International Journal of Mathematical Archive-8(9), 2017, 173-178
IJMA Available online through www.ijma.info ISSN 2229 – 5046

A Pál Type (0, 1; 0) Interpolation Process on Laguerre Polynomial

R. SRIVASTAVA[1], GEETA VISHWAKARMA*[2]

Department of Mathematics and Astronomy, Lucknow University, Lucknow, INDIA – 226007.

(Received On: 14-06-17; Revised & Accepted On: 12-07-17)

ABSTRACT

In the present paper ,we have considered the problem in which $\{\xi_i\}_{i=1}^n$ and $\{\xi_i^*\}_{i=1}^n$ be the two sets of interscaled nodal points on the interval [0,∞) . Here we deal with the problem in which one set consists of the nodes of $L_n^k(x)$ and other consists of the nodes of $L_n^{k-1}(x)$. We investigate the existence, uniqueness explicit representation of interpolatory polynomial. Estimation of the fundamental polynomials have also been obtained.

Keywords: *lacunary Interpolation, Pál - Type Interpolation, Laguerre Polynomial.*

MSC 2000: *41 A 05 65 D 32.*

1. INTRODUCTION

J. Balázs [2] was the first to give the solution of the problem with the nodes as the zeros of ultra spherical polynomial $P_n^{(\alpha)}(x)$ $(\alpha > -1)$ and the weight function $(x) = (1-x^2)^{\frac{(1+\alpha)}{2}}$, $x \in [-1,-1]$. He proved that generally there do exist any polynomial $R_n(x)$ of degree ≤ 2n-1 satisfying the conditions:

$$R_n(\xi_i^*) = g_i^*,\ (\omega R_n)'(\xi_i^*) = g_i^{**} \quad for\ i = 1(1)n$$

where g_i^* and g_i^{**} are arbitrary real numbers. However taking an additional condition

$$R_n(0) = \sum_{i=1}^n \alpha_i l_i^2(0)$$

where 0 is not a nodal point. In 1984, L. Szili [13] studied analogous problem with the nodes as the roots of $H_n(x)$, the Hermite polynomial and weight function $\omega(x) = e^{(-\frac{1}{2}x^2)}$. Pál [10] proved that for a given arbitrary numbers $\{\alpha_i^*\}_{i=1}^n$ and $\{\beta_i^*\}_{i=1}^n$ there exists a unique polynomial of degree ≤ 2n-1 satisfying the conditions:

$$R_n(\xi_i^*) = \alpha_i^*,\ for\ i = 1(1)n \qquad (\omega R_n)'(\xi_i^*) = \beta_i^* \ for\ i = 1(1)n-1,$$

with an initial condition $R_n(a) = 0$ where a is a given point, different from the nodal points $\{x_i\}_{i=1}^n$ and $\{y_i\}_{i=1}^n$. In this paper we study Pál – type interpolational polynomial with $\omega_{n+k}(x) = x^k L_n^{(k)}(x)$.we have determined the existence, uniqueness, explicit representation and estimation of fundamental polynomials of such special kind of mixed type of interpolation on interval [0,∞). Let $\{\xi_i\}_{i=1}^n$ and $\{\xi_i^*\}_{i=1}^n$ be the two sets of interscaled nodal points on the interval [0, ∞). We seek to determine a polynomial $R_n(x)$ of minimal possible degree ≤ 3n+k satisfying the interpolatory conditions:

$$R_n(\xi_i) = g_i,\ R_n{}'(\xi_i) = g_i^*, \quad R_n(\xi_i^*) = g_i^{**}, \qquad for\ i = 1(1)n$$

(1.3) $$R_n^{(j)}(\xi_0) = g_0^{(j)} \qquad , \quad j = 0,1,\dots,k$$

where g_i, g_i^*, $g_i^{**}$and $g_0^{(j)}$ are arbitrary real numbers. Here Laguerre polynomials $L_n^{(k)}(x)$ and $L_n^{(k-1)}(x)$ have zeroes $\{\xi_i\}_{i=1}^n$ and $\{\xi_i^*\}_{i=1}^n$ respectively and $x_0 = 0$. We prove existence, uniqueness, explicit representation and estimation of fundamental polynomials.

