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PATH RELATED NEAR MEAN CORDIAL GRAPHS

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ABSTRACT

*Let G = (V, E) be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1} such that the induced map f* defined by*

$$f^*(uv) = \begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

*and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.*

*In this paper, It is to be proved that $P_n \times K_2$, $P_n@2K_{1,n}$ and H_n^+ are **Near Mean Cordial** graphs.*

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Keywords and Phrases: *Cordial labeling, Near Mean Cordial Labeling and Near Mean Cordial Graph.*

1. INTRODUCTION

By a graph, it means a finite undirected graph without loops or multiple edges. For graph theoretic terminology, Harary [4] and G.J. Gallian[1] are referred.

A vertex labeling of a graph G is an assignment of labels to the vertices of G that induces for each edge *uv* a label depending on the vertex labels of *u* and *v*.

A graph *G* is said to be labeled if the *n* vertices are distinguished from a given set, which induces distinguish edge values satisfying certain conditions. The concept of graceful labeling was introduced by Rosa [3] in 1967 and subsequently by Golomb [2].

In this paper, It is to be proved that $P_n \times K_2$, $P_n@2K_{1,n}$ and H_n^+ are **Near Mean Cordial** graphs.

2. PRELIMINARIES

Definition 2.1: Let G = (V, E) be a simple graph. Let f:V(G)→{0,1} and for each edge uv, assign the label |f(u) − f(v)|. f is called a **cordial labeling** if the number of vertices labeled 0 and the the number of vertices labeled 1 differ by atmost 1 and also the number of edges labeled 0 and the the number of edges labeled 1 differ by atmost 1. A graph is called **Cordial** if it has a cordial labeling.

Definition 2.2: Let G = (V, E) be a simple graph. A **Near Mean Cordial Labeling** of G is a function in f : V(G) → {1, 2, 3, . . . , p−1, p+1}such that the induced map f* defined by

$$f^*(uv) = \begin{cases} 1 & if(f(u)+f(v)) \equiv 0\ (mod 2) \\ 0 & else \end{cases}$$

and it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ and $e_f(1)$ represent the number of edges labeled with 0 and 1 respectively. A graph is called a **Near Mean Cordial Graph** if it admits a near mean cordial labeling.

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Definition 2.3: $P_n@2K_{1,n}$ is a graph which is obtained by joining the root of the star $K_{1,n}$ to the end vertex of the path P_n.

Definition 2.4: Define the product $G_1 \times G_2$, by consider any two vertices u = (u_1 ,u_2), and v = (v_1 , v_2) in $V_1 \times V_2$ Then u and v are adjacent in $G_1 \times G_2$.whenever ($u_1 = v_1$ and u_2 adj to v_2) or ($u_2 = v_2$ and u_1 adj to v_1).

The product $P_m \times P_n$ is called planar grids and $\mathbf{K_2 \times P_n}$ is called Ladder. The product $C_m \times P_n$ is called Grids on cylinder of order mn. In particular, $D_n = C_n \times K_2$ is called a prism and $B_m = K_{1,m} \times K_2$ is called a book.

Definition 2.5: G^+ is a graph obtained from G by attaching a pendant vertex from each vertex of the graph G.

3. MAIN RESULTS

Theorem 3.1: $P_n \times K_2$ is a Near Mean Cordial Graph $\forall\, n \geq 2$.

Proof: Let V($P_n \times K_2$) = $\{u_i: 1 \leq i \leq n\,,\ v_i: 1 \leq i \leq n\}$.
Let E ($P_n \times K_2$) = $\{(u_i v_i) : 1 \leq i \leq n\} \cup \{(u_i u_{i+1}) :\ 1 \leq i \leq n-1\} \cup \{(v_i v_{i+1}) :\ 1 \leq i \leq n-1\}$

