International Journal of Mathematical Archive-8(9), 2017, 135-139
IJMA Available online through www.ijma.info ISSN 2229 – 5046

LRS BIANCHI TYPE I ANTI- STIFF FLUID COSMOLOGICAL MODEL WITH ELECTROMAGNETIC FIELD AND VACUUM ENERGY DENSITY

SWATI PARIKH*[1], ATUL TYAGI[2] AND BARKHA RANI TRIPATHI[3]

[1,2,3]Department of Mathematics and Statistics,
University College of Science, MLS University, Udaipur – 313001, (Raj.), India.

(Received On: 09-08-17; Revised & Accepted On: 11-09-17)

ABSTRACT

In this paper we have investigated LRS Bianchi type I anti- stiff fluid cosmological model with electromagnetic field and vacuum energy density. We have assumed that the magnetic field is due to an electric current produced along z-axis. So, F_{12} *is the only non-vanishing component of electromagnetic field tensor* F_{ij}*. To get the deterministic solution, it has been assumed that* $p+\rho=0$ *, where p is the pressure and ρ is the rest energy density, and a relation between metric potentials* $A=B^n$*, where A and B are functions of t alone. It is observed that the model has a point type singularity at* $T=0$ *provided* $n>1$*. Physical and geometrical properties of the model are also discussed.*

Keywords and Phrases: *LRS Bianchi type I, electromagnetic field, anti-stiff fluid, vacuum energy density.*

1. INTRODUCTION

The cosmological problem within the frame work of general relativity consists of finding a model of the physical universe which correctly predicts the result of astronomical observations and which is determined by those physical laws that describe the behavior of matter on scales up to those of clusters of galaxies. The simplest models of the expanding universe are those which are spatially homogeneous and isotropic at each instance of time. The Bianchi cosmologies which are spatially homogeneous and anisotropic play an important role in theoretical cosmology. For simplification and description of large scale behavior of actual universe, LRS Bianchi models have great importance. Lidsey[13] showed that these models are equivalent to FRW universe. Bianchi type I cosmological models include an FRW model with zero curvature. The famous Einstein and de- Sitter universe are Bianchi type I models, but these are static models.

The occurrence of magnetic field on galactic scale is well-established fact today and their importance for a variety of astrophysical phenomena is generally acknowledged, as pointed out by Zeldovich *et al.* [28]. Also Harrison [11] has suggested that magnetic field could have a cosmological origin. As a natural consequence, we should include magnetic field in energy momentum tensor of early universe.

One of the most interesting puzzles concerning our current understanding of the physical world is the tiny value of the cosmological constant $\Lambda < 10^{-120} M^2$ p. This problem is reviewed by Weinberg [27] some time ago. Several authors have tried independently to account for the present value of Λ considering it as a variable rather than a constant. Several cosmologists suggested that the variable cosmological term is a function of scale factor. Among them the phenomenological developed power laws of scale factor by Ozer and Taha[15], Gasperini[9-10] and Chen and Wu[6] are worthy attention. Linde [14] has suggested that Λ is a function of temperature and is related to the spontaneous symmetry breaking process. Therefore, it could be a function of time in a spatially homogenous expanding Universe [26].

Corresponding Author: Swati Parikh[1],*
[1]*Department of Mathematics and Statistics,*
University College of Science, MLS University, Udaipur – 313001, (Raj.), India.

The large scale distribution of galaxies in our universe shows that the matter distribution can satisfactorily be described by perfect fluid. Singh [21] has obtained spatially homogeneous LRS Bianchi type-V cosmological model with perfect fluid in general relativity. Singh [22] has investigated Bianchi type-V cosmological model with a specific Hubble parameter in presence of perfect fluid. Roy and Singh [17-18] have investigated Bianchi type V models with electromagnetic field. Bali and Ali [2] have investigated Bianchi Type I Magnetized stiff fluid cosmological model with two degrees of freedom for perfect fluid distribution. Collins [7] gave a qualitative analysis of Bianchi Type I model with magnetic field. Jacobs [12] investigated Bianchi Type I cosmological model with barotropic fluid in the presence of a magnetic field. Bali and Tyagi [1] have investigated magnetized Bianchi type I orthogonal cosmological model for perfect fluid distribution in General Relativity. Bali and Meena[3] have investigated magnetized stiff fluid tilted universe for perfect fluid distribution in General Relativity.

Dunn and Tupper [8] discussed properties of Bianchi type VI_0 models with perfect fluid and magnetic field. An axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field have been investigated by Benerjee *et.al.*[5].Roy *et al.*[19] explored the effects of cosmological constant in Bianchi type I and VI_0 models with perfect fluid and homogeneous magnetic field in axial direction. Bali and Upadhaya [4] have presented LRS Bianchi type-I string dust magnetized cosmological models in general relativity. Wang [24-25] has investigated LRS Bianchi I string cosmological models in general relativity in presence of bulk viscosity and electromagnetic field where constant coefficient of bulk viscosity is considered.

