

## CONVECTIVE HEAT TRANSFER FLOW OF A ROTATING NANOFLUID IN A VERTICAL CHANNEL WITH HEAT SOURCES

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### ABSTRACT

*The effect of heat sources on convective heat transfer flow of a nanofluid in a vertical channel. Analytical closed form solutions are obtained for both the momentum and the energy equations. Graphs are used to illustrate the significance of key parameters on the nanofluid velocity and temperature distributions.*

**Keywords:** Heat Sources, Nanofluid, Vertical channel, rotating fluid.

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### 1. INTRODUCTION

Present days, researchers are more concentrating on enhancement of heat transfer. The low thermal conductivity of conventional heat transfer fluids, such as water, is considered a primary limitation in enhancing the heat transfer performance. Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers, with sizes typically on the order of 1–100 nm, suspended in a liquid. Nanofluids are characterized by an enrichment of a base fluid like water, toluene, ethylene glycol or oil with nanoparticles in variety of types like Metals, Oxides, Carbides, Carbon, Nitrides, etc. Today nanofluid are sought to have wide range of applications in medical application, biomedical industry, detergency, power generation in nuclear reactors and more specifically in any heat removal involved industrial applications. The ongoing research ever since then has extended to utilization of nanofluids in microelectronics, fuel cells, pharmaceutical processes, hybrid-powered engines, engine cooling, vehicle thermal management, domestic refrigerator, chillers, heat exchanger, nuclear reactor coolant, grinding, machining, space technology, Defense and ships, and boiler flue gas temperature reduction [Agarwal *et al.* (2011)]. Indisputably, the nanofluids are more stable and have acceptable viscosity and better wetting, spreading, and dispersion properties on a solid surface [Akbarinia *et al.* (2011), Nguyen *et al.* (2007)]. Several reviews [Ghadimi *et al.* (2011), Mahabudul *et al.* (2012)] on nanofluids with respect to thermal and rheological properties have been reported.

Thus, nanofluids have an ample collection of potential applications in electronics, pharmaceutical processes, hybrid-powered engines, automotive and nuclear applications where enhanced heat transfer or resourceful heat dissipation is required. In view of these, [Kiblinki *et al.* (2002)] suggested four possible explanations for the anomalous increase in the thermal conductivity of nanofluids. These are nanoparticles clustering, Brownian motion of the particles, molecular level layering of the liquid/particles interface and ballistic heat transfer in the nanoparticles. Despite a vast amount of literature on the flow of nanofluid model proposed by [Buongiorno (2006)], we are referring to a few recent studies [Alsaedi *et al.* (2012), Hajipour and Dekhordi (2012), Rana and Bhargava (2012)] in this article. However, we are following the nanofluid model proposed by [Tiwari and Das (2007)], which is being used by many current researchers [Hamad and Ferdows (2012), Hamad and Pop (2011), Norifah *et al.* (2012)] on various flow fields.

The study of MHD flow and heat transfer due to the effect of a magnetic field in a rotating frame of reference has attracted the interest of many investigators in view of its applications in many industrial, astrophysical (dealing with the sunspot development, the solar cycle and the structure of a rotating magnetic stars), technological and engineering applications (MHD generators, ion propulsion, MHD pumps, etc.) and many other practical applications, such as in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet). Many authors have studied the flow and heat transfer in a rotating system with various geometrical situations [Hickman (1957), Hide (1960), Mazumder (2012)]. [Hamad (2011)] investigated the effect of a transverse magnetic field on free convection flow of a nanofluid past a vertical semi-infinite flat plate. Recently, [Satya Narayana *et al.* (2013)] studied the Hall current and radiation absorption effects on MHD micropolar fluid in a rotating system. Some other related works can also be found in recent papers [Kameswaran *et al.* (2012), Kesavaiah *et al.* (2011), Rushi kumar and Sivaraj (2013), Srinivas *et al.* (2012)].

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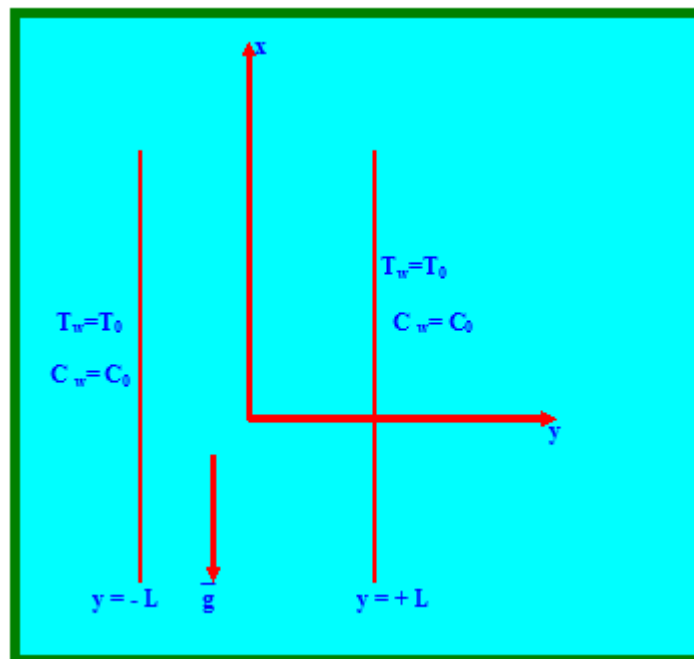
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The study of flow and heat transfer distinctiveness of a nanofluid past a vertical plate with thermal radiation in a rotating frame of reference. [Salem and Aziz (2008) have analyzed the effect of Hall currents and chemical reaction on the unsteady flow, heat and mass transfer laminar of a viscous, electrically conducting fluid over a continuously stretching surface in the presence of heat generation/absorption. Aziz (2010) has investigated the flow and heat transfer of a viscous fluid flow over a unsteady stretching surface with Hall effects. Sarojamma *et al.* (2015) have investigated the influence of Hall currents on cross diffusive convection in a MHD boundary layer flow on stretching sheet in porous medium with non-uniform heat source. Sarojamma *et al.* (2015) have discussed the effect of Hall currents on the flow induces by a stretching surface. Satyanarayana. *et. al* (2013) have analysed the effect of heat sources on convective heat transfer flow of a nanofluid past a vertical plate in a rotating system .

