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FUZZY CDS ALGORITHM FOR A FLOWSHOP SCHEDULING PROBLEM

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ABSTRACT

In job sequencing for a flowshop, processing times are generally not known exactly. Only values occur in estimated intervals. So Fuzzy numbers come in handy to represent these interval values. In this paper, the fuzzified Campbell, Dudek and Smith (CDS)job sequencing algorithm is employed with octagonal fuzzy processing times. The results are deterministic sequences but the sequence performance criteria like the makespan and mean flow time are fuzzy and are calculated using fuzzy arithmetic.

Keywords: *Flow-shop scheduling, CDS Algorithm, Octagonal fuzzy numbers.*

Classification: MSC 2010 No.: *90C70, 97M40.*

1. INTRODUCTION

Scheduling problems occurring in real life applications generally are flow-shop scheduling problems. Each job has the same routing through machines and the sequence of operations is fixed in a flow-shop. In the most studies concerned with the scheduling problems, processing times were taken as certain and fixed value. But in the real world application, information is often ambiguous, vague and imprecise. Several techniques are proposed for managing uncertainty. To solve vague situations in real problems, the first systemic approach related to fuzzy set theory was successfully applied in many areas such as in scheduling problems. In recent studies, scheduling problems were fuzzified by using the concept of fuzzy due date and processing times.Dumitru and Luban (1982) investigated the application of fuzzy mathematical programming model on the problem of the production scheduling.

Especially from the beginning of the 1990s fuzzy logic applications on the scheduling problems are increased. Han *et al*.(1994);Ishibuchi *et al.* (1994 a, b); Ishii *et al.*(1992) and Murata *et al.* (1997) used fuzzy due-date in their studies. Adamapoulos and Pappis (1996); Kuroda and Wang(1996);Hong *et al.* (1995); Hong and Chuang (1996); Ishibuchi *et al.* (1995), McCahon and Lee(1990,1992); Izzettin Temiz *et al*.(1994);Murata *et al*.(1996), Stanfield *et al*.(1996) fuzzified scheduling problems by using fuzzy processing times. Moreover Cheng *et al*.(1994); Dubois *et al.*(1995); Ishii and Tada (1995); Watanabe *et al*.(1992) used fuzzy precedence relations in scheduling problems.

In this paper we have adopted the well known flow shop job sequencing heuristic algorithm of Campbell, Dudek and Smith[2] that is modified to accept triangular(TFN) and trapezoidal fuzzy numbers (TrFN) as processing times by C.S.McCahon[5] to which is now employed with octagonal fuzzy numbers(OFN).The CDS algorithm is chosen because this approximate sequencing method provides a practical solution to large sequencing problems that cannot be solved by exact procedures such as branch and bound algorithm. Solutions produced by CDS algorithm are optimal or near optimal.

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2. FUZZY REPRESENTATION AND NOTATIONS

In this paper, normalized octagonal fuzzy numbers (OFNs) are used to represent the fuzzy processing times. Triangular (TFNs) and trapezoidal (TrFNs) could also be used as these are special cases of OFNs. The OFNs are represented by (a, b, c, d, e, f, g, h).The membership function is 1 at d to e, k at b to c and f to g & zero at the two end points a and h. The graph of a OFN is given in Figure 1, where $\mu(x)$ is the membership function and x is the processing time. The arithmetics of fuzzy numbers defined by A.Kaufmann [3] are used to define the fuzzy arithmetic of OFN's and can be found in [4, 6].

