-b), (s-c : 0 : s-a), (s-b : s-c : 0)
${}_AI_1$, ${}_BI_1$, ${}_CI_1$	(0 : s-b : s-c), (s-b : 0 : -s), (s-c : -s : 0)
${}_AI_2$, ${}_BI_2$, ${}_CI_2$	(0 : s-a : - s), (s-a : 0 : s-c), (-s : s-c : 0)
${}_AI_3$, ${}_BI_3$, ${}_CI_3$	(0 : -s : s-a), (-s : 0 : s-b), (s-a : s-b : 0)

Now let us quote few lemmas with out proofs which are useful in proving main theorems

3. BASIC LEMMA'S

Lemma 1: Carnot's Theorem: *Suppose a conic L intersect in the side line BC at* X , X' *, CA at* Y , Y' *and AB at* Z , Z' *then* $\left(\frac{BX}{XC}\right)\left(\frac{BX'}{X'C}\right)\left(\frac{CY}{YA}\right)\left(\frac{CY'}{Y'A}\right)\left(\frac{AZ}{ZB}\right)\left(\frac{AZ'}{Z'B}\right)=1$

Lemma 2: Converse of Carnot's Theorem: *If* X , X' , Y , Y' , Z , Z' *are points on the side lines such that* $\left(\frac{BX}{XC}\right)\left(\frac{BX'}{X'C}\right)\left(\frac{CY}{YA}\right)\left(\frac{CY'}{Y'A}\right)\left(\frac{AZ}{ZB}\right)\left(\frac{AZ'}{Z'B}\right)=1$ *then the six points are lie on a conic.*

Lemma 3: *If* X , Y , Z *are the traces of a point P and* X' , Y' , Z' *are the traces of a point Q then the equation of conic through the six points* X , X' , Y , Y' , Z , Z' *is given by* $\sum_{cyclic}\left[\frac{x^2}{uu'}-\left(\frac{1}{vw'}+\frac{1}{v'w}\right)yz\right]=0$ *where the homogeneous barycentric coordinates of P and Q with reference to ABC are* $P=(u:v:w)$ *and* $Q=(u':v':w')$.

4. MAIN THEOREMS

Theorem 1: *There is conic through the six points, The three intouch points* ${}_AI$, ${}_BI$ *and* ${}_CI$ *and three extouch points* ${}_AI_1$, ${}_BI_2$ *and* ${}_CI_3$ *(for recognition sake let us call this conic as Internal Maneeal's Pedal Conic of order one).*

Proof:

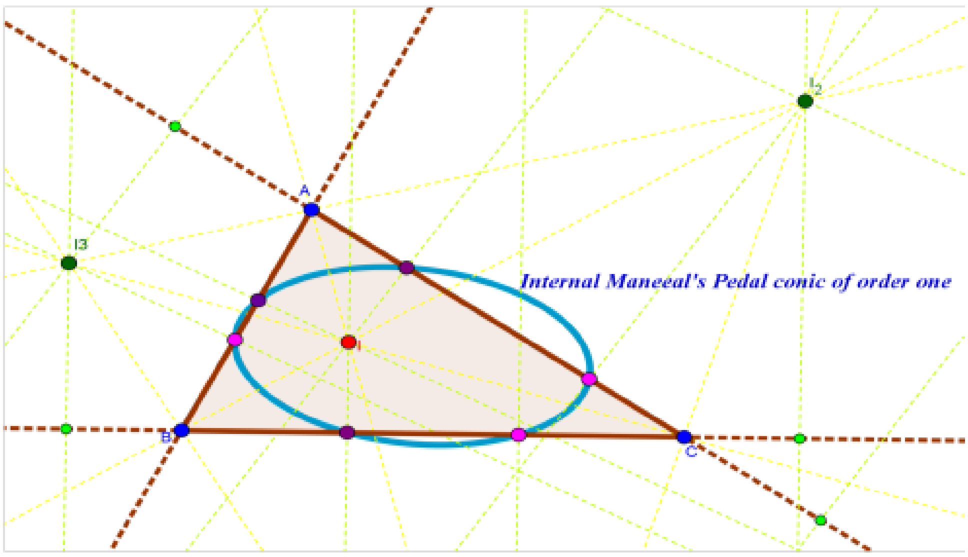

Figure -1: (Internal Maneeal's Pedal Conic of order one)

Clearly the points ${}_A\boldsymbol{I}$, ${}_A\boldsymbol{I}_1$ lies on the side BC, the points ${}_B\boldsymbol{I}$, ${}_B\boldsymbol{I}_2$ lies on the side CA and the points ${}_C\boldsymbol{I}$ ${}_C\boldsymbol{I}_3$ lies on the side AB. (see figure-1)