2. PRELIMINARIES

In this section we shall give some well-known results which are as follws:

As we know that the Laguerre polynomial is a constant multiple of a confluent hypergeometric function so the differential equation is given by

(2.1) $$xD^2L_n^k(x) + (1+k-x)DL_n^k(x) + nL_n^k(x) = 0$$

(2.2) $$L_n^{(k-1)'}(x) = -L_{n-1}^{(k)}(x)$$

Corresponding Author: Geeta Vishwakarma[2]*
Department of Mathematics and Astronomy, Lucknow University, Lucknow, INDIA – 226007.

Also using the identities

(2.3) $$L_n^{(k)}(x) = L_n^{(k+1)}(x) - L_{n-1}^{(k+1)}(x)$$

(2.4) $$xL_n^{(k)'}(x) = nL_n^{(k)}(x) - (n+k)L_{n-1}^{(k)}(x)$$

We can easily find a relation

(2.5) $$\frac{d}{dx}[x^k L_n^k(x)] = (n+k)x^{k-1}L_n^{(k-1)}(x)$$

By the following conditions of orthogonality and normalization we define Laguerre polynomial $L_n^{(k)}(x),\ for\ k > -1$

(2.6) $$\int_0^\infty e^{-x}x^k L_n^{(k)}(x)L_m^{(k)}(x)dx = \Gamma(k+1)\binom{n+k}{n}\delta_{nm} n, m = 0,1,2, \ldots .$$

(2.7) $$L_n^{(k)}(x) = \sum_{\mu=0}^{n}\binom{n+k}{n-\mu}\frac{(-x)^\mu}{\mu!}$$

The fundamental polynomials of Lagrange interpolation are given by

(2.8) $$l_j(x) = \frac{L_n^{(k)}(x)}{L_n^{(k)'}(x_j)(x-x_j)} = \delta_{i,j}$$

(2.9) $$l_j^*(x) = \frac{L_n^{(k-1)}(x)}{L_n^{(k-1)'}(x_j)(x-x_j)} = \delta_{i,j}$$

(2.10) $$l_j^{*'}(y_j) = \begin{cases} \dfrac{L_n^{(k-1)'}(y_i)}{L_n^{(k-1)'}(y_j)(y_i-y_j)} & i \neq j \\ -\dfrac{(k-y_j)}{2y_j} & i = j \end{cases} \quad i, j = 1(1)n$$

(2.12) $$l_j'(y_j) = \frac{1}{(y_j-x_j)}\left[\frac{L_n^{(k)'}(y_j)}{L_n^{(k)'}(x_j)} - \frac{L_n^{(k)}(y_j)}{L_n^{(k)'}(x_j)(y_j-x_j)}\right], \quad j = 1(1)n$$

For the roots of $L_n^{(k)}(x)$ we have

(2.13) $$x_k^2 \sim \frac{k^2}{n}$$

(2.14) $\eta(x)\left|S_n^{(l)}(x)\right| = O(1)$ where $\eta(x)$ is the weight function

(2.15) $$\left|L_n^{(k)'}(x_j)\right| \sim j^{-k-\frac{3}{2}}n^{k+1}, (0 < x_j \leq \Omega, n = 1,2,3, \ldots \ldots)$$

(2.16) $$\left|L_n^k(x_j)\right| = \begin{cases} x^{-\frac{k}{2}-\frac{1}{4}}O\left(n^{\frac{k}{2}-\frac{1}{4}}\right), & cn^{-1} \leq x \leq \Omega \\ O(n^k), & 0 \leq x \leq cn^{-1} \end{cases}$$

3. NEW RESULTS

Theorem 1: For n >1 fixed integer let $\{g_i\}_{i=1}^n$, $\{g_i^*\}_{i=1}^n$, $\{g_i^{**}\}_{i=1}^n$ and, $\{g_0^{(j)}\}_{j=0}^k$ are arbitrary real numbers then there exists a unique polynomial $R_n(x)$ of minimal possible degree ≤ 3n+k on the nodal points (1.1) satisfying the condition (1.2) and (1.3). The polynomial $R_n(x)$ can be written in the form

(3.1) $$\boldsymbol{R_n(x) = \sum_{j=1}^n U_j(x)g_j + \sum_{j=1}^n V_j(x)g_j^* + \sum_{j=1}^n W_j(x)g_j^{**} + \sum_{j=0}^k C_j(x)g_0^{(j)}}$$

where $U_j(x)$, $V_j(x)$, $W_j(x)$ and $C_j(x)$ are fundamental polynomials of degree ≤ 3n+k given by