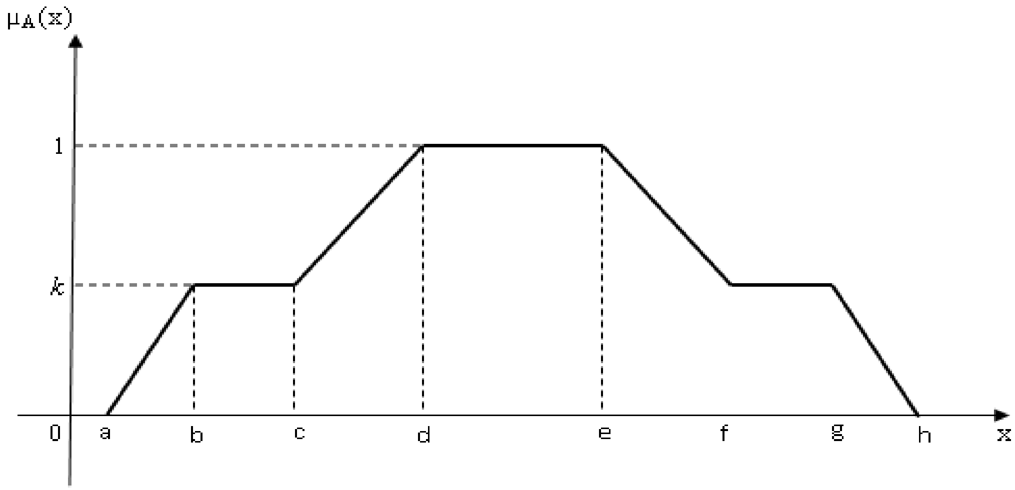

Figure-1: Membership Function of OFN

We use the following notations in the work.

$\tilde{A}$ = Fuzzy Number A

$\mu_{\tilde{A}}(x)$ = Membership function of $\tilde{A}$

J = Set of jobs to be processed

n = Number of jobs in J

m = Number or machines

$\tilde{p}_{ij}$ = Fuzzy processing time for the i-th job at machine j.

$\tilde{C}_{ij}$ = Fuzzy completion time for the i-th job at machine j.

$(+)$ = Fuzzy addition

$(-)$ = Fuzzy subtraction

$\overset{n}{\underset{i=1}{(+)}}$ = Fuzzy summation

$\widetilde{\max}$ = Fuzzy maximum

Comparing the fuzzy numbers using one of the ranking methods available for OFNs allows the flowshop job sequencing problem with fuzzy processing times to be solved completely. To compare the fuzzy numbers, the GMV for a OFN is calculated as found in Malini [4].

$$M_0^{oct}(\tilde{A}) = \frac{1}{4}\left[\left(a_1 + a_2 + a_7 + a_8\right)k + \left(a_3 + a_4 + a_5 + a_6\right)(1-k)\right] \tag{2.1}$$

where 0 < k < 1, is the Generalised Mean Value(GMV) of the octagonal fuzzy number.

The fuzzy number with the higher GMV is the ranked higher than the fuzzy number with the lower GMV. If the GMVs happen to be equal, the spread (found in Dhanalakshmi [1]), $s(\tilde{A})$, is calculated for each fuzzy number and the one with smaller spread is considered small.

3. THE N-JOB M-MACHINE FLOWSHOP AND THE PERFORMANCE CRITERIA:

The n-job m-machine flow shop problem is where n jobs must be processed through m machines. All jobs pass through machine 1 first, then machine 2 and so on up to machine m.Jobs may not pass each other, and it is assumed no new jobs arrive in the makespan of the n jobs. The goal is to sequence and schedule the jobs such that some sequence performance evaluators are optimized. In this work fuzzy makespan($\tilde{M}$) and fuzzy mean flow time ($M\tilde{F}T$) are the sequence performance evaluators used to compare alternative sequences and to interpret the impact of the fuzzy processing times on job completion time, flow time and makespan.

The fuzzy makespan is calculated as $\tilde{M} = \underset{i \in J}{\tilde{\max}}\, \tilde{C}_{im}$ (3.1)

Where each $\tilde{C}_{im}$ is the completion time for the i^{th} job at the m^{th} machine.

The fuzzy mean flow time is calculated as $$M\tilde{F}T = \frac{\overset{n}{\underset{i=1}{(+)}} \tilde{C}_{im}}{n} \quad (3.2)$$

In the process, the fuzzy completion time for job i at machine j, $\tilde{C}_{ij}$, is calculated as

$$\tilde{C}_{ij} = \tilde{\max}\left\{ \tilde{C}_{i-1,j} ; \tilde{C}_{i,j-1} \right\} (+)\ \tilde{p}_{ij} \quad (3.3)$$

assuming job i-1 precedes job i in the sequence.

4. THE FUZZIFIED CAMPBELL, DUDEK AND SMITH ALGORITHM

In this for the n-job and m-machines flowshop problem, we create a series of m-1 auxiliary n-job, 2 –machines problems and then apply Johnson's algorithm for each of the auxiliary problem as per the following steps.