Hence, To prove that there is a conic through the six points ${}_A\boldsymbol{I}$, ${}_B\boldsymbol{I}$, ${}_C\boldsymbol{I}$, ${}_A\boldsymbol{I}_1$, ${}_B\boldsymbol{I}_2$ and ${}_C\boldsymbol{I}_3$,

By the converse of Carnot's theorem it is enough to prove

$$\left(\frac{B_AI}{{}_AIC}\right)\left(\frac{B_AI_1}{{}_AI_1C}\right)\left(\frac{C_BI}{{}_BIA}\right)\left(\frac{C_BI_2}{{}_BI_2A}\right)\left(\frac{A_CI}{{}_CIB}\right)\left(\frac{A_CI_3}{{}_CI_3B}\right)=1$$

Consider

$$\left(\frac{B_A I}{{}_A IC}\right)\left(\frac{B_A I_1}{{}_A I_1 C}\right)\left(\frac{C_B I}{{}_B IA}\right)\left(\frac{C_B I_2}{{}_B I_2 A}\right)\left(\frac{A_C I}{{}_C IB}\right)\left(\frac{A_C I_3}{{}_C I_3 B}\right)=\left(\frac{s-b}{s-c}\right)\left(\frac{s-c}{s-b}\right)\left(\frac{s-c}{s-a}\right)\left(\frac{s-a}{s-c}\right)\left(\frac{s-a}{s-b}\right)\left(\frac{s-b}{s-a}\right)=1$$

Hence the six points ${}_AI$, ${}_BI$, ${}_CI$, ${}_AI_1$, ${}_BI_2$ and ${}_CI_3$ lie on a conic (***Internal Maneeal's Pedal Conic of order one***)

Lemma 4: It is clear that the three intouch points are the traces of Gergonne point and the three extouch points are the traces of Nagel point and their barycentric coordinates is given by

The Gergonne point

The points of tangency of the incircle with the side lines are

${}_AI = 0 : s-c : s-b$,
${}_BI = s-c : 0 : s-a$,
${}_CI = s-b : s-a : 0$.

These can be reorganized as

${}_AI = 0 : \frac{1}{s-b} : \frac{1}{s-c}$,
${}_BI = \frac{1}{s-a} : 0 : \frac{1}{s-c}$,
${}_CI = \frac{1}{s-a} : \frac{1}{s-b} : 0$.

It follows that A_AI, B_BI, *and* C_CI intersect at a point with coordinates $\left(\frac{1}{s-a} : \frac{1}{s-b} : \frac{1}{s-c}\right)$

This is called the ***Gergonne point*** G_e of triangle ABC.

The Nagel point

The points of tangency of the excircles with the corresponding sides have coordinates

${}_AI_1 = 0 : s-b : s-c$,
${}_BI_2 = s-a : 0 : s-c$,
${}_CI_3 = s-a : s-b : 0$.

These are the traces of the point with coordinates $(s-a: s-b: s-c)$.

This is the ***Nagel point*** N_a of triangle ABC.