In this paper we investigate the effect of Heat source on unsteady convective heat transfer flow of a rotating nanofluid in a vertical channel. Analytical closed form solutions are obtained for both the momentum and the energy equations. The velocity and temperature are analysed for different variations of parameters graphically. The skin friction and Nusselt number on the walls are evaluated numerically for different parametric variations. It is found that an increase in nanoparticle concentration  $\phi$  reduces the primary velocity, enhances the secondary and temperature in the flow region. The skin friction components reduce, the Nusselt number increases with increase in  $\phi$ .

## 2. FORMULATION OF THE PROBLEM

We analyse the steady, three dimensional flow of a nanofluid consisting of a base fluid and small nanoparticles in a vertical parallel channel under the influence of a uniform magnetic field of strength  $H_0$ . The flow is assumed to be in the x-direction which is taken along the plane in an upward direction and z-axis is normal to the plate. Also it is assumed that the whole system is rotating with a constant angular velocity vector  $\vec{\Omega}$  about the z-axis. Due to a semi-infinite plate surface assumption, the flow variables are functions of z and t only. Figure. 1 shows that the problem under consideration and the co-ordinate system.



**Figure-1:** Schematic diagram of the problem

:

The equations governing the flow and heat transfer of a nanofluids are Equation of Continuity

$$\frac{\partial w}{\partial z} = 0$$

**Equation of Momentum**

(1)

$$w \frac{\partial u}{\partial z} - 2\Omega v = \frac{1}{\rho_{nf}} \left( \mu_{nf} \frac{\partial^2 u}{\partial z^2} \right) + (\rho \beta_{nf}) g (T - T_\infty) - (\sigma \mu_e^2 H_o^2) u \quad (2)$$

$$w \frac{\partial v}{\partial z} + 2\Omega u = \frac{1}{\rho_{nf}} \left( \mu_{nf} \frac{\partial^2 v}{\partial z^2} - (\sigma \mu_e^2 H_o^2) v \right) \quad (3)$$

### Equation of Energy

$$w \frac{\partial T}{\partial z} = k_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{Q_H}{(\rho C_p)_{nf}} (T - T_\infty) \quad (4)$$

where u,v are the velocity components along x,y directions respectively, T is the temperature of the fluid, thermal conductivity of the nanofluid  $k_{nf}$ ,  $(\rho\beta)_{nf}$  is thermal expansion coefficient of nanofluid,  $Q_H$  is the heat source coefficient,  $(\rho C_p)_{nf}$  heat capacitance  $C_p$  of the nanofluid,  $\rho_{nf}$  is effective density of the nanofluid density and g is gravity.

The boundary conditions are

$$\begin{aligned} u(\pm L) &= 0, & v(\pm L) &= 0, \\ T(-L) &= T_1, & T(+L) &= T_2 \end{aligned} \quad (5)$$

The properties of the nanofluids are defined as follows

$$\left. \begin{aligned} \mu_{nf} &= \mu_f / (1 - \phi)^{2.5} & \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}} & \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s \\ (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s & (\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \\ k_{nf} &= \frac{k_f(k_s + 2k_f - 2\phi(k_f - k_s))}{(k_s + 2k_f + 2\phi(k_f - k_s))} \end{aligned} \right\} \quad (6)$$

**Table 1:** Thermo-physical properties of water and nanoparticles

Physical properties	Water	Cu	Al <sub>2</sub> O <sub>3</sub>
C <sub>p</sub> (J/kg K)	4179	385	765
ρ (kg/m <sup>3</sup> )	997.1	8933	3970
K (W/m K)	0.613	400	40
β X 10 <sup>-5</sup> (1/K)	21	1.67	0.85

We consider the solution of equation (1) as:

$$w = -w_0 \quad (7)$$

We introduce the following dimensionless variables:

$$\left. \begin{aligned} \eta &= \frac{z}{L}, \quad u' = \frac{u}{w_0}, \quad v' = \frac{v}{w_0}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad G = \frac{\beta g (T_2 - T_1) L^2}{\mu_f w_0} \\ S &= \frac{w_0 L}{\mu_f}, \quad M = \frac{\sigma \mu_e^2 H_0^2 L^2}{\rho_f \mu_f}, \quad Q = \frac{Q_H L^2}{k_f} \end{aligned} \right\} \quad (8)$$

Equations (2)-(4) in the non-dimensional form are

$$-S \frac{\partial u}{\partial \eta} - 2Rv = \frac{1}{A_1 A_3} \frac{\partial^2 u}{\partial \eta^2} + \frac{A_4}{A_3} G \theta - \frac{M^2}{A_3} u \quad (9)$$

$$-S \frac{\partial v}{\partial \eta} + 2Ru = \frac{1}{A_1 A_3} \frac{\partial^2 v}{\partial \eta^2} - \frac{M^2}{A_3} v \quad (10)$$

$$-S \frac{\partial \theta}{\partial \eta} = \frac{1}{P_r} \left( \frac{A_2}{A_5} \right) \frac{\partial^2 \theta}{\partial \eta^2} - \frac{1}{A_5} Q \theta \quad (11)$$

Where

$$A_1 = (1 - \phi)^{2.5}, \quad A_2 = \frac{k_{nf}}{k_f} + \frac{4F}{3}, \quad A_3 = 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right)$$

$$A_4 = 1 - \phi + \phi \left( \frac{(\rho\beta)_s}{(\rho\beta)_f} \right), \quad A_5 = 1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}$$

The boundary conditions (2.5) reduce to

$$u(\pm 1) = 0, v(\pm 1) = 0, \theta(-1) = 0, \theta(+1) = 1 \quad (12)$$

In view of the fluid velocity in the component form:

$$V(z, t) = u(z, t) + iv(z, t)$$

The equations (9) and (10) reduce to

$$-S \frac{\partial V}{\partial \eta} - 2iRV = \frac{1}{A_1 A_3} \frac{\partial^2 V}{\partial \eta^2} + \frac{A_4}{A_3} G\theta - \frac{M^2}{A_2} V \quad (13)$$

The boundary conditions (12) reduce to

$$V(\pm 1) = 0 \quad \theta(-1) = 0, \theta(+1) = 0, \quad (14)$$

### 3. METHOD SOLUTION

Solving the equations (11) and (12) we get

$$V(z) = \exp\left(-\frac{b_9 \eta}{2}\right) (B_5 \cosh(m_3 \eta) + B_6 \sinh(m_3 \eta) + b_{14} \exp((m_1 - b_1) \eta) + b_{15} \exp(-(m_1 + b_1) \eta))$$

$$\theta(z) = \exp(-b_1 \eta) \left( \frac{\cosh(m_1 \eta)}{\cosh(m_1)} \sinh(b_1) + \frac{\sinh(m_1 \eta)}{\sinh(m_1)} \cosh(b_1) \right)$$