Case (i): when n = 2 and n = 3

Define f : V($P_n \times K_2$) → {1, 2, 3, . . . , 2n−1 , 2n+1} by

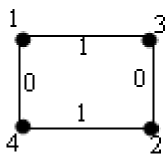

Figure: 3.1.1

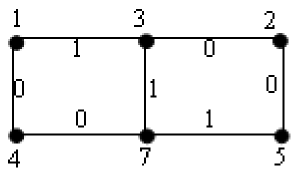

Figure: 3.1.2

Case (ii): when n > 3

Define f : V($P_n \times K_2$) → {1, 2, 3, . . . , 2n−1 , 2n+1} by

when n is even:

Let f(u_1) = 1

$f(u_{2i+1}) = \frac{n}{2} + i + 1, \quad 1 \leq i \leq \frac{n-2}{2}$

$f(u_{2i}) = 1 + i, \quad 1 \leq i \leq \frac{n}{2}$

$f(v_{2i-1}) = n + i, \quad 1 \leq i \leq \frac{n}{2}$

$f(v_{2i}) = \frac{3n}{2} + i, \quad 1 \leq i \leq \frac{n-2}{2}$

$f(v_n) = 2n+1$

when n is odd :

Let f(u_1) = 1, f(u_2) = 2

$f(u_{2i+1}) = 2+i,$ \ $\quad 1 \leq i \leq \frac{n-1}{2}$

$f(u_{2i}) = \frac{n+3}{2} + i - 1, \quad 2 \leq i \leq \frac{n-3}{2}$

$f(v_{2i-1}) = n + i, \quad 1 \leq i \leq \frac{n+1}{2}$

$f(v_{2i}) = \frac{3(n+1)}{2} + (i-1), \quad 1 \leq i \leq \frac{n-3}{2}$

$f(v_{n-1}) = 2n+1$

The induced edge labeling are

$$f^*(u_i\, v_i) = \begin{cases} 1 & \text{if } f(u_i) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \leq i \leq n$$

$$f^*(u_i\, u_{i+1}) = \begin{cases} 1 & \text{if } f(u_i) + f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \leq i \leq n-1$$

$$f^*(v_i\, v_{i+1}) = \begin{cases} 1 & \text{if } f(v_i) + f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \leq i \leq n-1$$

(i) Let $n = 2k,\ (k \epsilon N)$
Here, $e_f(1) = e_f(0) = n + k - 1$.

(ii) Let $n = 2k + 1$, $(2, 4, 6, \ldots \epsilon N)$
Here, $e_f(0) = n + k$ and $e_f(1) = n + k - 1$
(iii) Let $n = 2k + 1$, $(3, 5, \ldots \epsilon N)$
Here, $e_f(0) = n + k - 1$ and $e_f(1) = n + k$

Hence, it satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $P_n \times K_2$ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of $P_6 \times K_2$, $P_7 \times K_2$ and $P_9 \times K_2$ are shown in Figures 3.1.3 - 3.1.5.

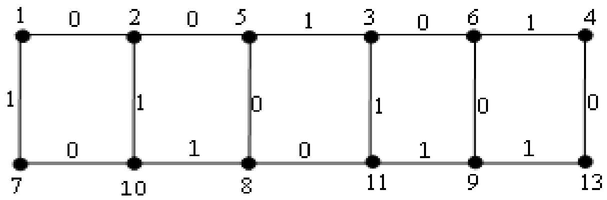

Figure: 3.1.3

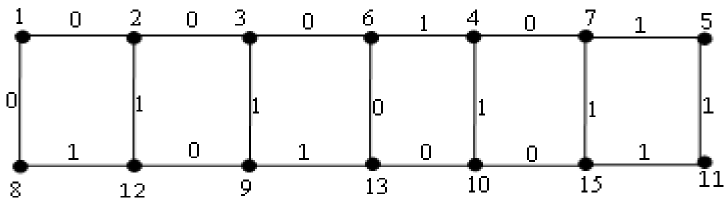

Figure: 3.1.4

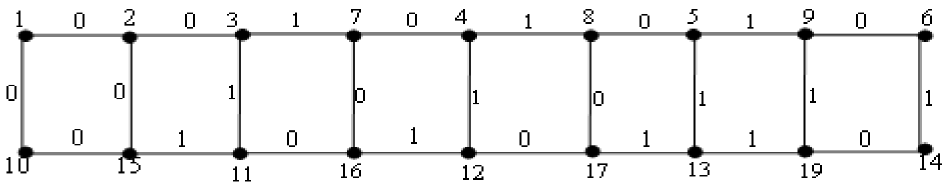

Figure: 3.1.5

Theorem 3.2: $P_n @2\, K_{1,n}$ is a Near Mean Cordial Graph.