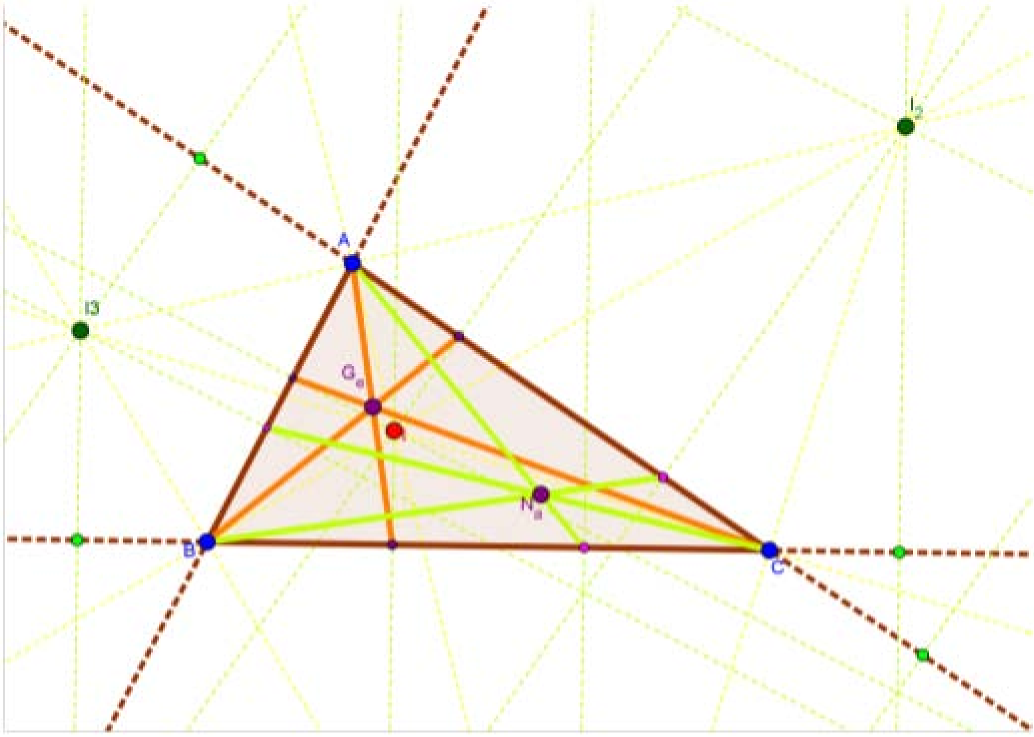

Figure-2: (G_e , N_a are Gergonne, Nagel points)

Theorem 2: The barycentric equation for the Internal Maneeal's Pedal Conic of order one is given by $x^2+y^2+z^2-\left(\frac{s-b}{s-c}+\frac{s-c}{s-b}\right)yz-\left(\frac{s-c}{s-a}+\frac{s-a}{s-c}\right)zx-\left(\frac{s-a}{s-b}+\frac{s-b}{s-a}\right)xy=0$

Proof: Clearly Internal Maneeal's pedal conic of order one passes through the six points which are the traces of Gergonne point and Nagel point

So using lemma-3 and 4.

The barycentric equation of the conic which passes through these six points is given by

$$\sum_{cyclic}\left[\frac{x^2}{\left(\frac{1}{s-a}\right)(s-a)}-\left(\frac{s-b}{s-c}+\frac{s-c}{s-b}\right)yz\right]=0$$

Further simplification gives $x^2+y^2+z^2-\left(\frac{s-b}{s-c}+\frac{s-c}{s-b}\right)yz-\left(\frac{s-c}{s-a}+\frac{s-a}{s-c}\right)zx-\left(\frac{s-a}{s-b}+\frac{s-b}{s-a}\right)xy=0$

Now by comparing with general conic with barycentric equation $Ax^2+By^2+Cz^2+2Hxy+2Gxz+2Fyz=0$

We get A= B = C =1 and

$$H=-\frac{1}{2}\left(\frac{s-a}{s-b}+\frac{s-b}{s-a}\right),\ G=-\frac{1}{2}\left(\frac{s-c}{s-a}+\frac{s-a}{s-c}\right)\ \text{and}\ F=-\frac{1}{2}\left(\frac{s-b}{s-c}+\frac{s-c}{s-b}\right)$$

And by (I),

The adjoint matrix of M,

Namely $M^{\#}=\begin{pmatrix} BC-F^2 & FG-CH & HF-BG \\ FG-CH & AC-G^2 & GH-AF \\ HF-BG & GH-AF & AB-H^2 \end{pmatrix}$

$$\begin{aligned} characterstic(M)=\lambda(M)&=GM^{\#}G^T \\ &=2(AF+BG+CH)+(G^2+H^2+F^2-2GH-2HF-2FG)-(AB+BC+CA)\end{aligned}$$

Center of the conic =

$$C(M)=GM^{\#}=\begin{pmatrix}[F(G+H-F)+BC-BG-CH]:[G(F+H-G)+AC-AF-CH]: \\ [H(G+F-H)+AB-AF-BG]\end{pmatrix}$$

By replacing the values of A, B, C , G, H, F we can find the center of this conic.

Theorem 3: *There is conic through the six points, The two extouch points ${}_BI_1$, and ${}_CI_1$ in which the A-excircle meets the sides CA and AB, the two extocuh points ${}_AI_2$, and ${}_CI_2$ in which the B-excircle meets the sides BC and AB, the two extocuh points ${}_AI_3$, and ${}_BI_3$, in which the C-excircle meets the sides BC and CA, in total six points. (for recognition sake let us call this conic as External Maneeal's Pedal Conic of order one).*

Proof: Clearly the points ${}_AI_2$, ${}_AI_3$ lies on the side BC, the points ${}_BI_1$, ${}_BI_3$ lies on the side CA and the points ${}_CI_1$, ${}_CI_2$ lies on the side AB.