The physical quantities of interest are skin friction and Nusselt number which are, respectively, defined as:

$$C_f = \frac{\tau_w}{\rho_f U_o^2}$$

$$Nu = \frac{x q_w}{k_f (T_w - T_\infty)} \quad (15)$$

Where  $\tau_w$  and  $q_w$  are the wall shear and the wall heat flux from the plate respectively, which are given by

$$\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial z} \right)_{z=0} \quad \text{and} \quad q_w = -k_{nf} \left( \frac{\partial T}{\partial z} \right)_{z=0}$$

In view of Equation (15) we obtain

$$C_f = \frac{1}{A_1} V'(+1) = b_{18} \cosh(m_3) + b_{19} \sinh(m_3) + b_{20} \exp(m_1 - b_1) + b_{21} \exp(-(m_1 + b_1))$$

$$= \frac{1}{A_1} V'(-1) = b_{22} \cosh(m_3) + b_{23} \sinh(m_3) + b_{20} \exp(-(m_1 - b_1)) + b_{21} \exp(-(m_1 + b_1))$$

$$Nu = -\frac{k_{nf}}{k_f} \theta'(\pm 1) = -A_2 \theta'(\pm 1)$$

$$Nu(+1) = -b_1 \exp(-b_1) (\sinh(b_1) + \cosh(b_1)) + m_1 \exp(-b_1) (\sinh(b_1) \tanh(m_1) + \cosh(b_1) \coth(m_1))$$

$$Nu(-1) = -b_1 \exp(b_1) (\sinh(b_1) - \cosh(b_1)) + m_1 \exp(b_1) (-\sinh(b_1) \tanh(m_1) + \cosh(b_1) \coth(m_1))$$

Where

$$m_1 = \frac{\sqrt{(A_5 S \text{Pr})^2 + 4 A_2 Q}}{2 A_2}, \quad b_1 = \frac{S \text{Pr} A_5}{2 A_2}, \quad b_2 = \frac{\cosh(b_1)}{\sinh(m_1)}$$

$$b_3 = \frac{m_1 \cosh(b_1)}{\sinh(m_1)}, \quad B_1 = \frac{\sinh(b_1)}{\cosh(m_1)}, \quad B_2 = \frac{\cosh(b_1)}{\sinh(m_1)}$$

$$b_7 = b_4 \cosh(m_1 - b_1) + b_5 \cosh(m_1 + b_1),$$

$$\begin{aligned}
 b_8 &= b_4 Sh(m_1 - b_1) - b_5 Sh(m_1 + b_1), B_3 = \frac{Ch(b_4) - b_7}{Ch(m_2)}, B_4 = \frac{Sh(b_4) - b_8}{Sh(m_2)}, \\
 b_9 &= SA_1 A_3, b_{10} = M^2 + iRA_1 A_3, b_{11} = A_1 A_4, \\
 m_3 &= -b_9 \pm \sqrt{b_9^2 + 4b_{10}}, \\
 b_{12} &= -b_{11} \left( \frac{Sh(b_1)}{Ch(m_1)} + \frac{Ch(b_1)}{Sh(m_1)} \right), b_{13} = b_{11} \left( -\frac{Sh(b_1)}{Ch(m_1)} + \frac{Ch(b_1)}{Sh(m_1)} \right), \\
 b_{14} &= \frac{b_{12}}{(m_1 - b_1)^2 + m_3(m_1 - b_1) - b_{10}}, b_{15} = \frac{b_{13}}{(m_1 + b_1)^2 - m_3(m_1 + b_1) - b_{10}}, \\
 b_{16} &= \exp(b_9)((b_{14} \exp(-(m_1 - b_1)) + b_{15} \exp(m_1 + b_1)), \\
 b_{17} &= \exp(b_9)((b_{14} \exp((m_1 - b_1)) + b_{15} \exp(-(m_1 + b_1))), \\
 b_{18} &= -(0.5b_9 b_5 + m_3)b_6, b_{19} = (m_3 b_5 - 0.5b_9 b_6), b_{20} = b_{14}(m_1 - b_1) \exp(m_1 - b_1) \\
 b_{21} &= -b_{15}(m_1 + b_1) \exp(-(m_1 + b_1)), b_{22} = -(0.5b_9 b_5 + m_3 b_6) Ch(m_3) \exp(0.5b_9) \\
 b_{23} &= -(0.5b_9 b_6 + m_3 b_5) Sh(m_3) \exp(0.5b_9) \\
 B_5 &= -\frac{(b_{16} + b_{17})}{Ch(m_3)}, B_6 = \frac{(b_{16} - b_{17})}{Sh(m_3)},
 \end{aligned}$$

#### 4. DISCUSSION OF THE NUMERICAL RESULTS

Figs.2a and 2b represent the primary and secondary velocity components with respect to Grashof number (G). It can be seen from the profiles that the primary velocity enhances while the secondary velocity reduces with increase in G.

Fig.3a represents the effect of magnetic field on the nanofluid velocity profile. It is found that the nanofluid velocity field enhances with increase of magnetic field parameter M along the surface. These effects are much significant near the surface of the plate. This shows that the fluid velocity is enhanced by increasing the magnetic field and confirms the fact that the application of the magnetic field to an electrically conducting fluid produces a drag like force which causes an enhancement in the fluid velocity. From Fig.3b, it is interesting to note that the effect of magnetic parameter on the secondary velocity is to enhance the magnitude of v. It is seen that the velocity rapidly increases attaining maximum at y=3 and then reduces to attain the prescribed value zero far away from the boundary.

Fig.4a represents the nanofluid velocity profiles for different values of rotational parameter R. This result displays that the nanofluid velocity reduces with an increase in R, as noted in reference [Hamad *et al.* (2011)]. Fig.4b depicts the variation of the secondary velocity with rotation parameter (R). It is observed from the profiles that the magnitude of v enhances with increase in R.