Pradhan *et.al.* [16] discussed some homogeneous cosmological model with electromagnetic field in presence of perfect fluid with variable Λ. Singh and Kumar [20] have investigated some spatially homogeneous and anisotropic Bianchi I perfect fluid cosmological model with variable cosmological constant. Tyagi and Singh [23] has investigated magnetized anti-stiff fluid cosmological model in LRS Bianchi type V universe with time dependent Λ and variable magnetic permeability.

In this paper we have investigated LRS Bianchi type I anti- stiff fluid cosmological model with electromagnetic field and vacuum energy density. We have assumed that F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij} . To get the deterministic model, it has been assumed that $p+\rho=0$, where p is the pressure and ρ is the rest energy density, and a relation between metric potentials $A=B^n$, where A and B are functions of t alone. It is observed that the model has a point type singularity at $T=0$ provided $n>1$. Physical and geometrical properties of the model are also discussed.

2. THE METRIC AND FIELD EQUATIONS

We consider LRS Bianchi type I metric of the form

$$ds^2=-dt^2+A^2(dx^2+dy^2)+B^2dz^2 \tag{1}$$

where A and B are functions of time only. In this paper we have considered distribution of matter to consist of anti-stiff perfect fluid with an infinite electrical conductivity and magnetic field. So, the energy-momentum tensor is taken in the form of

$$T_i^j=(\rho+p)\,\mathrm{v_i\,v^j}+pg_i^j+E_i^j \tag{2}$$

where ρ is the energy density, p is cosmological pressure and v^i is the fluid four-velocity vector satisfying the condition,

$$g_{ij}v^iv^j=-1 \tag{3}$$

and E_i^j is the electromagnetic field given by Lichnerowicz as

$$E_i^j==\bar{\mu}\left[h_lh^l\left(v_iv^j+\frac{1}{2}g_i^j\right)-h_ih^j\right] \tag{4}$$

where $\bar{\mu}$ is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_i=\frac{\sqrt{-g}}{2\bar{\mu}}\varepsilon_{ijkl}F^{kl}v^j \tag{5}$$

where F^{kl} is the electromagnetic field tensor and ε_{ijkl} is the Levi-Civita tensor density. Here, the comoving coordinates are taken to be

$$v^i=(0,0,0,1) \tag{6}$$

We take incident magnetic field to be in direction of z-axis so that

$$\mathrm{h}_1=h_2=h_4=0;h_3\neq 0 \tag{7}$$

The first set of Maxwell's equations is

$$F_{[ij;k]} = 0 \tag{8}$$

which leads to

$$F_{12} = M = constant \tag{9}$$

which is the only non-vanishing component.

The components of electromagnetic field with the help of equations (4), (5) and (9) are obtained as

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{M^2}{2\bar{\mu}A^4} \tag{10}$$

The Einstein's field equation (in the gravitational units $8\pi G = 1, c = 1$) with time dependent cosmological term is given by

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \tag{11}$$

The Einstein's field equation (11) for metric (1) together with equation (10) reduces to

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \Lambda = -p - \frac{M^2}{2\bar{\mu}A^4} \tag{12}$$

$$\frac{2A_{44}}{A} + \frac{A_4^{\ 2}}{A^2} + \Lambda = -p + \frac{M^2}{2\bar{\mu}A^4} \tag{13}$$

$$\frac{A_4^{\ 2}}{A^2} + \frac{2A_4 B_4}{AB} + \Lambda = \rho + \frac{M^2}{2\bar{\mu}A^4} \tag{14}$$

where the sub indices 4 in A and B denotes differentiation with respect to t.

Equations (12)-(14) are three equations in five unknowns A, B, ρ, p and Λ. To obtain the determinate solution, we assume that shear (σ) is proportional to expansion (θ) which leads to

$$A = B^n \tag{15}$$

We also assume that vacuum energy density

$$\Lambda = \frac{a}{R^3} \tag{16}$$

where R is scale factor.

So, we have

$$\Lambda = \frac{a}{A^2 B} = \frac{a}{B^{2n+1}} \tag{17}$$

On applying anti-stiff condition

$$p + \rho = 0 \tag{18}$$

in equations (13) and (14) we have

$$\frac{A_{44}}{A} = \frac{A_4 B_4}{AB} \tag{19}$$

Equation (19) leads to

$$A_4 = LB \tag{20}$$

where L is constant of integration.

Now, using equation (15) in equation (20), we have

$$B^{n-2}dB = \frac{L}{n}dt \tag{21}$$

On integration equation (21) leads to

$$\frac{B^{n-1}}{n-1} = \frac{L}{n}t + C \tag{22}$$

Equation (22) gives

$$B^{n-1} = \beta t + \gamma \tag{23}$$

where $\beta = \frac{L(n-1)}{n}$

$$\gamma = C(n-1) \tag{24}$$

From equation (23) we have

$$B = T^{\frac{1}{n-1}} \tag{25}$$

where $T = (\beta t + \gamma)$

And from equation (15),

$$A = T^{\frac{n}{n-1}} \tag{26}$$

So, the metric (1) reduces to the form

$$ds^2 = \frac{-dT^2}{\beta^2} + T^{\frac{2n}{n-1}}(dx^2 + dy^2) + T^{\frac{2}{n-1}}dz^2 \tag{27}$$