Hence, To prove that there is a conic through the six points ${}_AI_2$, ${}_AI_3$, ${}_BI_1$, ${}_BI_3$ and ${}_CI_1$, ${}_CI_2$

By the converse of Carnot's theorem it is enough to prove

$$\left(\frac{B\,{}_AI_3}{{}_AI_3\,C}\right)\left(\frac{B\,{}_AI_2}{{}_AI_2\,C}\right)\left(\frac{C\,{}_BI_1}{{}_BI_1\,A}\right)\left(\frac{C\,{}_BI_3}{{}_BI_3\,A}\right)\left(\frac{A\,{}_CI_2}{{}_CI_2\,B}\right)\left(\frac{A\,{}_CI_1}{{}_CI_1\,B}\right)=1$$

Consider

$$\left(\frac{B\,{}_AI_3}{{}_AI_3\,C}\right)\left(\frac{B\,{}_AI_2}{{}_AI_2\,C}\right)\left(\frac{C\,{}_BI_1}{{}_BI_1\,A}\right)\left(\frac{C\,{}_BI_3}{{}_BI_3\,A}\right)\left(\frac{A\,{}_CI_2}{{}_CI_2\,B}\right)\left(\frac{A\,{}_CI_1}{{}_CI_1\,B}\right)=\left(\frac{s-a}{s}\right)\left(\frac{s}{s-a}\right)\left(\frac{s-b}{s}\right)\left(\frac{s}{s-b}\right)\left(\frac{s-c}{s}\right)\left(\frac{s}{s-a}\right)=1$$

Hence the six points ${}_AI_2$, ${}_AI_3$, ${}_BI_1$, ${}_BI_3$ and ${}_CI_1$, ${}_CI_2$ lie on conic (***External Maneeal's pedal Conic of order one)***

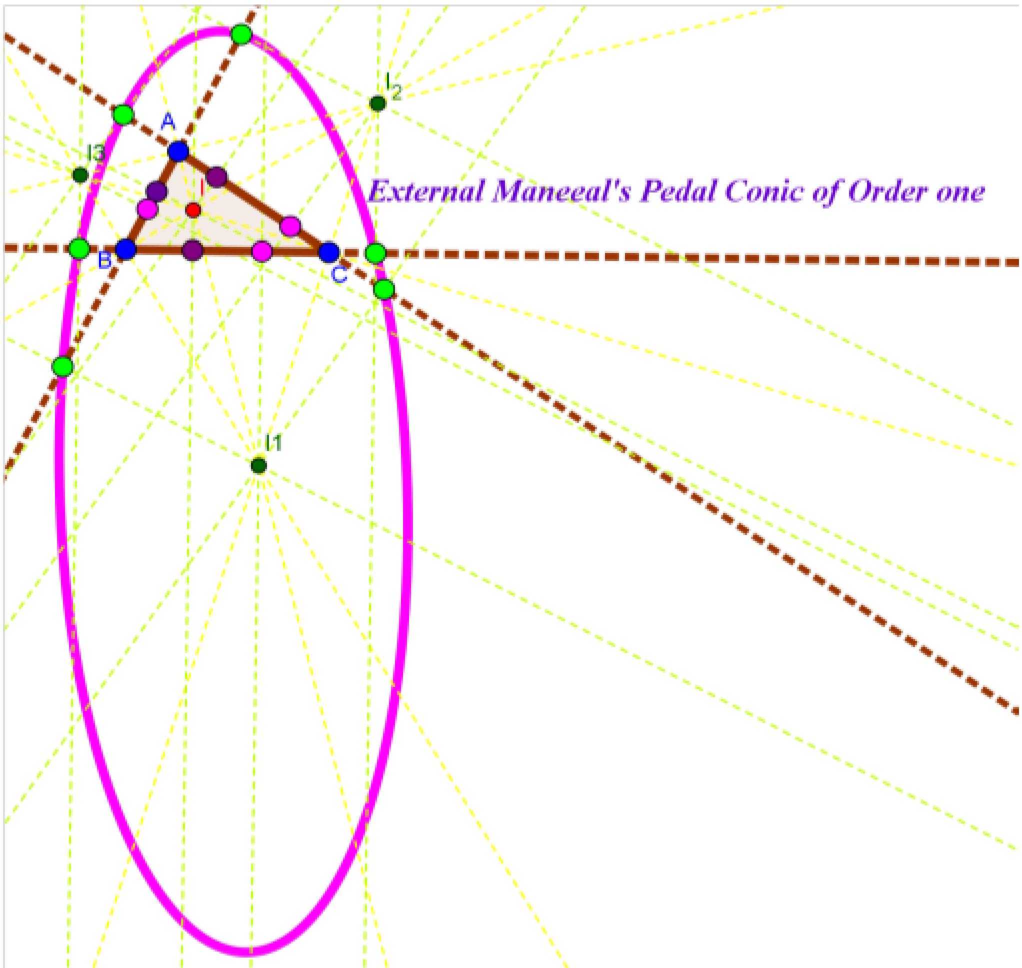

Figure -3: *(External Maneeal's Pedal Conic of order one)*

Theorem 4: The barycentric equation for the External Maneeal's Pedal Conic of order one is given by

$$x^2+y^2+z^2+\left(\frac{s}{s-a}+\frac{s-a}{s}\right)yz+\left(\frac{s}{s-b}+\frac{s-b}{s}\right)zx+\left(\frac{s}{s-c}+\frac{s-c}{s}\right)xy=0$$

Proof: If x = 0 the equation becomes as $y^2+z^2+\left(\frac{s}{s-a}+\frac{s-a}{s}\right)yz=0$

Which can be factorized as $\left[(s-a)y+sz\right]\left[sy+(s-a)z\right]=0$ (1)

This means that the External Maneeal's Pedal Conic intersects the line BC at the points $(0:-s:s-a)$ and $(0:s-a:-s)$.