(3.2) $$U_j(x) = \frac{x^{(k+1)}\ L_n^{(k-1)}(x)\,[l_j(x)]^2[1-2(x-x_j)]}{x_j^{(k+1)}L_n^{(k-1)}(x_j)}$$

(3.3) $$V_j(x) = \frac{x^{(k+1)}\ l_j(x)\,L_n^{(k)}(x)L_n^{(k-1)}(x)}{x_j^{k+1}\ L_n^{k-1}(x_j)\,L_n^{(k)'}(x_j)}$$

(3.4) $$W_j(x) = \frac{x^{k+1} l_j^*(x) [L_n^{(k)}(x)]^2}{y_j^{k+1} [L_n^k(y_j)]^2},$$

(3.5) $$C_j(x) = p_j(x) x^j \left[L_n^{(k-1)}(x)\right]^2 L_n^{(k)}(x)\, x^k L_n^{(k)}(x) L_n^{(k-1)}(x) \left[c_j \ - \frac{L_n^{(k-1)}(x) p_j(x) + q_j(x) L_n^{(k)}(x)}{x^{k-j}}\right],$$

$$j = 0,1,\ldots,k-1$$

(3.6) $$C_k(x) = \frac{1}{k! L_n^k(0) [L_n^{(k-1)}(0)]^2} x^k L_n^{(k-1)}(x) [L_n^{(k)}(x)]^{\,2}$$

where $p_j(x)$ and $q_j(x)$ are polynomials of degree at most k-j-1.

Theorem 2: Let the interpolatory function $f: \mathcal{R} \to \mathcal{R}$ be continuously differentiable such that,

$$C(m) = \{f(x): f \text{ is continuous in } [0,\infty), f(x) = O(x^m) \text{ as } x \to \infty;$$

where $m \geq 0$ *is an integer* , then for every $f \in C(m)$ and $k \geq 0$

(3.7) $$R_n(x) = \sum_{j=1}^n \alpha_j^{**} U_j(x) + \sum_{j=1}^n \beta_j^{**} V_j(x) + \sum_{j=1}^n \gamma_j^{**} W_j(x) + \sum_{j=0}^k \varphi_0^{**(j)} C_j(x)$$

satisfies the relations:

(3.8) $$|R_n(x) - \mathrm{f(x)}| = O(1)\omega\left(f, \frac{\log n}{\sqrt{n}}\right), \quad \text{for } 0 \leq x \leq cn^{-1}$$

(3.9) $$|R_n(x) - \mathrm{f(x)}| = O(1)\omega\left(f, \frac{\log n}{\sqrt{n}}\right), \quad \text{for } cn^{-1} \leq x \leq \Omega$$

where ω is the modulus of continuity.

4. PROOF OF THEOREM 1

Let $U_j(x)$, $V_j(x)$, $W_j(x)$ and $C_j(x)$ are polynomials of degree ≤ 3n+k satisfying conditions (4.1), (4.2), (4.3) and (4.4) respectively.

(4.1) For j = 1 ,2, … . , n

(4.1) $$\begin{cases} U_j(x_i) = \begin{cases} 0 & for \quad i \neq j \\ 1 & \quad\quad i = j \end{cases}, \quad U_j{}'(x_i) = 0, \quad U_j(y_i) = 0, \\ and \\ U_j^{(l)}(0) = 0, \quad i = 1(1)n, \quad l = 0,1,\ldots,k \end{cases}$$

For j = 1,2, … . , n

(4.2) $$\begin{cases} V_j(x_i) = 0, \quad V_j{}'(x_i) = \begin{cases} 0 & for \quad i \neq j \\ 1 & \quad\quad i = j \end{cases}, \quad V_j(y_i) = 0 \\ and \\ V_j^{(l)}(0) = 0, \quad i = 1(1)n \ , \ l = 0,1,\ldots,k \end{cases}$$

For j = 1,2, … . , n

(4.3) $$\begin{cases} W_j(x_i) = 0, \quad W_j{}'(x_i) = 0, \quad W_j(y_i) = \begin{cases} 0 & for \quad i \neq j \\ 1 & \quad\quad i = j \end{cases}, \\ and \\ W_j^{(l)}(0) = 0, \quad i = 1(1)n \ , \ l = 0,1,\ldots,k \end{cases}$$

and for l = 0,1, … … , k

(4.4) $$\begin{cases} C_k(x_i) = 0, \quad C_k{}'(x_i) = 0, \quad C_k(y_i) = 0 \\ and \\ C_k^{(l)}(0) = \begin{cases} 0 & for \quad i \neq j \\ 1 & \quad\quad i = j \end{cases}, \quad i = 1(1)n \end{cases}$$