Proof: Let V($P_n @2\, K_{1,n}$) $= \{u_i : 1 \leq i \leq n,\ v_i : 1 \leq i \leq n,\ w_i : 1 \leq i \leq n\}$.
Let E ($P_n @2\, K_{1,n}$) $= \{(u_i w_i) : 1 \leq i \leq n\} \cup \{(w_i w_{i+1}) : 1 \leq i \leq n-1\} \cup \{w_i v_i : 1 \leq i \leq n\}$.

When n $\equiv$ 0 $(mod\ 4)$:

Define f: V($P_n @2\, K_{1,n}$) $\rightarrow$ {1, 2, 3, . . . , 3n−1 , 3n+1} by

$f(u_i) = 2i - 1, \quad 1 \leq i \leq n$
$f(v_i) = 2i, \quad 1 \leq i \leq n$
$f(w_{2i-1}) = 2n + i, \quad 1 \leq i \leq \frac{n}{2}$
$f(w_2) = 3n+1$
$f(w_{2(i+1)}) = 3n - i, \quad 1 \leq i \leq \frac{n-2}{2}$

When n $\equiv$ 1 $(mod\ 4)$:

Define f : V($P_n @2\, K_{1,n}$) $\rightarrow$ {1, 2, 3, . . . , 3n−1 , 3n+1} by

$f(u_i) = 2i - 1, \quad 1 \leq i \leq n$
$f(v_i) = 2i, \quad 1 \leq i \leq n$
$f(w_{2i-1}) = 2n + i, \quad 1 \leq i \leq \frac{n+1}{2}$
$f(w_2) = 3n+1$
$f(w_{2(i+1)}) = 3n - i, \quad 1 \leq i \leq \frac{n-3}{2}$

When $n \equiv 2\ (mod\ 4)$:

Define f : V(P_n@2 $K_{1,n}$) → {1, 2, 3, . . . , 3n−1 , 3n+1} by

$f(u_i) = 2n + i, \quad 1 \le i \le n-1$

$f(u_n) = 3n+1$

$f(v_i) = n + i, \quad 1 \le i \le n$

$f(w_{2i-1}) = i, \quad 1 \le i \le \frac{n}{2}$

$f(w_{2i}) = \frac{n}{2} + i, \quad 1 \le i \le \frac{n}{2}$

When $n \equiv 3\ (mod\ 4)$:

Define f : V(P_n@2 $K_{1,n}$) → {1, 2, 3, . . . , 3n−1 , 3n+1} by

$f(u_i) = 2n + i, \quad 1 \le i \le n-1$

$f(u_n) = 3n+1$

$f(v_i) = n + i, \quad 1 \le i \le n$

$f(w_{2i-1}) = i, \quad 1 \le i \le \frac{n+1}{2}$

$f(w_{2i}) = \frac{n+1}{2} + i, \quad 1 \le i \le \frac{n-1}{2}$

From all the cases, The induced edge labelings are

$f^*(u_i\, w_i) = \begin{cases} 1 & \text{if } f(u_i) + f(w_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$

$f^*(w_i\, w_{i+1}) = \begin{cases} 1 & \text{if } f(w_i) + f(w_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$

$f^*(w_i\, v_i) = \begin{cases} 1 & \text{if } f(w_i) + f(v_i) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$

Let $n = 2k + 1,\ (k \epsilon N)$

Here, $e_f(1) = e_f(0) = n + k$.

Let $n = 2k,\ (k \epsilon N)$

Here, $e_f(1) = n + k - 1$ and $e_f(0) = n + k$, (when $k \equiv 0\ (mod\ 2)$) .

Here, $e_f(0) = n + k - 1$ and $e_f(1) = n + k$, (when $k \equiv 1\ (mod\ 2)$).

