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EXTENDED EDGE VERTEX CORDIAL LABELING OF GRAPH

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ABSTRACT

A binary labeling that assigns 0 or 1 to each vertex of a graph with certain condition called as pairity condition is called as vertex binary labeling. Here we discuss a labeling that give some natural numbers as labels to edges but results in binary labeling of vertices. This graph labeling is called as extended edge vertex cordial (eevc) labeling and we show that path Pn, Cycles C_n, $K_{1,n}$, K_2,n,snskes on C3 i.e. S(C3,n) have eevc labeling.

Key words: *edge, vertex, cordial, graph, wheel, path, label.*

Subject Classification (AMS): *05C78.*

1. INTRODUCTION

All graphs considered here are simple , finite, connected and undirected. For graph terminology we refer [9]. Let G be a (p, q) graph. Define bijective function f: E(G)→{0,1,..,q-1}. This introduces binary vertex label function f*: V(G)→{0,1} given by f*(u) = $\sum_{(uv)\in E(G)} f(uv)(mod 2)$ with further condition that| $v_f(0)$-$v_f(1)$|≤1. $v_f(i)$ stands for number of vertices labeled with i= 0,1.Then f is called as extended edge vertex cordial (eevc) labeling. The graph that admits eevc labeling is called as eevc graph.

We consider following points that gives us liberty to label any edge just as an even unmber or an odd number. This gives versatile eevc labeling of any graph. The particular odd number or even number must be from range 0 to q-1 and can be used only once.

i) odd sum of odd numbers is always congruent to 1(mod 2)
ii) even sum of odd numbers is always congruent to 0(mod2)
iii) sum of even numbers is congruent to 0(mod2).

2. DEFINITIONS

2.1 path P_n is a sequence of vertices and edges given by $v_1e_1v_2e_2\ldots..v_{n-1}e_{n-1}v_n$

2.2 cycle C_n is a closed path with $v_n = v_1$. It has n vertices and n edges.

2.3 Snake S(c3,n) . A graph S(C_3,n) is a snake of length n on C_3 . It is obtained from a path P_{n+1}=(v_1,v_2,..v_{n+1}) by joining vertices v_i and v_{i+1} to new vertex w_i (i= 1,2,…n) giving edges p_i =(v_iw_i) and edge q_i=(w_iv_{i+1}).S(C3,n) It has |E| =3n and |V| =2n+1

2.4 K1, n is commonly called as Star graph has n pendent edges incident to the same veretex.It has one vertex of degree n and all other vertices of degree one.

2.5 $K_{2,n}$ Known as bistar has two copies of K1,n whose n-degree vertex is joined by an edge. |E| = 2n+1 and |V| = 2n+2 .

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3. THEOREMS WITH PROOF

Theorem 3.1: Pn is eevc except for n≡2(mod4)

Proof: For n≥we define label as follows. The numbers are from 0 to n-2 and without repeatition. f(ej) = even number for j ≡ 0, 1(mod4) and an odd number otherwise.

The resultant binary vertex distribution is as follows:

Case n ≡ 0(mod4) Then n = 4t, $v_f(0)=2t=v_f(1)$

Case n ≡ 1(mod4) Then n = 1+4t, $v_f(0) = 2t+1, v_f(1)= 2t$

Case n ≡ 3(mod4) Then n = 3+ 4t, $v_f(0) =2t+1$ and $v_f(1) =2t+2$#

Theorem 3.2: Cycle Cn is eevc for all values of n≥3 other than n ≡ 2(mod4).

Proof: A cycle is given by $v_1e_1v_2e_2\ldots..v_{n-1}e_{n-1}v_n =v_{1.}$

We give below the particular labelling of consecutive edges on C3, C4, C5, C7 as follows. 1, 2, 0 for C3
1, 3, 2, 0 for C4
1, 3, 2, 0, 4 for C5
1, 3, 2, 0, 4, 5, 6 for C7.
1, 3, 2, 0, 4, 5, 6, 7 for C8.Note that here $f(e_7) = 6$, $f(e_8) = 7$.

Case n ≡ 0(mod4)
Let n =4x.and t = x-2
Insert n-8 new edges p1,p2,p3…pk between e7 and e8.
Last t new edges are labelled as odd number each. For rest of new edges we define
f(pi) = even number for i≡0,1(mod3)
f(pi) = odd number otherwise.

The resultant binary vertex distribution is as follows:

Case n ≡ 0(mod4) Then n = 4t, $v_f(0) = 2t = v_f(1)$

Case n ≡ 1(mod4) Then n = 1+4t, $v_f(0) = 2t+1$, $v_f(1) = 2t$

Case n ≡ 3(mod4) Then n = 3+ 4t, $v_f(0) = 2t+1$ and $v_f(1) = 2t +2$#

Theorem 3.3: $S(C_3,n)$ is eevc.

Proof: **f**(vivi+1) = even number
f(pi) = odd number
f(qi) = even number for even i and odd number otherwise.
The resultant vertex binary numbers are for n ≡ 1 (mod2) $v_f(0) = n$ and $v_f(1) = n+1$
for n ≡ 0(mod2) $v_f(0) = n+1$ and $v_f(1) = n$. #

Theorem 3.4: $K_{1,n}$ is eevc except for n ≡ 1(mod 4)

Proof: We just have to label edges as numbers from 0 to n. The binary vertex distribution is given by

Case n ≡ 0(mod4) Then n = 4t, |V| = 4t+1, $v_f(0) = 2t+1, v_f(1) = 2t$

Case n ≡ 2(mod4) Then n = 4t+2, |V| = 4t+3, $v_f(0) = 2t+1, v_f(1) = 2t+2$

Case n ≡ 3(mod4) Then n = 3+ 4t, |V| = 4t+4 $v_f(0) = 2t+2$ and $v_f(1) = 2t+2$ #

Theorem 3.5: $K_{2,n}$ is eevc only for odd n.

Proof: For n = 1 it is a path P_3. Label consecutive edges as 0, 2, 1.
For rest of the n≠1 we label all edges incident to one of the two vertices as odd numbers and all edges that are incident to other vertex are labelled as even numbers(other than 0). The edge joining the two n degree vertices is labelled as 0.

The binary label distribution is $v_f(0) = n = v_f(1)$#

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