These are the points ${}_AI_3$ and ${}_AI_2$.

Similiarly the conic intersects CA at ${}_BI_1$, ${}_BI_3$ and AB at ${}_CI_1$, ${}_CI_2$.

To get center of this conic we proceed as discussed in Theorem-3.

Theorem 5: *There are three conics through the three sets of six points, The three intouch points $_AI$, $_BI$, $_CI$ and any one set of three extouch points like the extouch points $_AI_1$, $_BI_1$, $_CI_1$ by A-excircle with the sides BC, CA, AB or the extouch points $_AI_2$, $_BI_2$, $_CI_2$ by B-excircle with the sides BC, CA, AB or the extouch points $_AI_3$, $_BI_3$, $_CI_3$ by C-excircle with the sides BC, CA, AB in total twelve points. (for recognition sake let us call these conics as Partial Maneeal Pedal Conics of order one).*

Proof: To prove there is a conic through three in touch points and three extouch points which are obtained by A-excircle, we proceed as follows

Clearly the points $_AI$, $_AI_1$ lies on the side BC, the points $_BI$, $_BI_1$ lies on the side CA and the points $_CI$, $_CI_1$ lies on the side AB.

Hence, To prove that there is a conic through the six points $_AI$, $_AI_1$, $_BI$, $_BI_1$ and $_CI$, $_CI_1$

By the converse of Carnot's theorem it is enough to prove

$$\left(\frac{B\,_AI}{_AI\,C}\right)\left(\frac{B\,_AI_1}{_AI_1\,C}\right)\left(\frac{C\,_BI}{_BI\,A}\right)\left(\frac{C\,_BI_1}{_BI_1\,A}\right)\left(\frac{A\,_CI}{_CI\,B}\right)\left(\frac{A\,_CI_1}{_CI_1\,B}\right)=1$$

Consider

$$\left(\frac{B\,_AI}{_AI\,C}\right)\left(\frac{B\,_AI_1}{_AI_1\,C}\right)\left(\frac{C\,_BI}{_BI\,A}\right)\left(\frac{C\,_BI_1}{_BI_1\,A}\right)\left(\frac{A\,_CI}{_CI\,B}\right)\left(\frac{A\,_CI_1}{_CI_1\,B}\right)=\left(\frac{s-c}{s-b}\right)\left(\frac{s-b}{s-c}\right)\left(\frac{s-c}{s-a}\right)\left(\frac{s-b}{s}\right)\left(\frac{s-a}{s-b}\right)\left(\frac{s}{s-c}\right)=1$$

Hence the six points $_AI$, $_AI_1$, $_BI$, $_BI_1$ and $_CI$, $_CI_1$ lie on conic (*A-Partial Maneeal's pedal conic of order one).*

similiarly we can prove that the six points $_AI$, $_AI_2$, $_BI$, $_BI_2$ and $_CI$, $_CI_2$ lie on conic (*B-Partial Maneeal's Pedal Conic of order one)* and also the six points $_AI$, $_AI_3$, $_BI$, $_BI_3$ and $_CI$, $_CI_3$ lie on conic (*C-Partial Maneeal's Pedal Conic of order one).*

Theorem 6: *The barycentric equation for Partial Maneeal Pedal Conic of order one is given by*

A-Partial Maneeal's Pedal Conic of order one is….

$$s(s-a)x^2-(s-b)(s-c)(y^2+z^2)+\left[(s-b)^2+(s-c)^2\right]yz-\left[2s(s-c)-ab\right]zx-\left[2s(s-b)-ac\right]xy=0$$

B-Partial Maneeal's Pedal Conic of order one is…

$$s(s-b)y^2-(s-c)(s-a)(z^2+x^2)+\left[(s-c)^2+(s-a)^2\right]zx-\left[2s(s-a)-bc\right]xy-\left[2s(s-c)-ba\right]yz=0$$

C-Partial Maneeal's Pedal Conic of order one is…

$$s(s-c)z^2-(s-a)(s-b)(x^2+y^2)+\left[(s-a)^2+(s-b)^2\right]xy-\left[2s(s-b)-ca\right]yz-\left[2s(s-a)-cb\right]zx=0$$

Proof:

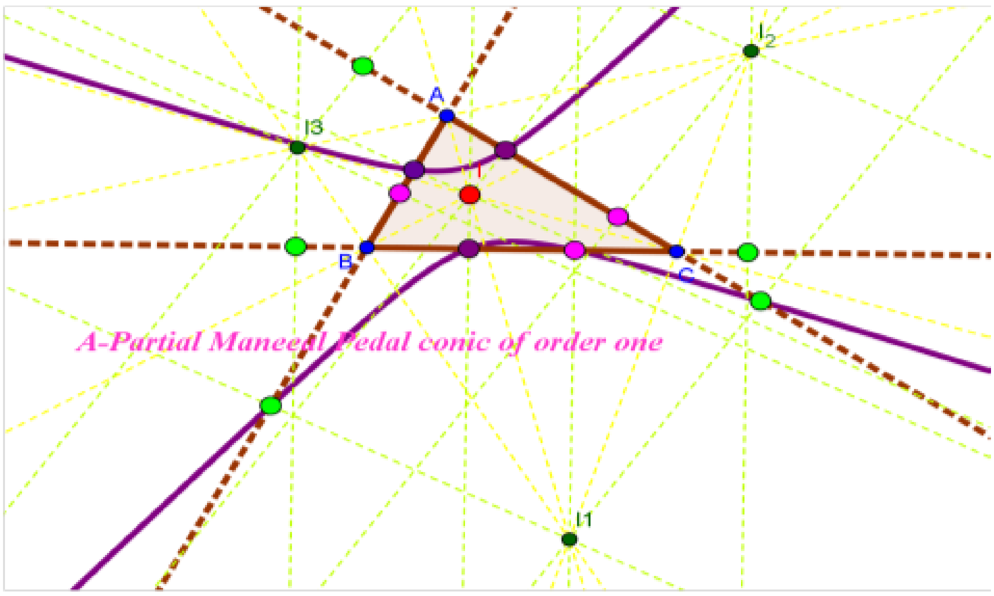

Figure-4: (A-Partial Maneel Pedal Conic of order one**)**

Clearly A-Partial Maneeal Pedal Conic passes through the three points ${}_AI$, ${}_BI$, ${}_CI$ which are the traces of the Gergonne point (G_e) whose bary centric coordinates are

$$\left(\frac{1}{s-a} \;:\; \frac{1}{s-b} \;:\; \frac{1}{s-c}\right)$$

And also A- Partial Maneeal Pedal Conic passes through the another three points ${}_AI_1$, ${}_BI_1$, ${}_CI_1$

Which are the traces of the External Gergonne Point whose bary centric coordinates are $\left(\frac{-1}{s} \;:\; \frac{1}{s-c} \;:\; \frac{1}{s-b}\right)$

So to find the barycentric equation of the A-Partial Maneeal Pedal conic of order one,

we can make use of lemma-3,

By considering $Q=(u':v':w')=\left(\frac{-1}{s} \;:\; \frac{1}{s-c} \;:\; \frac{1}{s-b}\right)$ $Q=(u':v':w')=\left(\frac{-1}{s} \;:\; \frac{1}{s-c} \;:\; \frac{1}{s-b}\right)$

i.e., The equation of the conic is $\sum_{cyclic}\left[\frac{x^2}{uu'}-\left(\frac{1}{vw'}+\frac{1}{v'w}\right)yz\right]=0$

Further simplification gives

$$s(s-a)x^2-(s-b)(s-c)(y^2+z^2)+\left[(s-b)^2+(s-c)^2\right]yz-\left[2s(s-c)-ab\right]zx-\left[2s(s-b)-ac\right]xy=0$$

similiarly we can find the barycentric equations of B-Partial, C-Partial Maneeal Pedals Conic of order one.