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, P_n@2 $K_{1,n}$ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of P_8@2 $K_{1,8}$, P_9@2 $K_{1,9}$, P_6@2 $K_{1,6}$ and P_7@2 $K_{1,7}$ are shown in Figures 3.2.1 - 3.2.4.

When n ≡ 0 (mod 4):

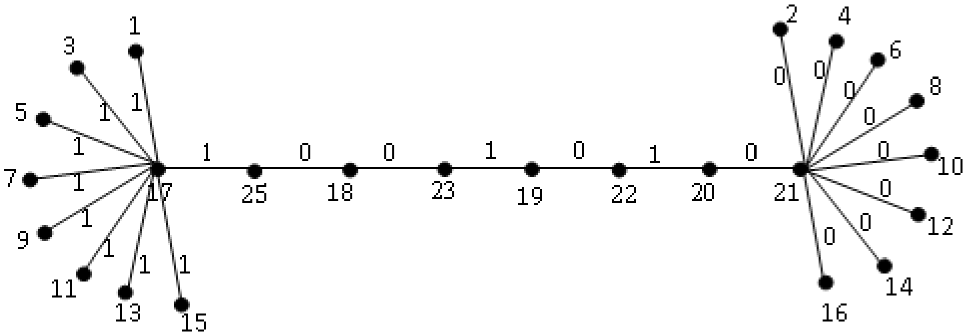

Figure: 3.2.1

When n $\equiv$ 1 (mod 4):

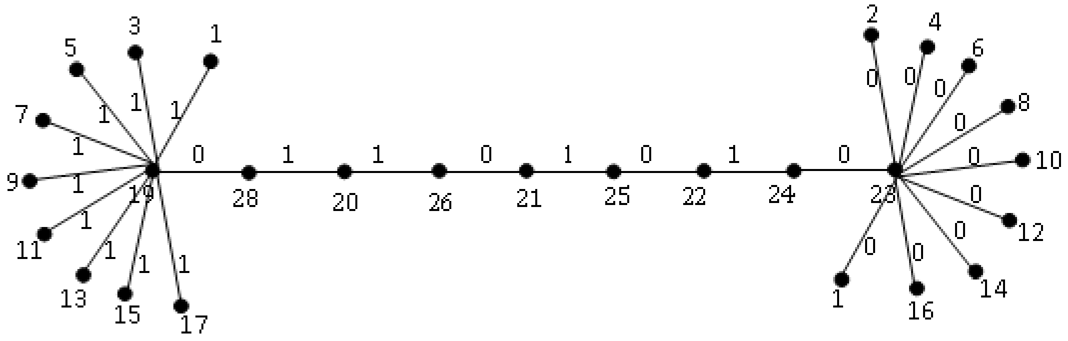

Figure: 3.2.2

When n $\equiv$ 2 (mod 4):

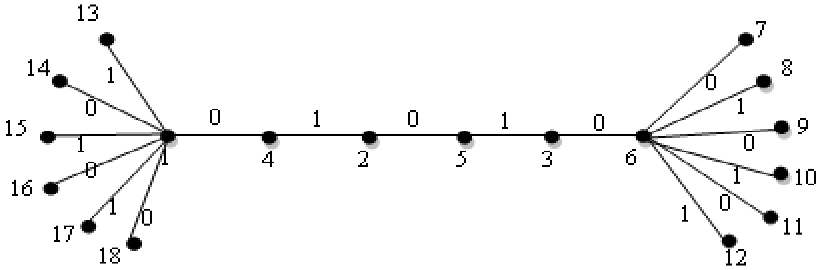

Figure: 3.2.3

When n $\equiv$ 3 (mod 4):

