INTUITIONISTIC LEFT OPERATOR SEMIGROUP OF AN ORDERED Γ-SEMIGROUPS

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ABSTRACT

In this paper we obtain operator ordered semigroup of an ordered Γ-semigroups have been made to work by obtaining various relationship between the intuitionistic fuzzy ordered filters of an ordered Γ-semigroups and that of its left operator semigroups. Also we obtain some theorem related to such left operator semigroups

Keywords: Ordered Γ- semigroup, intuitionistic fuzzy ordered filters, left operator Semigroups.

1. INTRODUCTION

The concept of a fuzzy set given by L.A. Zadeh in his classic paper of 1965 [13] has been used by many authors to generalize some of the basic notions of algebra. Fuzzy semigroups have been first considered by N. Kuroki [8], and fuzzy ordered groupoids and ordered semigroups, by Kehayopulu and Tsingelis [6] [7]. The notion of a Γ-semigroup was introduced by Sen [10]. Many classical notions of semigroups have been extended to Γ-semigroups. The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [2][3][4]. In [11], M. Shabir and A. Khan introduced fuzzy filters in ordered semigroups. In [12] Sujith kumar, Pavel pal, Samith kumar, Majumder and Parimal Das, operator semigroups in their paper Atanassov’s intuitionistic fuzzy ideals of a po- Γ- semigroups. In this paper we obtain left operator ordered semigroup of an ordered Γ-semigroups and also study about the relation between left operator ordered semigroup and intuitionistic fuzzy ordered filters of an ordered Γ-semigroups. Also we obtain some theorem related to such operator semigroups.

2. PRELIMINARIES

Definition 2.1: Let S be a Γ-semigroup. Let us define a relation ρ on S×Γ as follows: (x, α) ρ (y, β) iff xαs = yβs for all s ϵ S and γxα = γyβ for all γ ϵ Γ. Then ρ is an equivalence relation. Let [x,α] denote the equivalence class containing (x,α). Let L={[x,α] : xϵS,αϵΓ}. Then L is a semigroup with respect to the multiplication defined by [x,α][y,β]=[xαy, β]. This semigroup L is called left operator semigroup of the Γ-semigroup S.

Definition 2.2: Let S be a Γ-semigroup. Let us define a relation ρ on S×Γ as follows: (x, α) ρ (y, β) iff xαs = yβs for all s ϵ S and γxα = γyβ for all γ ϵ Γ. Then ρ is an equivalence relation. Let [x,α] denote the equivalence class containing (x,α). Let L={[x,α] : xϵS,αϵΓ}. Then L is a semigroup with respect to the multiplication defined by [x,α][y,β]=[xαy, β]. This semigroup L is called left operator semigroup of the Γ-semigroup S.

Definition 2.3: Let (S,Γ,≤) be an ordered Γ-semigroup we define a relation ≤ on L by [a,α] ≤ [b,β] iff aαs ≤ bβs for all s ϵ S and γaα ≤ γbβ for all γ ϵ Γ. Then L becomes ordered Γ-semigroup.

Definition 2.4: If there exists an element [a,α] ϵ L such that aαs = s for all s ϵ S then [a,α] is called the left unity of S.
Definition 2.5: For an intuitionistic fuzzy subset $A = \langle \mu_A, \nu_A \rangle$ of $L$, define an intuitionistic fuzzy subset $A^+ = \langle \mu_A^+, \nu_A^+ \rangle$ of $S$ by

$$
\mu_A^+(x) = \bigwedge_{\alpha \in \Gamma} \mu_A([x, \alpha])
$$

and

$$
\nu_A^+(x) = \bigvee_{\alpha \in \Gamma} \nu_A([x, \alpha]),
$$

where $x \in S$. For an intuitionistic fuzzy subset $B = \langle \mu_B, \nu_B \rangle$ of $S$, define an intuitionistic fuzzy subset $B^+ = \langle \mu_B^+, \nu_B^+ \rangle$ by

$$
\mu_B^+(x) = \bigwedge_{\alpha \in \Gamma} \mu_B([a, \alpha])
$$

and

$$
\nu_B^+(x) = \bigvee_{\alpha \in \Gamma} \nu_B([a, \alpha]),
$$

where $[a, \alpha] \in L$.