Hence proved

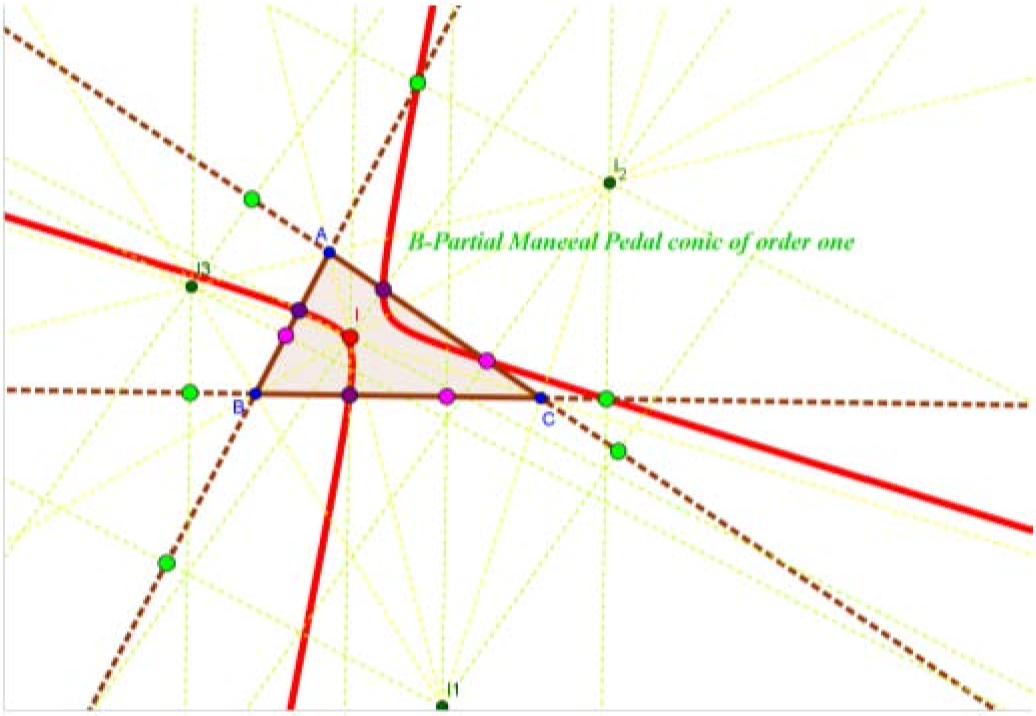

Figure-5: (B-Partial Maneeal's Pedal Conic of order one)

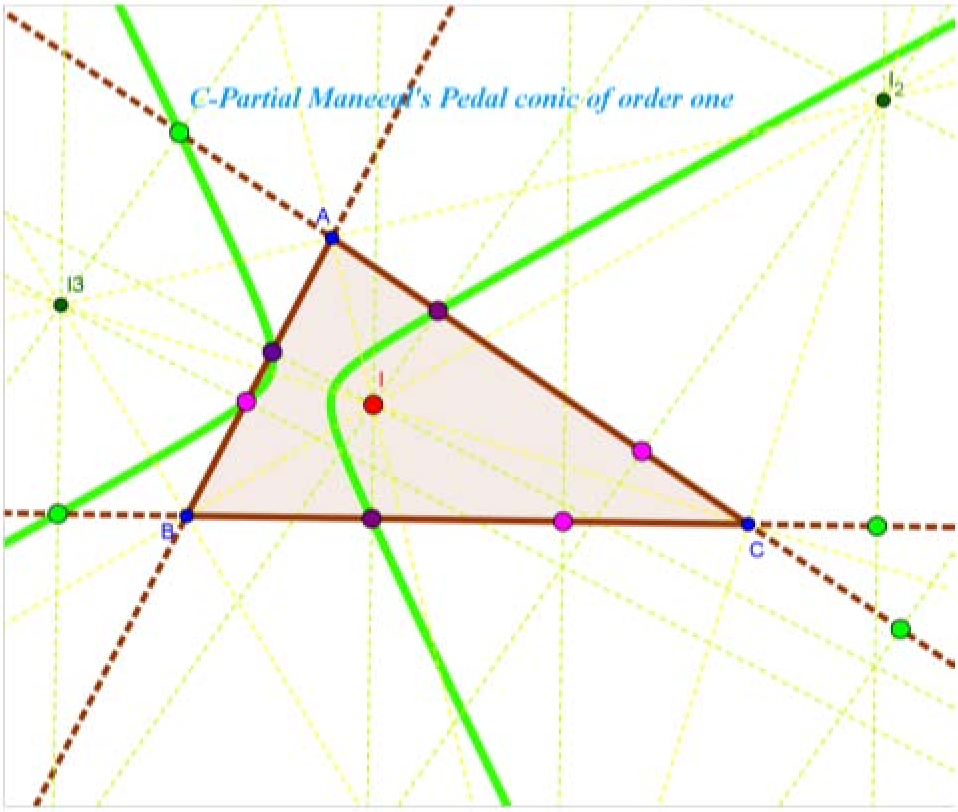

Figure-6: (C-Partial Maneeal's Pedal Conic of order one)