Step-1: calculate the pseudo –machine processing times for each l^{th} auxiliary problem, l = 1, 2,…,m-1 as:

$$\beta_{i1}^{l} = \overset{l}{\underset{j=1}{(+)}}\ \tilde{p}_{ij} \quad (4.1)$$

$$\beta_{i2}^{l} = \overset{m}{\underset{j=m+1-l}{(+)}}\ \tilde{p}_{ij} \quad (4.2)$$

where $\tilde{p}_{ij}$ is the processing time of job i at machine j and (+) denotes fuzzy addition.

Step-2: GMVs of the fuzzy processing times are found before applying Johnson's algorithm to the two pseudo-machines using β_{i1}^{l} and β_{i2}^{l} as the processing times to get an optimal sequence.

Step-3: Calculate the makespan of the l-th sequence found in Step 2.

Step-4: Compare the makespans of the m-1 sequences. Select the minimum makespan.

5. ILLUSTRATION

The octagonal processing times of a four job, four machines scheduling problem is presented in Table 1.

Table-1: The illustrative problem

Job	Machines			
	M1	M2	M3	M4
1	(2,3,4,5,6,7,8,9)	(1,3,5,6,8,10,11,12)	(7,9,11,12,14,16,17,19)	(5,6,7,8,10,11,12,13)
2	(5,7,9,11,12,13,14,15)	(8,9,10,11,12,13,14,15)	(11,12,13,14,16,17,18,19)	(9,10,11,12,14,15,16,17)
3	(3,5,7,8,10,11,14,15)	(1,2,5,7,8,9,10,12)	(13,14,16,17,19,21,22,23)	(2,4,5,7,8,10,11,12)
4	(1,3,5,7,9,11,13,15)	(2,3,5,7,8,10,11,13)	(7,8,9,10,11,13,16,17)	(3,5,6,7,8,10,11,13)

Using the fuzzified Campbell, Dudek and Smith algorithm, when l=1 yields the results of Table2.Using the ranking method of octagonal fuzzy numbers for k = 0.5 in equation (2.1), the GMV of the above pseudo-machines processing times are given in Table3.

Table-2

Job	Pseudo-machine	
	1	2
	$\beta_{i1}^{1} = \tilde{p}_{i1}$	$\beta_{i2}^{1} = \tilde{p}_{i4}$
1	(2,3,4,5,6,7,8,9)	(5,6,7,8,10,11,12,13)
2	(5,7,9,11,12,13,14,15)	(9,10,11,12,14,15,16,17)
3	(3,5,7,8,10,11,14,15)	(2,4,5,7,8,10,11,12)
4	(1,3,5,7,9,11,13,15)	(3,5,6,7,8,10,11,13)

Table-3

Job	Pseudo-machine	
	1	2
	$M^{oct}(\beta^1_{i1})$	$M^{oct}(\beta^1_{i2})$
1	5.5	9
2	10.75	13
3	9.125	7.375
4	8	7.875

Using Johnson's algorithm, the optimal sequence is S1: 1-2-4-3 when *l*=1.

Continuing when *l*=2, we get Table4 and the GMVs are given in Table 5.The optimal sequence when *l* = 2, is therefore S2:4-3-2-1.

Table-4

Job	Pseudo-machine	
	1	2
	$\beta^2_{i1} = \tilde{p}_{i1}(+)\tilde{p}_{i2}$	$\beta^2_{i2} = \tilde{p}_{i3}(+)\tilde{p}_{i4}$
1	(3,6,9,13,14,17,19,21)	(12,15,18,20,24,27,29,32)
2	(13,16,19,22,24,26,28,30)	(20,22,24,26,30,32,34,36)
3	(4,7,12,16,18,20,24,27)	(15,18,21,24,27,31,33,35)
4	(3,6,10,14,17,21,24,28)	(10,13,15,17,19,23,27,30)

Table-5

Job	Pseudo-machine	
	1	2
	$M^{oct}(\beta^2_{i1})$	$M^{oct}(\beta^2_{i2})$
1	12.75	22.125
2	22.25	28
3	16	25.5
4	15.375	19.25

Proceeding as above when *l*=3, we get the results in Table6.Yhe GMVs are listed in Table 7.The optimal sequence when *l*=3 is then S3:1-2-3-4 by Johnson's algorithm.