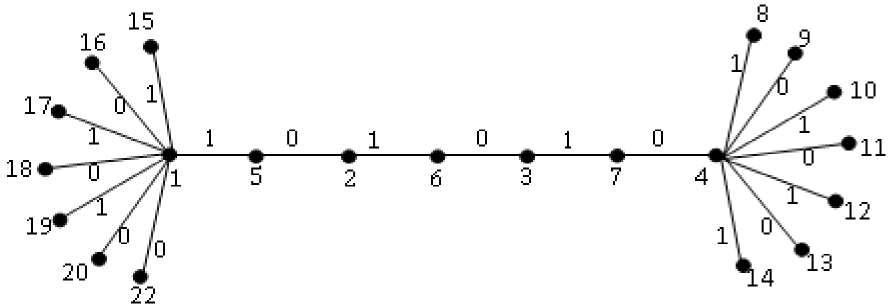

Figure: 3.2.4

Theorem 3.3: H_n^+ (n : odd) is a Near Mean Cordial Graph.

Proof: Let V(H_n^+) = $\{u_i, v_i : 1 \le i \le n, u_i', v_i' : 1 \le i \le n\}$.
Let E(H_n^+) = $\{(u_i u_{i+1}), (v_i v_{i+1}) : 1 \le i \le n-1\} \cup \{u_i u_i', v_i v_i' : 1 \le i \le n\} \cup \{u_{\left(\frac{n+1}{2}\right)} v_{\left(\frac{n+1}{2}\right)}\}$.

Define f : V(H_n^+) → {1, 2, 3, . . . , 4n−1 , 4n+1} by

f(u_{2i-1}) = $2n - 2(i-1)$, $1 \le i \le \frac{n+1}{2}$

f(u_{2i}) = $2i$, $1 \le i \le \frac{n-1}{2}$

f(u_{2i-1}') = $2i - 1$, $1 \le i \le \frac{n+1}{2}$

f(u_{2i}') = $2n - 1 - 2(i-1)$, $1 \le i \le \frac{n-1}{2}$

f(v_1) = $4n + 1$

f(v_{2i+1}) = $4n - 2 - 2(i-1)$, $1 \le i \le \frac{n-1}{2}$

f(v_{2i}) = $2n + 2 + 2(i-1)$, $1 \le i \le \frac{n-1}{2}$

f(v_{2i-1}') = $2n + 1 + 2(i-1)$, $1 \le i \le \frac{n+1}{2}$

f(v_{2i}') = $4n - 1 - 2(i-1)$, $1 \le i \le \frac{n-1}{2}$

The induced edge labelings are

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & \text{if } f(u_i)+f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(u_iu_i') = \begin{cases} 1 & \text{if } f(u_i)+f(u_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(v_iv_{i+1}) = \begin{cases} 1 & \text{if } f(v_i)+f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(v_iv_i') = \begin{cases} 1 & \text{if } f(v_i)+f(v_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(u_{\left(\frac{n+1}{2}\right)}v_{\left(\frac{n+1}{2}\right)}) = 1$$

Here, $e_f(0) = 2n$ and $e_f(1) = 2n-1$

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, H_n^+ is a Near Mean Cordial Graph .

For example, the Near Mean Cordial Labeling of H_7^+ is shown in the Figure 3.3.1.

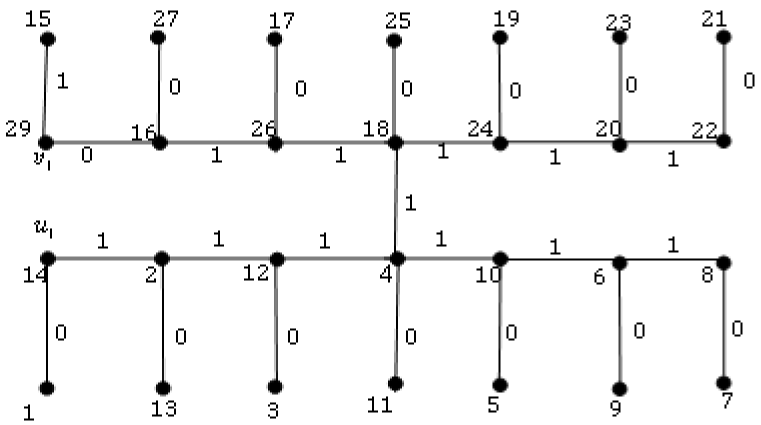

Figure: 3.3.1

Theorem 3.4: H_n^+ (n : even) is a Near Mean Cordial Graph .