Fig.5a shows that the nanofluid velocity with suction parameter S. It can be seen from the profiles that the nanofluid primary velocity reduces for higher S which indicates that the suction stabilizes the boundary layer growth. The free convection effect is also apparent in this Figure. The effect of S on the nanofluid velocity is the reverse in the case of [Hamad *et al.* (2013)]. This is due to the dominate role of the radiation in the fluid field. Fig.5b depicts v with Suction parameter S. It is noticed that |v| enhances with increase in S. These results are clearly supported from the physical point of view. The variation of temperature with suction parameter S is exhibited in Fig.5c. It can be seen from the profiles that the temperature reduces with increase in S.

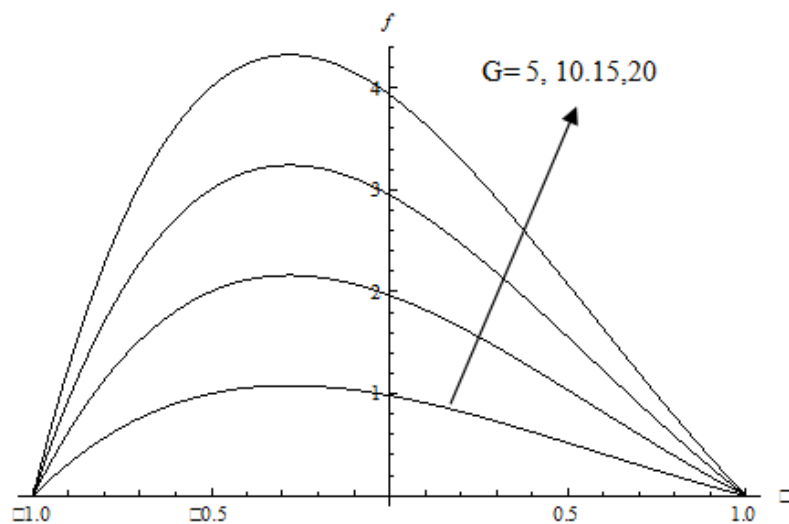
Figs.6a&6b depict the behavior of the primary and secondary velocities with heat source parameter Q. It is found that the both components of velocities exhibit an increasing tendency with increase in the strength of the heat source. This is due to the fact that when heat is generated, the buoyancy forces increase which enhances the flow rate and thereby give rise to an enhancement in the velocity component profiles. An increase in the strength of the heat absorbing source, the velocity components reduce as the heat is absorbed in the boundary layer. Fig.6c represents the temperature (T) with Heat source parameter Q. It is observed from the profiles that an increase in Q reduces the temperature and hence the thickness of the thermal boundary layer reduces with increase in Q in Cu-water nanofluids.

Figs.7a&7b display the effect of nanoparticle volume fraction  $\phi$  on the nanofluid velocity and secondary velocity respectively. It is found that as the nanoparticle volume fraction increases the nanofluid primary velocity experiences a depreciation while the secondary velocity component enhances in the boundary layer. These Figures illustrate this agreement with the physical behavior. When the volume of the nanoparticle increases, the thermal conductivity and the thermal boundary layer thickness increase. Fig.7c shows that the with  $\phi$ . It can be seen from the profiles that an

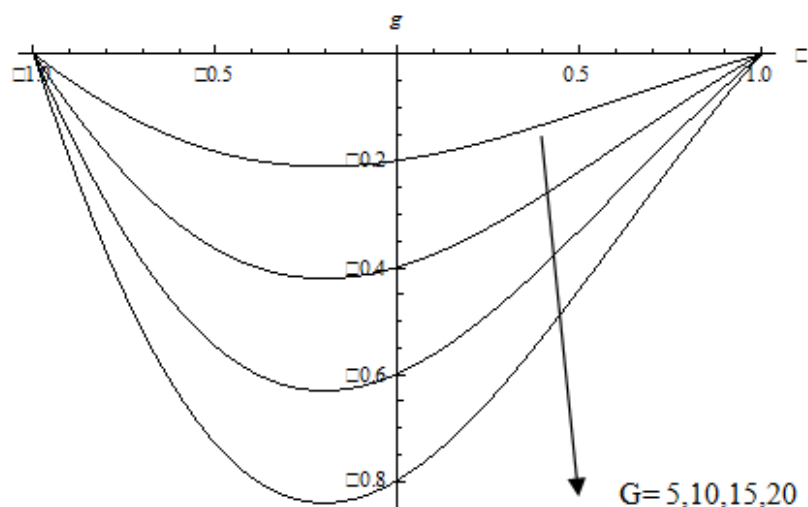
increase in the nanoparticle volume fraction enhances the temperature in the boundary layer. This is due to the fact that the thickness of the thermal boundary layer increases with increase in The variation of primary and secondary velocities with Prandtl number  $Pr$  is exhibited in Figs.8a&8b..It is found that the primary velocity component shows a depreciation while the secondary velocity experiences an increasing tendency with increasing the values of Prandtl number and hence the thickness of the boundary layer also reduces in Cu-water nanofluids. Fig.9c shows the variation of temperature with  $Pr$ . It is found that an increase in  $Pr$  enhances the temperature in the thermal boundary layer Thus lesser the thermal diffusivity larger the temperature in the boundary layer.

Table.2 displays the behavior of local skin friction component  $\tau_x$  and Nusselt number  $Nu$  at the plates  $\eta=\pm 1$ . It is found that an increase in the Hartmann number  $M$  reduces  $\tau_x$  at  $\eta=-1$  and increases it at  $\eta=1$  while an increase in the rotation parameter  $R$  reduces  $\tau_x$  at both the walls.. Also  $\tau_x$  reduces with increase in the suction parameter  $S$  and the radiation parameter  $F$  at  $\eta=\pm 1$ . An increase in  $Q>0$  enhances  $\tau_x$  at  $\eta=\pm 1$  and reduces with increase in  $Q<0$  at both the walls. An increase in the nanoparticle volume fraction  $\phi$  reduces  $\tau_x$  for Cu-water nanofluid. Lesser the thermal diffusivity smaller the skin friction component at  $\eta=\pm 1$ .