3. SOME PHYSICAL AND GEOMETRICAL FEATURES

The energy density (ρ) and pressure (*p)* for the model (27) are given by,

$$\rho = \frac{(2n+n^2)\beta^2}{(n-1)^2 T^2} + \Lambda - \frac{M^2}{2\bar{\mu}A^4} \tag{28}$$

$$p = -\frac{(2n+n^2)\beta^2}{(n-1)^2 T^2} - \Lambda + \frac{M^2}{2\bar{\mu}A^4} \tag{29}$$

The Scalar expansion (θ) of the model is given by,

$$\theta = \frac{(2n+1)\beta}{(n-1)T} \tag{30}$$

The Shear Scalar (σ) of the model is given by,

$$\sigma^2 = \frac{1}{3}\frac{\beta^2}{T^2} \tag{31}$$

The scale factor (R) is given by,

$$R^3 = A^2 B = B^{(2n+1)} = T^{\frac{(2n+1)}{n-1}} \tag{32}$$

The vacuum energy density (Λ) is given by

$$\Lambda = \frac{a}{T^{\frac{(2n+1)}{n-1}}} \tag{33}$$

The deceleration parameter (q) of the model is given by,

$$q = \frac{(n-4)}{(2n+1)} \tag{34}$$

From equation (30) and (31) we obtain

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\frac{(n-1)}{(2n+1)} = const. \tag{35}$$

4. DISCUSSION

The model starts with big bang singularity at $T=0$ when $n>1$ and the expansion decreases with time. The expansion stops at $T\to\infty$. Since $T\to\infty, \frac{\sigma}{\theta}\neq 0$ so the model does not approach isotropy for large values of T, however when $n=1$ the model isotropizes. Also, the model has a point type singularity as $T\to 0$ provided $n>1$. The scale factor $R^3\to\infty$ as $T\to\infty$. The vacuum energy density decreases with time. The deceleration parameter

$$q<0 \text{ if } n<4$$
$$q>0 \text{ if } n>4$$

Thus the model represents decelerating and accelerating phases of universe which matches with recent observations.

In general the model represents expanding, shearing and non rotating universe.

REFERENCES

1. Bali, R. and Tyagi, A. (1987). *Int. J. Theor. Phys.* 27 627
2. Bali, R. and Ali, M. (1996). *Pramana J. Phys.* 47 25
3. Bali, R. and Meena, B. L. (1999). *Astrophys. Space Sci.* 262 89
4. Bali, R. and Upadhaya, R. D. (2003). *Astrophys. Space Sci.* 283 97
5. Banerjee, A., Sanyal, A. K. and Chakraborti, S. (1990). *Pramana J. Phys.* 34 1
6. Chen, W. and Wu, Y. S. (1990). *Phys. Rev. D* 41 695
7. Collins, C. S. (1972). *Comm. Maths. Phys.* 27 37
8. Dunn, K. A. and Tupper, B. O. J. (1976). *Astrophys. J.* 204 322
9. Gasperin, M. (1987). *Phys. Lett. B* 194 347
10. Gasperin, M. (1988). *Class. Quant. Gravit.* 5 521
11. Harrison, E. R. (1973). *Phy. Rev. Lett.* 30 188.
12. Jacobs, K. C. (1968). *Astrophys. J.* 153 661
13. Lidsey, J. E. (1992). *Class.Quant.Gravit.* 9 1239
14. Linde, A. D. (1974). *ZETP Lett.* 19 183
15. Ozer, M. and Taha, M. O. (1987). *Nucl. Phys.B* 287 776
16. Pradhan, A., Pandey, P. and Rai, K. K. (2006). *Czech. J. Phys.* 56 303
17. Roy, S. R. and Singh, J. P. (1983). *Astrophys. Space Sci.* 96 303
18. Roy, S. R. and Singh, J. P. (1985). *Aust. J. Phys.* 38 763
19. Roy, S. R., Singh, J. P. and Narin, S. (1985). *Astrophys. Space Sci.* 111 389
20. Singh, C. P. and Kumar, S. (2008). *Int. J. Theor. Phys.* 47 3171
21. Singh, C. P. (2009). *Pramana J. Phys.*72 429
22. Singh, J. P. (2009). *Int. J. Theor. Phys.* 48 2041
23. Tyagi, A. and Singh, G. P. (2011). *J. Chem. Bio. Phys. Sci.* 1 112
24. Wang, X. X. (2009). *Chin. Phys. Lett.* 26 109804
25. Wang, X. X. (2004). *Chin. Phys. Lett.* 293 433
26. Weinberg, S. (1967). *Phy. Rev. Lett.* 19 1264
27. Weinberg, S. (1989). *Rev.Mod.Phy.* 61 1
28. Zeldovich, Ya. B., Ruzmainkin, A. A. and Sokoloff, D. D. (1983). *Magnetic Field in Astrophys* Gordon and Breach New York

Source of support: Nil, Conflict of interest: None Declared.