Table-6

Job	Pseudo-machine	
	1	2
	$\beta^3_{i1} = \tilde{p}_{i1}(+)\tilde{p}_{i2}(+)\tilde{p}_{i3}$	$\beta^3_{i2} = \tilde{p}_{i2}(+)\tilde{p}_{i3}(+)\tilde{p}_{i4}$
1	(10,15,20,25,28,33,36,40)	(13,18,23,26,32,37,40,44)
2	(24,28,32,36,40,43,46,49)	(28,31,34,37,42,45,48,51)
3	(17,21,28,33,37,41,46,50)	(16,20,26,31,35,40,43,47)
4	(10,14,19,24,28,34,40,45)	(12,16,20,24,27,33,38,43)

Table-7

Job	Pseudo-machine	
	1	2
	$M^{oct}(\beta^3_{i1})$	$M^{oct}(\beta^3_{i2})$
1	25.875	29.125
2	37.25	39.5
3	34.125	32.25
4	26.75	26.625

The $\tilde{M}$ s , $\widetilde{MFT}$ s and the GMVs of the three sequences S1,S2,S3 obtained in the pseudo-machine problems are calculated using equations(3.1) – (3.3) and are listed in Table 8.

Table-8

Sequence	$\tilde{M}$	$M^{oct}(\tilde{M})$	$\widetilde{MFT}$	$M^{oct}(\widetilde{MFT})$
S1:1-2-4-3	(48,57,66,75,84,94,103,110)	79.625	(34,41.25,48.25,54.75,62.5,69.5,75.5,81.5)	58.41
S2:4-3-2-1	(48,56,66,75,87,97,108,117)	81.75	(32.25,39.25,47.5,55.25,63.75,72.5,82,89.75)	60.28
S3:1-2-3-4	(44,54,63,72,81,91,98,106)	76.375	(33.75,41.25,48.5,55.25,63,70.25,75.75,81.25)	58.68

The fuzzy processing and completion times for each of these sequencesS1,S2,S3 in support of these evaluations can be found in Tables (9) – (11)respectively.Here both S3 and S1 happen to be best sequences since S3 has the smallest GMV of makespan and S1 has the smallest GMV of mean flow time.Note that S3's results are very close to S1's. It shows that it is important to have the first two jobs in the sequence be 1 , 2 in that order.The solutions are near-optimal.

Table-9

Parameter	Job			
	1	2	4	3
$\tilde{p}_{i1}$	(2,3,4,5,6,7,8,9)	(5,7,9,11,12,13,14,15)	(3,5,7,8,10,11,14,15)	(1,3,5,7,9,11,13,15)
$\tilde{C}_{i1}$	(2,3,4,5,6,7,8,9)	(7,10,13,16,18,20,22,24)	(10,15,20,24,28,31,36,39)	(11,18,25,31,37,42,49,54)
$\tilde{p}_{i2}$	(1,3,5,6,8,10,11,12)	(8,9,10,11,12,13,14,15)	(2,3,5,7,8,10,11,13)	(1,2,5,7,8,9,10,12)
$\tilde{C}_{i2}$	(3,6,9,11,14,17,19,21)	(15,19,23,27,30,33,36,39)	(17,22,28,34,38,43,47,52)	(18,24,33,41,46,52,57,64)
$\tilde{p}_{i3}$	(7,9,11,12,14,16,17,19)	(11,12,13,14,16,17,18,19)	(7,8,9,10,11,13,16,17)	(13,14,16,17,19,21,22,23)
$\tilde{C}_{i3}$	(10,15,20,23,28,33,36,40)	(26,31,36,41,46,50,54,58)	(33,39,45,51,57,63,70,75)	(46,53,61,68,76,84,92,98)
$\tilde{p}_{i4}$	(5,6,7,8,10,11,12,13)	(9,10,11,12,14,15,16,17)	(3,5,6,7,8,10,11,13)	(2,4,5,7,8,10,11,12)
$\tilde{C}_{i4}$	(15,21,27,31,38,44,48,53)	(35,41,47,53,60,65,70,75)	(38,46,53,60,68,75,81,88)	(48,57,66,75,84,94,103,110)