Proof: Let V $(H_n^+) = \{u_i, v_i : 1 \le i \le n, u_i', v_i' : 1 \le i \le n\}$.

Let E $(H_n^+) = \{(u_iu_{i+1}), (v_iv_{i+1}) : 1 \le i \le n-1\} \cup \{u_iu_i', v_iv_i' : 1 \le i \le n\} \cup \{u_{\left(\frac{n}{2}+1\right)}v_{\left(\frac{n}{2}\right)}\}$.

Define f : V$(H_n^+) \rightarrow \{1, 2, 3, \ldots, 4n-1, 4n+1\}$ by

$$f(u_{2i-1}) = 2n-2(i-1), \quad 1 \le i \le \tfrac{n}{2}$$
$$f(u_{2i}) = 2i, \quad 1 \le i \le \tfrac{n}{2}$$
$$f(u_{2i-1}') = 2i-1, \quad 1 \le i \le \tfrac{n}{2}$$
$$f(u_{2i}') = 2n-1-2(i-1), \quad 1 \le i \le \tfrac{n}{2}$$
$$f(v_1) = 4n+1$$
$$f(v_{2i+1}) = 4n-2-2(i-1), \quad 1 \le i \le \tfrac{n-2}{2}$$
$$f(v_{2i}) = 2n+2+2(i-1), \quad 1 \le i \le \tfrac{n}{2}$$
$$f(v_{2i-1}') = 2n+1+2(i-1), \quad 1 \le i \le \tfrac{n}{2}$$
$$f(v_{2i}') = 4n-1-2(i-1), \quad 1 \le i \le \tfrac{n}{2}$$

The induced edge labelings are

$$f^*(u_iu_{i+1}) = \begin{cases} 1 & \text{if } f(u_i)+f(u_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(u_iu_i') = \begin{cases} 1 & \text{if } f(u_i)+f(u_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n$$

$$f^*(v_iv_{i+1}) = \begin{cases} 1 & \text{if } f(v_i)+f(v_{i+1}) \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases}, \quad 1 \le i \le n-1$$

$$f^*(v_i v_i') = \begin{cases} 1 & \text{if } f(v_i) + f(v_i') \equiv 0 \pmod 2 \\ 0 & \text{else} \end{cases} \quad , \; 1 \le i \le n$$

$$f^*(u_{\left(\frac{n}{2}+1\right)} v_{\left(\frac{n}{2}\right)}) = 1$$

Here, $e_f(0) = 2n$ and $e_f(1) = 2n - 1$

So, it satisfies the condition $|e_f(0) - e_f(1)| \le 1$.

Hence, H_n^+ is a Near Mean Cordial Graph.

For example, the Near Mean Cordial Labeling of H_8^+ is shown in the Figure 3.4.1.

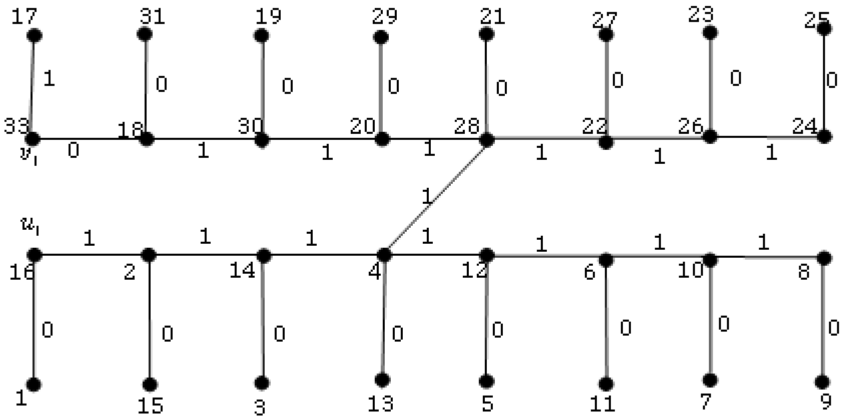

Figure: 3.4.1

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