3. LEFT OPERATOR SEMIGROUP OF AN ORDERED $\Gamma$-SEMIGROUPS

Theorem 3.1: If $\{A_i/i \in I\}$ is a collection of intuitionistic fuzzy subsets of $L$. Then

$$(\bigcap_{i \in I} \mu_{A_i})^+ = (\bigcap_{i \in I} \mu_{A_i})^+ \quad \text{and} \quad (\bigcup_{i \in I} \nu_{A_i})^+ = (\bigcup_{i \in I} \nu_{A_i})^+$$

Proof: Let $x \in S$. Now

$$(\bigcap_{i \in I} \mu_{A_i})^+(x) = \bigwedge_{\alpha \in \Gamma} \left[ (\bigcap_{i \in I} \mu_{A_i})([x, \alpha]) \right]$$

Also

$$(\bigcup_{i \in I} \nu_{A_i})^+(x) = \bigvee_{\alpha \in \Gamma} \left[ (\bigcup_{i \in I} \nu_{A_i})([x, \alpha]) \right]$$

Hence

$$(\bigcap_{i \in I} \mu_{A_i})^+ = (\bigcap_{i \in I} \mu_{A_i})^+ \quad \text{and} \quad (\bigcup_{i \in I} \nu_{A_i})^+ = (\bigcup_{i \in I} \nu_{A_i})^+$$

Theorem 3.2: If $A = \langle \mu_A, \nu_A \rangle$ is an intuitionistic fuzzy ordered filter of $L$, then the intuitionistic fuzzy set $A^+ = \langle \mu_A^+, \nu_A^+ \rangle$ is an intuitionistic fuzzy ordered filter of $S$.

Proof: Let $A$ be an intuitionistic fuzzy ordered filter of $L$. Then $\mu_A(1_L) = 1$ and $\nu_A(1_L) = 0$.

Also

$$\mu_A^+(1_S) = (\bigcap_{i \in I} \mu_{A_i})(1_S) = 1$$

and

$$\nu_A^+(1_S) = (\bigcup_{i \in I} \nu_{A_i})(1_S) = 0.$$ 

So $A^+$ is non-empty.

$$
\mu_A^+(a \circ b) = \bigwedge_{\gamma \in \Gamma} \mu_A([a \circ b, \gamma]) = \bigwedge_{\gamma \in \Gamma} \mu_A([a, \alpha] [b, \gamma])
$$

$$
\leq \bigwedge_{\gamma \in \Gamma} \mu_A([a, \alpha])
$$

and

$$
\nu_A^+(a \circ b) = \bigvee_{\gamma \in \Gamma} \mu_A([a \circ b, \gamma]) = \bigvee_{\gamma \in \Gamma} \mu_A([a, \alpha] [b, \gamma])
$$

$$
\leq \bigvee_{\gamma \in \Gamma} \mu_A([b, \gamma]) = \nu_A^+([a, \alpha])$$

Also

$$
\mu_A^+(a \circ b) = \bigwedge_{\gamma \in \Gamma} \mu_A([a \circ b, \gamma]) = \bigwedge_{\gamma \in \Gamma} \mu_A([a, \alpha] [b, \gamma])
$$

$$
\leq \bigwedge_{\gamma \in \Gamma} \mu_A([b, \gamma]) = \nu_A^+([a, \alpha])$$

and

$$
\nu_A^+(a \circ b) = \bigvee_{\gamma \in \Gamma} \mu_A([a \circ b, \gamma]) = \bigvee_{\gamma \in \Gamma} \mu_A([a, \alpha] [b, \gamma])
$$

$$
\leq \bigvee_{\gamma \in \Gamma} \mu_A([b, \gamma]) = \nu_A^+([a, \alpha]).$$
and L is the left operator of the ordered $\Gamma$-semigroup S.