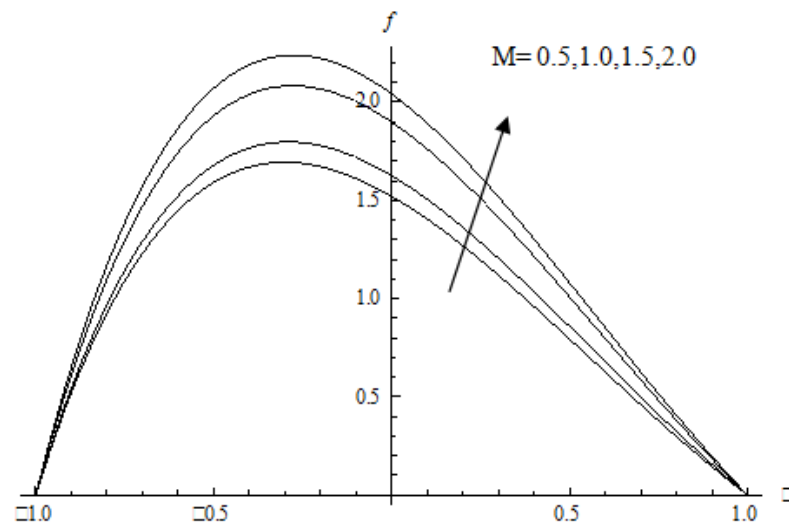
The local Nusselt number ( $Nu$ ) at  $\eta=+1$  is found to reduce with increase in  $S$  or  $Q$  or  $\phi$  or Prandtl number  $Pr$  while at  $\eta=-1$ , it enhances with increase in  $Q > 0$  or  $S$  or  $\phi$  or  $Pr$  and reduces with  $Q < 0$  in Cu-water nanofluid.



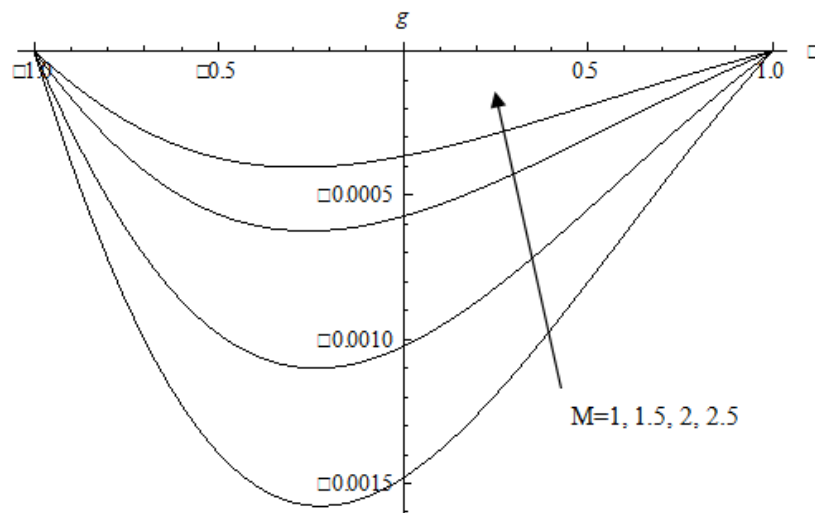
**Figure-2a:** Variation of Primary velocity  $f$  with  $G$   $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



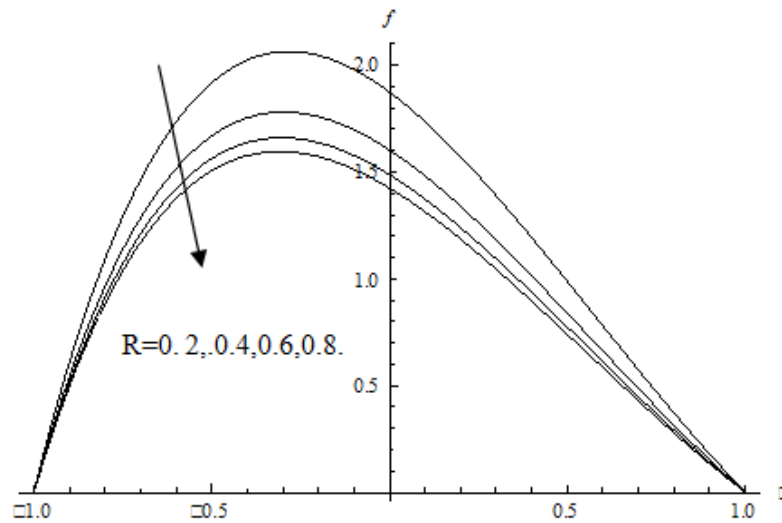
**Figure-2b:** Variation of Secondary velocity( $g$ ) with  $G$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



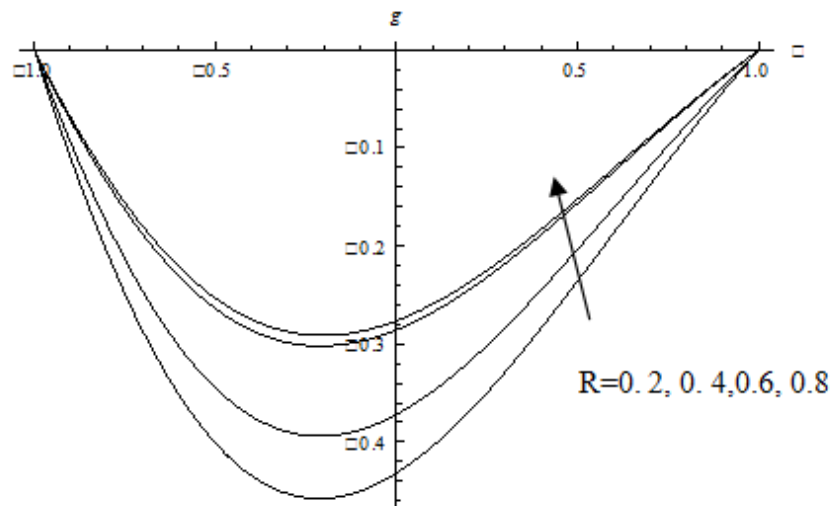
**Figure-3a:** Variation of  $f$  with  $M$ ,  $G=10$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