Table-10

Parameter	Job			
	4	3	2	1
$\tilde{p}_{i1}$	(1,3,5,7,9,11,13,15)	(3,5,7,8,10,11,14,15)	(5,7,9,11,12,13,14,15)	(2,3,4,5,6,7,8,9)
$\tilde{C}_{i1}$	(1,3,5,7,9,11,13,15)	(4,8,12,15,19,22,27,30)	(9,15,21,26,31,35,41,45)	(11,18,25,31,37,42,49,54)
$\tilde{p}_{i2}$	(2,3,5,7,8,10,11,13)	(1,2,5,7,8,9,10,12)	(8,9,10,11,12,13,14,15)	(1,3,5,6,8,10,11,12)
$\tilde{C}_{i2}$	(3,6,10,14,17,21,24,28)	(5,10,17,22,27,31,37,42)	(17,24,31,37,43,48,55,60)	(18,27,36,43,51,58,66,72)
$\tilde{p}_{i3}$	(7,8,9,10,11,13,16,17)	(13,14,16,17,19,21,22,23)	(11,12,13,14,16,17,18,19)	(7,9,11,12,14,16,17,19)
$\tilde{C}_{i3}$	(10,14,19,24,28,33,40,45)	(23,28,35,41,47,54,62,68)	(34,40,48,55,63,71,80,87)	(41,49,59,67,77,87,97,106)
$\tilde{p}_{i4}$	(3,5,6,7,8,10,11,13)	(2,4,5,7,8,10,11,12)	(9,10,11,12,14,15,16,17)	(5,6,7,8,10,11,12,13)
$\tilde{C}_{i4}$	(13,19,25,31,36,43,51,58)	(25,32,40,48,55,64,73,80)	(43,50,59,67,77,86,96,104)	(48,56,66,75,87,97,108,117)

Table-11

Parameter	Job			
	1	2	3	4
$\tilde{p}_{i1}$	(2,3,4,5,6,7,8,9)	(5,7,9,11,12,13,14,15)	(3,5,7,8,10,11,14,15)	(1,3,5,7,9,11,13,15)
$\tilde{C}_{i1}$	(2,3,4,5,6,7,8,9)	(7,10,13,16,18,20,22,24)	(10,15,20,24,28,31,36,39)	(11,18,25,31,37,42,49,54)
$\tilde{p}_{i2}$	(1,3,5,6,8,10,11,12)	(8,9,10,11,12,13,14,15)	(11,12,13,14,16,17,18,19)	(9,10,11,12,14,15,16,17)
$\tilde{C}_{i2}$	(3,6,9,11,14,17,19,21)	(15,19,23,27,30,33,36,39)	(26,31,36,41,46,50,54,58)	(35,41,47,53,60,65,70,75)
$\tilde{p}_{i3}$	(7,9,11,12,14,16,17,19)	(11,12,13,14,16,17,18,19)	(13,14,16,17,19,21,22,23)	(2,4,5,7,8,10,11,12)
$\tilde{C}_{i3}$	(10,15,20,23,28,33,36,40)	(26,31,36,41,46,50,54,58)	(39,45,52,58,65,71,76,81)	(41,49,57,65,73,81,87,93)
$\tilde{p}_{i4}$	(5,6,7,8,10,11,12,13)	(9,10,11,12,14,15,16,17)	(2,4,5,7,8,10,11,12)	(3,5,6,7,8,10,11,13)
$\tilde{C}_{i4}$	(15,21,27,31,38,44,48,53)	(35,41,47,53,60,65,70,75)	(41,49,57,65,73,81,87,93)	(44,54,63,72,81,91,98,106)

6. CONCLUSION

In the situations where one cannot exactly specify the job processing times as deterministic numbers, the processing times can naturally be expressed as fuzzy numbers. In this work, the CDS job sequencing algorithm which is modified to accept fuzzy processing times is used with octagonal fuzzy numbers. Deterministic sequences were developed and then fuzzy makespan and fuzzy job mean flow time were calculated using fuzzy arithmetic and we could find optimal or near-optimal solutions. When the flow shop problem is fairly large, exact methods are not found to be handy. In this case, we can rely on this algorithm to get a solution. We can also interpret the result using more than one approximation.

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