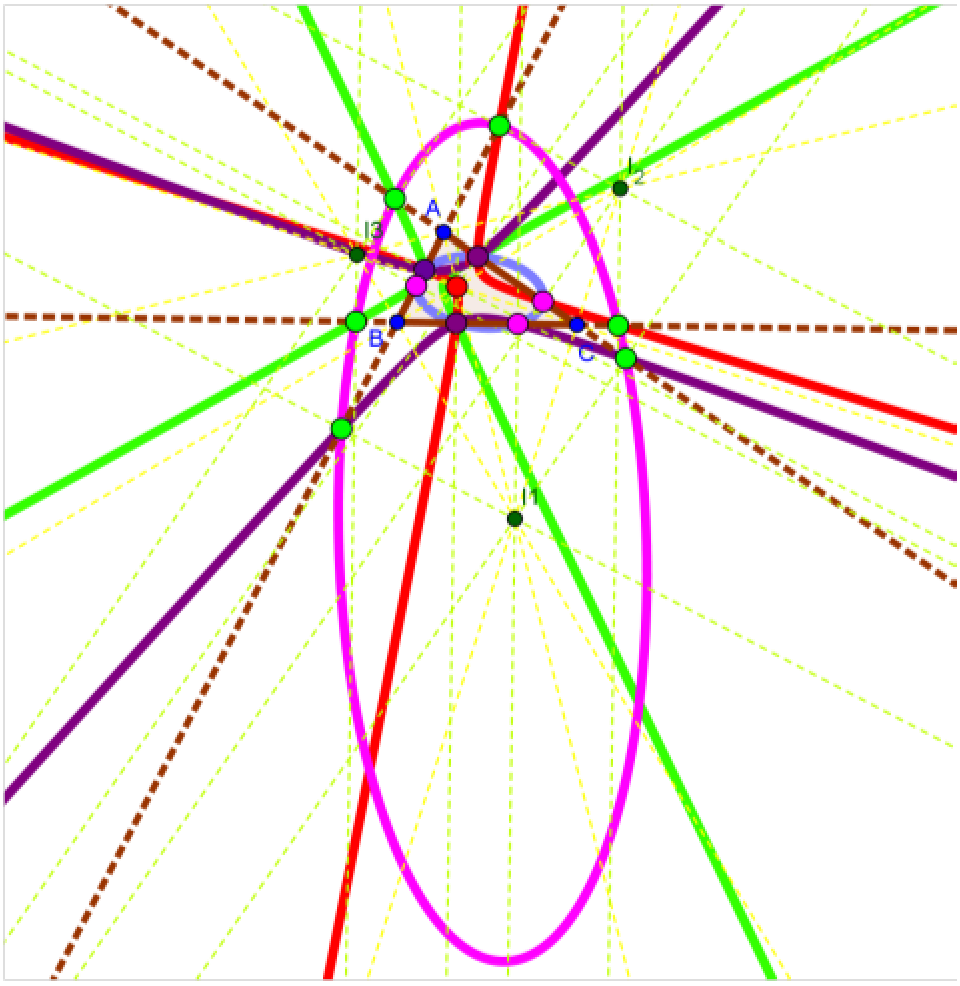

Figure-7: (Maneeal Pedal Conics of order one)

MANEEAL'S CONIC

Theorem 7: *Let M_p, M_q are two Maneeal's Points of order p, q then there always exists a conic through the traces of M_p, M_q, and the conic is called as Maneeal's conic.*

Proof: Let D_p, E_p, F_q and D_q, E_q, F_q are the traces of M_p, M_q respectively on the sides BC, CA, AB.

By the definition of Maneeal's Point of order p, q

We have $\frac{BD_p}{D_pC}=\frac{c^p}{b^p}, \frac{CE_p}{E_pA}=\frac{a^p}{c^p}, \frac{AF_p}{F_pB}=\frac{b^p}{a^p}$ and $\frac{BD_q}{D_qC}=\frac{c^q}{b^q}, \frac{CE_q}{E_qA}=\frac{a^q}{c^q}, \frac{AF_q}{F_qB}=\frac{b^q}{a^q}$

Hence, To prove there is conic through the six points D_p, E_p, F_q , D_q, E_q, F_q,

By the converse of Carnot's theorem it is enough to prove

$$\left(\frac{BD_p}{D_pC}\right)\left(\frac{BD_q}{D_qC}\right)\left(\frac{CE_p}{E_pA}\right)\left(\frac{CE_q}{E_qA}\right)\left(\frac{AF_p}{F_pB}\right)\left(\frac{AF_q}{F_qB}\right)=1$$

Consider

$$\left(\frac{BD_p}{D_pC}\right)\left(\frac{BD_q}{D_qC}\right)\left(\frac{CE_p}{E_pA}\right)\left(\frac{CE_q}{E_qA}\right)\left(\frac{AF_p}{F_pB}\right)\left(\frac{AF_q}{F_qB}\right)=\left(\frac{c^p}{b^p}\right)\left(\frac{a^p}{c^p}\right)\left(\frac{b^p}{a^p}\right)\left(\frac{c^q}{b^q}\right)\left(\frac{a^q}{c^q}\right)\left(\frac{b^q}{a^q}\right)=1$$

Hence the six points D_p, E_p, F_q , D_q, E_q, F_q, lie on a conic (***Maneeal's conic)***

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