To determine $W_j(x)$ let

(4.5) $$W_j(x) = C_1 x^{k+1} l_j^*(x) [L_n^k(x)]^2$$

where C_1 is a constant. $l_j^*(x)$ is defined in (2.8). $W_j(x)$ is a polynomial of degree ≤ 3n+k

By using (2.9) and (4.3) we determine

(4.6) $$C_1 = \frac{1}{y_j^{(k+1)}[L_n^k(y_j)]^2}$$

Hence we find the third fundamental polynomial $W_j(x)$ of degree ≤ 3n+k

To find second fundamental polynomial let

(4.7) $$V_j(x) = C_2 x^{k+1} L_n^{(k)}(x) L_n^{(k-1)}(x) l_j(x)$$

where C_2 is arbitrary constants. By using (2.8) and (4.2) we determine

(4.8) $$C_2 = \frac{1}{x_j^{(k+1)} L_n^{(k)'}(x_j) L_n^{(k-1)}(x_j)}$$

Hence we find the second fundamental polynomial $V_j(x)$ of degree ≤ 3n+k

Again let

(4.9) $$U_j(x) = C_3\, x^{k+1}[l_j(x)]^2 L_n^{(k-1)}(x) + C_4\, x^{k+1}(x - x_j)\,[l_j(x)]^2 L_n^{(k-1)}(x)$$

where C_3 and C_4 are arbitrary constanst, $l_j(x)$ is defined in (2.8). $U_j(x)$ is polynomial of degree ≤ 3 n+k satisfying the conditions (4.1) by which we obtain

(4.10) $$C_3 = \frac{1}{x_j^{(k+1)} L_n^{(k-1)}(x_j)}$$

(4.11) $$C_4 = -\frac{2}{x_j^{(k+1)} L_n^{(k-1)}(x_j)}$$

Hence we find the first fundamental polynomial $U_j(x)$ of degree ≤ 3n+k

To find $C_j(x)$, we assume $C_j(x)$ for fixed $j \epsilon \{0,1, \dots\dots, k-1\}$ in the form

(4.12) $$C_j(x) = p_j(x) x^j [L_n^{k-1}(x)]^2 L_n^k(x) + x^k L_n^{(k)}(x) L_n^{(k-1)}(x) g_n(x)$$

Where $p_j(x)$ and $g_n(x)$ are polynomials of degree k-j-1 and n respectively. Now it is clear that $C_j^{(l)}(0) = 0\ for$ $(l = 0, \dots\dots, j-1)$ and since $L_n^{(k)}(x_i) = 0$ and $L_n^{(k-1)}(y_i) = 0$ we get $C_j(x_i) = 0\ and\ C_j(y_i) = 0\ for\ i = 1(1)n$. The coefficient of the polynomial $p_j(x)$ are calculated by the system

(4.13) $$C_j^{(l)}(0) = \frac{d^l}{dx^l}\left[p_j(x) x^j [L_n^{k-1}(x)]^2 L_n^k(x)\right]_{x=0} = \delta_{i,j} \quad (l = j, \dots\dots, k-1)$$

Now from the equation $C_j^{(k)}(0) = 0$, we get

(4.14) $$c_j = g_n(0) = \frac{-1}{\binom{n+k}{k} k! L_n^{(k-1)}(0)} \frac{d^k}{dx^k}\left[p_j(x) x^j [L_n^{k-1}(x)]^2 L_n^k(x)\right]_{x=0}$$

Now using the condition $C_j'(x_i) = 0$ of (4.7), we get

(4.15) $g_n(x_i) = -(x_i)^{j-k} L_n^k(x_i) p_j(x_i)$ which implies $g_n(x)$ as follows

(4.16) $$g_n(x) = -\frac{L_n^{k-1}(x) p_j(x) + q_j(x) L_n^k(x)}{x^{k-j}}$$

where $q_j(x)$ is a polynomial of degree k-j

Using (4.12) and (4.14) we obtain $C_j(x)$ of degree ≤ 3n+k satisfying the conditions (4.4)

5. ESTIMATION OF THE FUNDAMENTAL POLYNOMIALS

Lemma 5.1: Let $the\ fundamental\ polynomial\ U_j(x),\ for\ \ j = 1,2, \dots, n\ be\ given\ by\ (3.2)\ then\ we\ have$