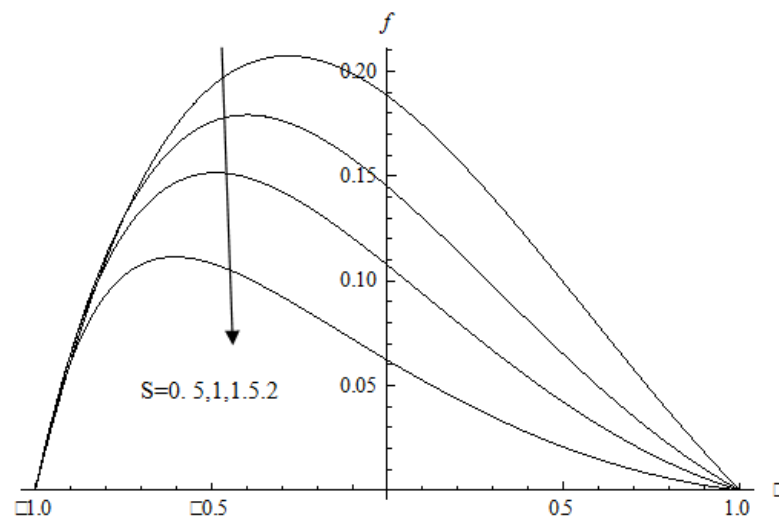
**Figure-3b:** Variation of  $g$  with  $M$ ,  $G=10$ ,  $R=0.2$ ,  $Q=25$ ,  $Pr=6.2$ ,  $S=0.5$



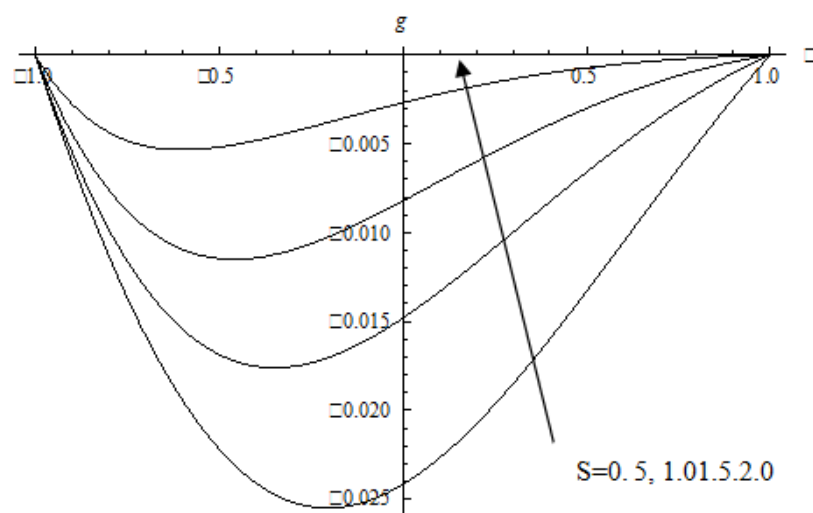
**Figure-4a:** Variation of  $f$  with  $R$ ,  $M=0.5$ ,  $G=10$ ,  $Q=2$ ,  $Pr = 6.2$ ,  $S = 0.5$