$$\mu_{A}(a \alpha b) = \min\{\mu_{A}(a), \mu_{A}(b)\}$$

Similarly

$$v_{A}(a \alpha b) = \forall_{\gamma \in \Gamma} v_{A}([a \alpha b, \gamma])$$

$$= \forall_{\gamma \in \Gamma} v_{A}([a, \alpha][b, \gamma])$$

$$\geq \forall_{\gamma \in \Gamma} v_{A}([a, \alpha])$$

$$= v_{A}(a, \alpha) \geq \forall_{\gamma \in \Gamma} v_{A}([a, \gamma]) = v_{A}(a) .$$

Also

$$v_{A}(a \alpha b) = \forall_{\gamma \in \Gamma} v_{A}([a \alpha b, \gamma])$$

$$= \forall_{\gamma \in \Gamma} v_{A}([a, \alpha][b, \gamma])$$

$$\geq \forall_{\gamma \in \Gamma} v_{A}([b, \gamma]) = \nu_{A}(b).$$

$$\nu_{A}(a \alpha b) = \max\{\nu_{A}(a), \nu_{A}(b)\} .$$

Let $a, b \in S$ be such that $a \leq b$. Then $[a, \alpha] \leq [b, \alpha]$, for all $\alpha \in \Gamma$.

Since A is intuitionistic fuzzy ordered filter of L, $\mu_{A}(a, \alpha) \leq \mu_{A}(b, \alpha)$, for all $\alpha \in \Gamma$.

This implies $\inf_{\alpha \in \Gamma} \mu_{A}(a, \alpha) \leq \inf_{\alpha \in \Gamma} \mu_{A}(b, \alpha)$. Therefore $\mu_{A}(a) \leq \mu_{A}(b)$.

Now $\nu_{A}(a, \alpha) \geq \nu_{A}(b, \alpha)$, for all $\alpha \in \Gamma$. This implies $\sup_{\alpha \in \Gamma} \nu_{A}(a, \alpha) \geq \sup_{\alpha \in \Gamma} \nu_{A}(b, \alpha)$.

Therefore $\nu_{A}(a) \geq \nu_{A}(b)$. Hence $A^{+} = <\mu_{A}, \nu_{A}>$ is an intuitionistic fuzzy ordered filter of S.

**Theorem 3.3:** If $A = <\mu_{A}, \nu_{A}>$ is an intuitionistic fuzzy ordered filter of S, then the intuitionistic set $A^{+} = <\mu_{A}^{+}, \nu_{A}^{+}>$ is an intuitionistic fuzzy ordered filter of L.

**Proof:** Let A be an intuitionistic fuzzy ordered filter of S. Then $\mu_{A}(1_{S}) = 1$ and $\nu_{A}(1_{S}) = 0$. Therefore now,

$$\mu_{A}(1_{S}, \gamma) = \bigwedge_{s \in S} \mu_{A}(1_{S}, \gamma) = \mu_{A}(1_{S}) = 1$$

and

$$\nu_{A}(1_{S}, \gamma) = \bigvee_{s \in S} \nu_{A}(1_{S}, \gamma) = \nu_{A}(1_{S}) = 0.$$ So $A^{+}$ is non-empty.

Let $[a, \alpha], [b, \beta] \in L$

$$\mu_{A^{+}}([a, \alpha], [b, \beta]) = \mu_{A^{+}}([a \alpha b, \beta]) = \bigwedge_{s \in S} \mu_{A}(a \alpha b | s \beta)$$

$$\leq \bigwedge_{s \in S} \mu_{A}(b | s \beta) = \bigwedge_{s \in S} \mu_{A}([b, \beta])$$

$$\mu_{A^{+}}([a, \alpha], [b, \beta]) = \mu_{A^{+}}([a \alpha b, \beta]) = \bigwedge_{s \in S} \mu_{A}(a \alpha b | s \beta)$$

$$= \bigwedge_{s \in S} \left( \min\{\mu_{A}(a), \mu_{A}(b | s \beta)\} \right)$$

$$= \bigwedge_{s \in S} \left( \min\{\mu_{A}(a), \min\{\mu_{A}(b), \mu_{A}(s)\}\} \right)$$