(5.1) $$\sum_{j=1}^n |U_j(x)| = O(1), \qquad \text{for } 0 \le x \le cn^{-1}$$

(5.2) $$\sum_{j=1}^n |U_j(x)| = O(1), \qquad \text{for } cn^{-1} \le x \le \Omega$$

where $U_j(x)$ is given in equation (3.2)

Proof: From (3.2) we have

(5.3) $$|U_j(x)| \le \frac{|x^{k+1}|\ [l_j^*(x)]^2 \left|L_n^{(k-1)}(x)\right|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k-1)}(x_j)\right|} + \frac{2|x^{k+1}| \left|l_j^*(x)\right| \left|L_n^{(k-1)}(x)\right| \left|L_n^k(x)\right|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k)'}(x)\right| \left|L_n^{(k-1)}(x_j)\right|}$$

(5.4) $$\sum_{j=1}^{n}\left|U_j(x)\right| \le \sum_{j=1}^{n}\frac{|x^{k+1}|\ [l_j^*(x)]^2 \left|L_n^{(k-1)}(x)\right|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k-1)}(x_j)\right|} + \sum_{j=1}^{n}\frac{2|x^{k+1}| \left|l_j^*(x)\right| \left|L_n^{(k-1)}(x)\right| |L_n^k(x)|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k)'}(x)\right| \left|L_n^{(k-1)}(x_j)\right|}$$
$$= \zeta_1 + \zeta_2$$

where

$$\zeta_1 = \sum_{j=1}^{n}\frac{|x^{k+1}|\ [l_j^*(x)]^2 \left|L_n^{(k-1)}(x)\right|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k-1)}(x_j)\right|}$$
$$\zeta_2 = \sum_{j=1}^{n}\frac{2|x^{k+1}| \left|l_j^*(x)\right| \left|L_n^{(k-1)}(x)\right| |L_n^k(x)|}{\left|x_j^{(k+1)}\right| \left|L_n^{(k)'}(x)\right| \left|L_n^{(k-1)}(x_j)\right|}$$

Thus by (3.2) and (2.16) equations (5.1) and (5.2) follows at once.

Lemma 3.3.2: Let $the\ fundamental\ polynomial\ V_j(x),\ for\ \ j = 1,2,\dots,n\ be\ given\ by\ (3.3)\ then\ \ we\ have$

(5.4) $$\sum_{j=1}^{n}\left|V_j(x)\right| = O(n^{-1}), \qquad \text{for } 0 \le x \le cn^{-1}$$

(5.5) $$\sum_{j=1}^{n}\left|V_j(x)\right| = O(1), \qquad \text{for } cn^{-1} \le x \le \Omega,$$

where $V_j(x)$ is given in equation (3.3)

Proof: From (3.3) we have

$$\left|V_j(x)\right| \le \frac{|x^{(k+1)}||l_j(x)|\left|L_n^{(k)}(x)\right|\left|L_n^{(k-1)}(x)\right|}{\left|x_j^{k+1}\right||L_n^{k-1}(x_j)|\left|L_n^{(k)'}(x_j)\right|}$$

(5.6) $$\sum_{j=1}^{n}\left|V_j(x)\right| \le \sum_{j=1}^{n}\frac{|x^{(k+1)}||l_j(x)|\left|L_n^{(k)}(x)\right|\left|L_n^{(k-1)}(x)\right|}{\left|x_j^{k+1}\right||L_n^{k-1}(x_j)|\left|L_n^{(k)'}(x_j)\right|}$$

Using (2.16), we get the result.

Lemma 5.3: Let $the\ fundamental\ polynomial\ W_j(x),\ for\ \ \ j = 1,2,\dots,n\ be\ given\ by\ (3.4)\ then\ \ we\ have$

(5.7) $$\sum_{j=1}^{n}\left|W_j(x)\right| = O(n^{-1}), \qquad \text{for } 0 \le x \le cn^{-1}$$

(5.8) $$\sum_{j=1}^{n}\left|W_j(x)\right| = O(1), \qquad \text{for } cn^{-1} \le x \le \Omega$$

where $W_j(x)$ is given in equation (3.4).

Proof: From (3.4) we have

(5.9) $$\sum_{j=1}^{n}\left|W_j(x)\right| \le \sum_{i=1}^{n}\frac{|x^{k+1}| \left|l_j^*(x)\right| \left[L_n^{(k)}(x)\right]^2}{\left|y_j^{k+1}\right| \ [L_n^k(y_j)]^2}$$

By equations (5.9) and (2.16), we yield the result.