**Figure-4b:** Variation of  $g$  with  $R$ ,  $M=0.5$ ,  $G=10$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$

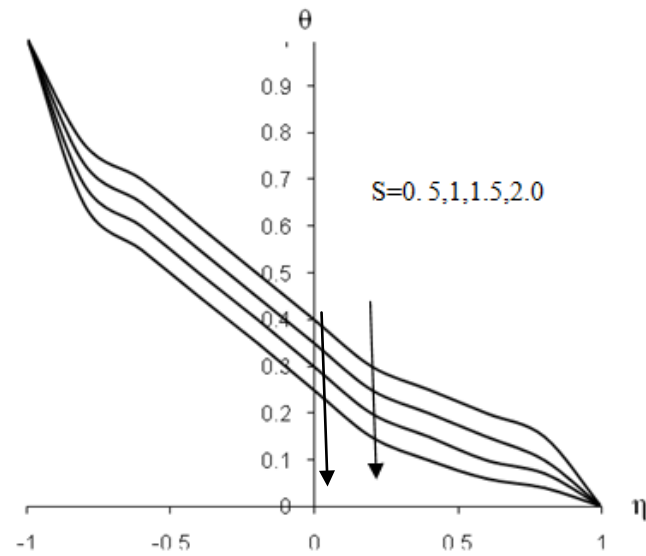


**Figure-5a:** Variation of  $f$  with  $S$ ,  $M=0.52$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $G=10$

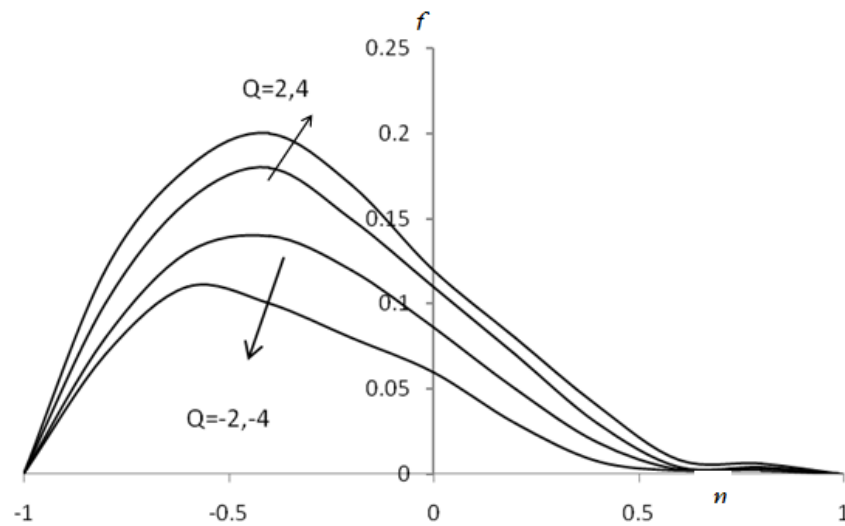


**Figure-5b:** Variation of  $g$  with  $S$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $G=10$

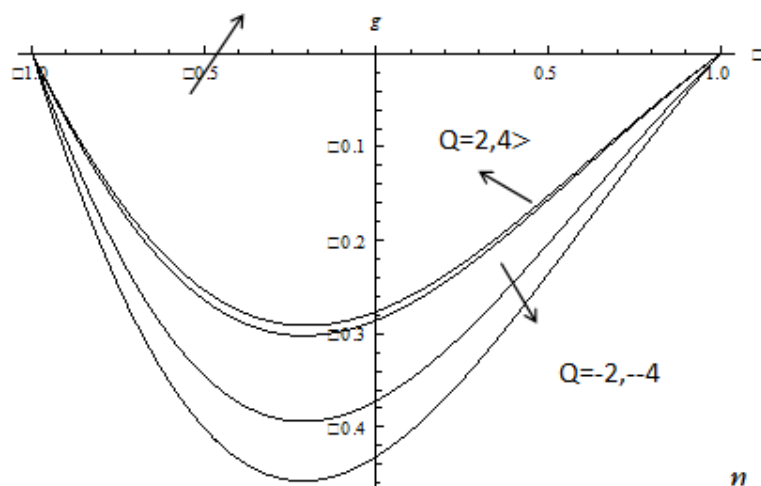




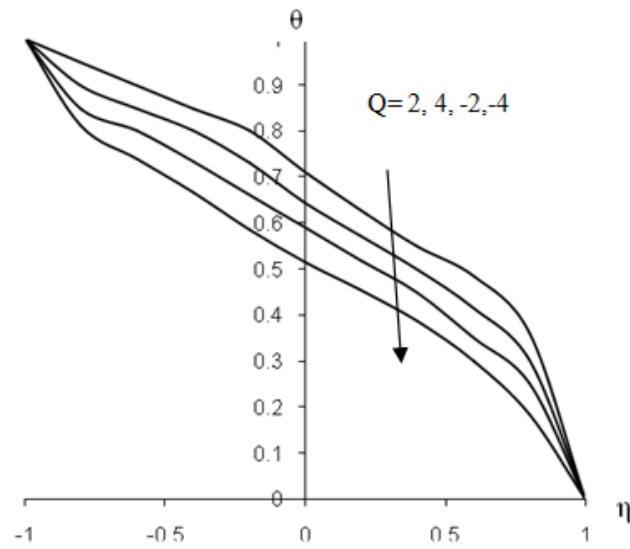
**Figure-5c:** Variation of Temperature ( $\theta$ ) with  $S$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $G=10$



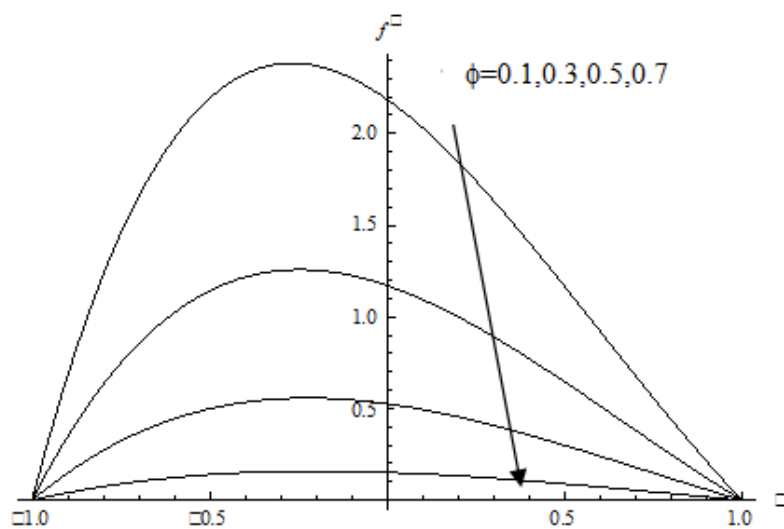
**Figure-6a:** Variation of  $f$  with  $Q$ ,  $G=10$ ,  $M=0.5$ ,  $R=0.2$ ,  $S=0.5$ ,  $\phi=0.1$ ,  $Pr=6.2$



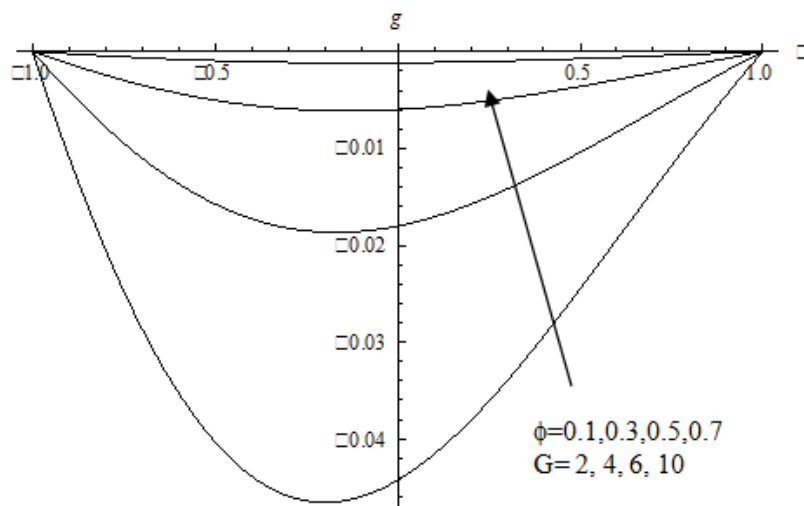
**Figure-6b:** Variation of  $g$  with  $Q$ ,  $M=0.5$ ,  $R=0.2$ ,  $G=10$ ,  $Pr=6.2$ ,  $S=0.5$



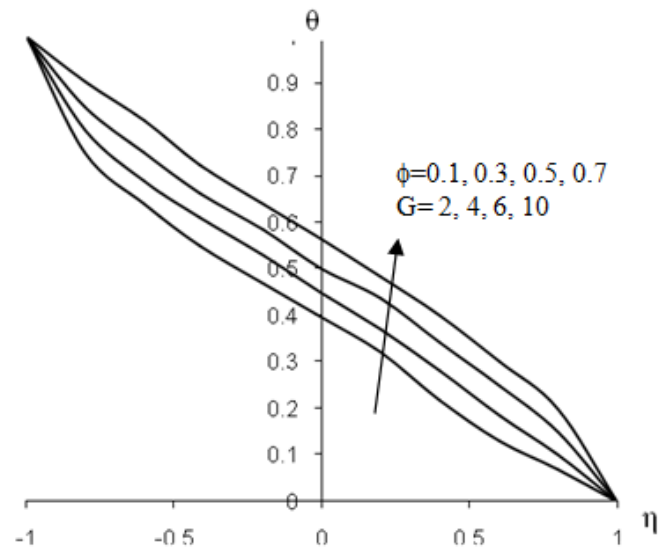
**Figure-6c:** Variation of  $\theta$  with  $Q$ ,  $M=0.5$ ,  $R=0.2$ ,  $G=10$ ,  $Pr=6.2$ ,  $S=0.5$