$$\leq \bigwedge_{s \in S} \left( \min\{\mu_{A}(a), \mu_{A}(s)\} \right)$$

$$= \bigwedge_{s \in S} \mu_{A}(a | s \alpha) = \bigwedge_{s \in S} \mu_{A}([a, \alpha]).$$

$$\nu_{A^{+}}([a, \alpha], [b, \beta]) = \nu_{A^{+}}([a \alpha b, \beta]) = \bigvee_{s \in S} \nu_{A}(a \alpha b | s \beta)$$

$$\geq \bigvee_{s \in S} \nu_{A}(b | s \beta) = \bigvee_{s \in S} \nu_{A}([b, \beta])$$
\[ \nu_A \cdot ([a, \alpha], [b, \beta]) = \nu_A \cdot [a b \alpha \beta] = \bigvee_{s \in S} \nu_A (a b \alpha \beta s) \]

\[ = \bigvee_{s \in S} \left( \min \{ \nu_A (a), \nu_A (b) s \} \right) \]

\[ \geq \bigvee_{s \in S} \left( \min \{ \nu_A (a), \min \{ \nu_A (b), \nu_A (s) \} \} \right) \]

\[ \geq \bigvee_{s \in S} \nu_A (a s) = \bigvee_{s \in S} \nu_A [a, a] \]

Hence \( A^+ \) is an intuitionistic fuzzy ordered filter of \( S \)

**Theorem 3.4:** Let \( S \) be an ordered \( \Gamma \)-semigroup with unities and \( L \) be its left operator semigroup. Then there exists an ordered \( \Gamma \)-isomorphism via inclusion preserving \( A \rightarrow A^+ \) between the set of all intuitionistic fuzzy ordered filters of \( S \) and the set of all intuitionistic fuzzy ordered filters of \( L \), where \( A \leq \mu_A \nu_A^+ \) is an intuitionistic fuzzy ordered filter of \( S \).

**Proof:** Let \( A \) be an intuitionistic fuzzy ordered filter of \( S \). Then clearly \( A^+ \) is also an intuitionistic fuzzy ordered filter of \( S \). Let \( a \in S \). Then

\[ (\mu_A^+)^+(a) = \bigvee_{\gamma \in \Gamma} \mu_A^+([a, \gamma]) = \bigvee_{\gamma \in \Gamma} \left[ \bigwedge_{s \in S} \mu_A ([a, \gamma] s) \right] \]

\[ \geq \bigwedge_{s \in S} \left[ \bigvee_{\gamma \in \Gamma} \mu_A ([a, \gamma] s) \right] = \mu_A (a) \]

Also \( \nu_A (x) = \nu_A (e x) \geq \bigvee_{\gamma \in \Gamma} \left[ \bigwedge_{s \in S} \nu_A ([a, \gamma] s) \right] = \bigvee_{\gamma \in \Gamma} \left( \nu_A (a) \right) \)

Hence \( A \leq (\mu_A^+)^+ \)

Let \( b \in S \), \( a \in \Gamma \) and \( [b, a] \in L \).

\[ (\mu_A^+)^+(b, a) = \bigvee_{s \in S} \mu_A^+([b, a] s) = \bigvee_{s \in S} \left[ \bigwedge_{\gamma \in \Gamma} \mu_A ([b, a] s, \gamma) \right] \]

\[ = \bigvee_{s \in S} \left[ \bigwedge_{\gamma \in \Gamma} \mu_A ([b, a] s, \gamma) \right] \]

\[ \leq \bigwedge_{s \in S} \left[ \bigvee_{\gamma \in \Gamma} \mu_A ([b, a] s, \gamma) \right] = \mu_A ([b, a]) \]

Also \( \nu_A (x) = \nu_A (e x) \geq \bigvee_{\gamma \in \Gamma} \left[ \bigwedge_{s \in S} \nu_A ([b, a] s) \right] = \bigvee_{\gamma \in \Gamma} \left( \nu_A (b, a) \right) \)

\[ \vdash (\nu_A^+)^+ \leq A \]