Now we prove our main theorem in § 6.

6. PROOF OF MAIN THEOREM 3.2

Since $R_n(x)$ given by equation (3.1) is exact for all polynomial $S_n(x)$ of degree ≤ 3n+k, we have

(6.1) $$Q_n(x) = \sum_{j=1}^{n} Q_n(x_j)U_j(x) + \sum_{j=1}^{n} Q_n{}'(x_j)V_j(x) + \sum_{j=1}^{n} Q_n(y_j)W_j(x) + \sum_{j=0}^{k} Q_n(x_0)C_j(x)$$

From equation (3.2.1) and (3.4.1) we get

(6.2) $$\begin{aligned}|f(x) - R_n(x)| &\le |f(x) - Q_n(x)| + |Q_n(x) - R_n(x)|\\ &\le |f(x) - Q_n(x)| + \sum_{j=1}^{n}\left|f(x_j) - Q_n(x_j)\right| \left|U_j(x)\right|\\ &\quad + \sum_{j=1}^{n}\left|f'(x_j) - Q_n{}'(x_j)\right| \left|V_j(x)\right|\\ &\quad + \sum_{j=1}^{n}\left|f(y_j) - Q_n(y_j)\right| \left|W_j(x)\right|\\ &\quad + \sum_{j=0}^{k}\left|f^l(x_0) - Q_n{}^l(x_0)\right| \left|C_j(x)\right|\end{aligned}$$

Thus (6.2) and Lemmas 5.1, 5.2, 5.3 completes the proof of the theorem.

REFERENCES

1. Balázs, J. and Turán .P., Notes on interpolation, II. Acta Math. Acad. Sci. Hunger., 8 (1957), pp. 201- 215.
2. Balázs, J., Weighted (0; 2)- interpolation on the roots of the ultraspherical polynomials, (in Hungarian: Súlyozott (0; 2)-interpoláció ultraszférikus polinom gyökein), Mat. Fiz. Tud. Oszt. Oszt. Közl., 11 (1961), 305-338
3. Balázs, J.P., Modified weighted (0, 2) interpolation, Approx. Theory, Marcel Dekker Inc., New York, 1998, 1-73
4. Chak, A.M. and Szabados, J.: On (0, 2) interpolation for Laguerre abscissas, Acta Math. Acad. Hung. 49 (1987) 415-455
5. Lénárd, M.: Pál type- interpolation and quadrature formulae on Laguerre abscissas, Mathematica Pannonica 15/2 (2004), 265-274
6. Lénárd, M., On weighted (0; 2)-type interpolation, Electron. Trans. Numer. Anal., 25 (2006), 206-223.
7. Lénárd, M., On weighted lacunary interpolation, Electron. Trans. Numer. Anal., 37 (2010), 113-122.
8. Mathur, P. and Datta S., On Pál -type weighted lacunary (0; 2; 0)- interpolation on infinite interval (-∞, +∞), Approx. Theor. Appl., 17 (4) (2001), 1-10.
9. Mathur, K.K. and Srivastava, R.: Pál –type Hermite interpolation on infinite interval, J.Math. Anal. and App. 192, 346-359 (1995)
10. Pál L. G., A new modification of the Hermite-Fejér interpolation, Anal. Math., 1 (1975), 197-205
11. Prasad, J. and Jhunjhunwala, N.: Some lacunary interpolation problems for the Laguerre abscissas, Demonstratio Math. 32 (1999), no. 4, 781-788.
12. Szegö G., *Orthogonal polynomials*, Amer. Math. Soc., Coll. Publ., 23, New York, 1939 (4th ed.1975).
13. Szili, L., Weighted (0, 2) interpolation on the roots of Hermite Polynomials Annals Univ. Sci. Budapest, De Rol Eot, 27 (1984), 155-166.
14. Szili, L., Weighted (0, 2) interpolation on the roots of classical orthogonal polynomials, Bull. Of Allahabad Math. Soc., 8-9 (1993-94), 1-10.
15. Srivastava, R.and Mathur, K.K.: weighted (0;0,2)- interpolation on the roots of Hermite polynomials, Acta Math. Hunger. 70 (1-2)(1996), 57-73..
16. Xie, T.F., On] Pál's problem, Chinese Quart. J. Math. 7 (1992), 48-2.

Source of support: Nil, Conflict of interest: None Declared.