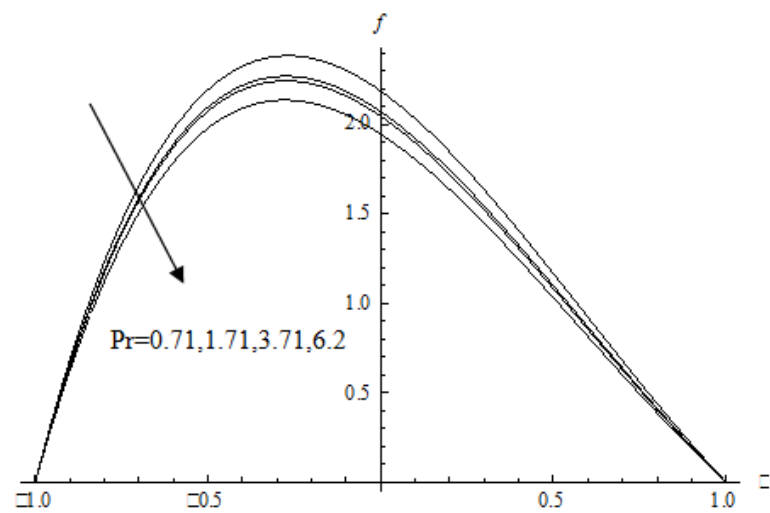
**Figure-7a:** Variation of  $f''$  with  $\phi$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



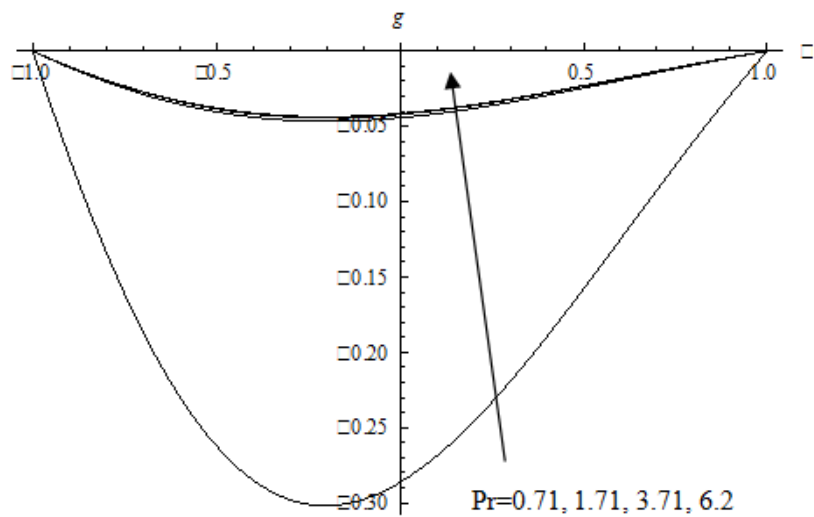
**Figure-7b:** Variation of  $g$  with  $\phi$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



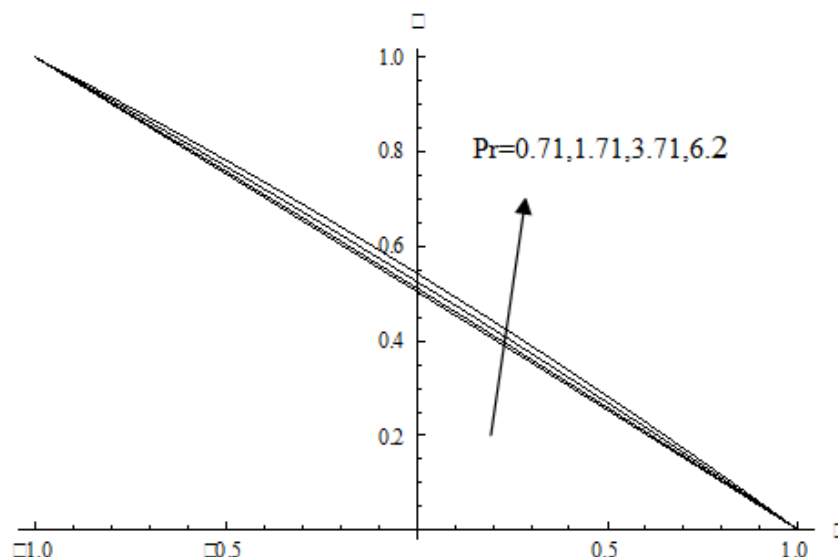
**Figure-7c:** Variation of  $\theta$  with  $\phi$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



**Figure-8a:** Variation of  $f$  with  $Pr$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



**Figure-8b:** Variation of  $g$  with  $Pr$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$



**Figure-8c:** Variation of  $\theta$  with  $Pr$ ,  $M=0.5$ ,  $R=0.2$ ,  $Q=2$ ,  $Pr=6.2$ ,  $S=0.5$

**Table – 2:** Skin friction ( $\tau$ ), Nusselt Number ( $Nu$  at  $\eta=\pm 1$ )

M	R	S	Q	$\phi$	Pr	$\tau_x(+1)$	$\tau_x(-1)$	$Nu(+1)$	$Nu(-1)$
0.5	0.2	0.5	2	0.05	6.2	-0.046486	0.67045	0.49466	0.51096
1	0.2	0.5	2	0.05	6.2	-0.056031	0.54662	-----	-----
1.5	0.2	0.5	2	0.05	6.2	-0.069979	0.55612	-----	-----
0.5	0.4	0.5	2	0.05	6.2	-0.042801	0.52575	-----	-----
0.5	0.6	0.5	2	0.05	6.2	-0.040121	0.49046	-----	-----
0.5	0.2	1.0	2	0.05	6.2	-0.038312	-0.06986	0.49359	0.51339
0.5	0.2	0.5	4	0.05	6.2	-0.170533	-0.64576	0.47524	0.51327
0.5	0.2	0.5	-2	0.05	6.2	-0.169884	0.65501	0.388766	0.45677
0.5	0.2	0.5	-4	0.05	6.2	-0.164556	0.66006	0.354843	0.43478
0.5	0.2	0.5	2	0.1	6.2	-0.013225	0.36442	0.48876	0.51017
0.5	0.2	0.5	2	0.3	6.2	-0.004228	0.12997	0.49676	0.51099
0.5	0.2	0.5	2	0.05	0.71	-0.043474	0.66589	0.49598	0.50555
0.5	0.2	0.5	2	0.05	1.71	-0.063464	0.67559	0.49375	0.50678

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