Also Let \( [a, \beta] \in L \). Let \([e, \delta] \) be the left unity of \( L \). Such that \([e, \delta] [a, \beta] = [a, \beta] \)

\[ \mu_A ([a, \beta]) = \mu_A ([e, \delta] [a, \beta]) \leq \bigwedge_{s \in S} \left[ \mu_A ([S, \delta] [a, \beta]) \right] \]

\[ \leq \bigwedge_{s \in S} \left[ \bigwedge_{\gamma \in \Gamma} \mu_A ([S, \gamma] [a, \beta]) \right] \]

\[ = (\mu_A^+)^+ ([a, \beta]) \]
Also
\[\nu_A([a, \beta]) = \nu_A([\sigma, \delta][a, \beta]) \geq \bigvee_{s \in S} \nu_A([S, \delta][a, \beta]) \geq \bigvee_{s \in S} \bigvee_{\gamma \in \Gamma} \nu_A([S, \gamma][a, \beta]) = (\nu_A^+)([a, \beta])\]

Hence \(A \subseteq (A^+)\).  

Hence the correspondence \(A \mapsto A^+\) is bijection. Now let \(A_1, A_2\) be intuitionistic fuzzy ordered filter of \(S\) such that \(A_1 \subseteq A_2\)  

Then
\[\mu_{A_2^+}([a, \alpha]) = \bigwedge_{\alpha \in \Gamma} H(A_2^+)(a \alpha) \leq \bigwedge_{\alpha \in \Gamma} H(A_2^+)(a \alpha) = \bigwedge_{\alpha \in \Gamma} H(A_2^+)(a \alpha) \text{ for all } [a, \alpha] \in \Gamma\]
\[\nu_{A_1^{-1}}([a, \alpha]) = \bigvee_{\alpha \in \Gamma} \nu_{A_1^{-1}}([a, \alpha]) \geq \bigvee_{\alpha \in \Gamma} \nu_{A_1^{-1}}([a, \alpha]) = \nu_{A_2^+}([a, \alpha]) \text{ for all } [a, \alpha] \in \Gamma\]

Therefore \(A_1^{-1} \subseteq A_2^+\).  

Now,
\[\left(\mu_{A_1^{-1}}\right)(a) = \bigwedge_{\alpha \in \Gamma} H(A_1^{-1})(a \alpha) = \bigwedge_{\alpha \in \Gamma} \bigwedge_{s \in S} H(A_1^{-1})(a \alpha) \leq \bigwedge_{\alpha \in \Gamma} \bigwedge_{s \in S} H(A_2^+)(a \alpha)\]
\[= \bigwedge_{\alpha \in \Gamma} H(A_2^+)(a \alpha) = (\mu_{A_2^+})'(a) \text{ for all } a \in L\]

\[\left(\nu_{A_1^{-1}}\right)(a) = \bigvee_{\alpha \in \Gamma} \nu_{A_1^{-1}}([a, \alpha]) \leq \bigvee_{\alpha \in \Gamma} \nu_{A_1^{-1}}([a, \alpha]) = \nu_{A_2^+}([a, \alpha]) \text{ for all } a \in L\]

So \((A_1^{-1}) \subseteq (A_2^+)\).  

Also \(A_1 \subseteq A_2\)
\[\mu_{A_1^{-1}}(a) = \bigwedge_{\gamma \in \Gamma} H(A_1^{-1})(a \gamma) \leq \bigwedge_{\gamma \in \Gamma} H(A_2^+)(a \gamma) = \mu_{A_2^+}(a) \text{ for all } a \in L\]
\[\nu_{A_1^{-1}}(a) = \bigvee_{\gamma \in \Gamma} \nu_{A_1^{-1}}([a, \gamma]) \geq \bigvee_{\gamma \in \Gamma} \nu_{A_1^{-1}}([a, \gamma]) = \nu_{A_2^+}(a) \text{ for all } a \in L\]

Therefore \(A_1^{-1} \subseteq A_2^+\).  

So the mapping \(A \rightarrow A^+\) is an ordered \(\Gamma\)-isomorphism.

REFERENCES


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