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RL AND RC CIRCUITS- SOLUTIONS BY USING ELZAKI TRANSFORM

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ABSTRACT

In this paper, we discuss the applications of Elzaki Transform [1] for solving RL and RC electrical circuit problems which are first order ordinary linear differential equations.

Keywords: *Elzaki Transform, RL and RC circuits, Differential equations.*

1. INTRODUCTION

In order to solve the differential equations, the integral transforms were extensively used. The importance of an Integral Transforms is that they provide powerful operational methods for solving initial value problems. There are various integral transforms such as Laplace, Fourier, Melin, Sumudu etc. Laplace Transform is very much useful in solving ordinary differential equations without finding general solution and particular solution to a initial value problems. Elzaki Transform [1] is the new integral transform which is modified form of Sumudu and Laplace Transforms.

Elzaki Transform was introduced by Tarig Elzaki [1] in 2011. We apply this new transform technique for solving the problems on RL and RC Circuits. Elzaki Transform defined for function of exponential order, we consider function on the set A defined by $A=\left\{ f(t):\exists M,k_1,k_2>0,\left| f(t)\right| < Me^{\frac{|t|}{k_j}},\ if\ t\in(-1)^j\times[0,\infty) \right\}$ For a given function in the set A the constant M must be a finite number, k_1 and k_2 may be finite or infinite. The Elzaki transform is defined by $E[f(t)]=v\int_0^\infty f(t)e^{\frac{-t}{v}}dt=T(V), t\geq 0, k_1\leq v\leq k_2$.The sufficient conditions for the existence of the Elzaki transform are that f (t) for $t\geq 0$ be piecewise continuous and of the exponential order otherwise Elzaki transform may (or) may not exist.

2. ELZAKI TRANSFORM OF SOME FUNCTIONS AND FOR FIRST DERIVATE

Here we consider some standard functions and the first derivative which are mostly occurred in the problems and their Elzaki transforms are given below.

(i) If f(t)=1, Now $E\{f(t)\}=v\int_0^\infty e^{\frac{-t}{v}}f(t)dt=v\int_0^\infty e^{\frac{-t}{v}}dt=v\left(\frac{e^{\frac{-t}{v}}}{\frac{-1}{v}}\right)_0^\infty=v^2 \qquad \therefore E\{1\}=v^2$

(ii) If f(t)=t, then $E\{f(t)\}=v\int_0^\infty e^{\frac{-t}{v}}f(t)dt \Rightarrow E\{t\}=v\int_0^\infty t\,e^{\frac{-t}{v}}dt$

$$=v\left\{t\left(\frac{e^{-t/v}}{-1/v}\right)-(1)\frac{e^{-t/v}}{1/v^2}\right\}_0^\infty=v(v^2)=v^3 \qquad \therefore E(t)=v^3$$

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(iii) Similarly, we get $E(t^n) = n!\, v^{n+2}$

(iv) $E(e^{at}) = v\int_0^\infty e^{at} e^{-\frac{t}{v}} dt = \dfrac{v^2}{1-av}$

(v) $E(\sin at) = v\int_0^\infty \sin at\; e^{-\frac{t}{v}} dt = \dfrac{av^3}{1+a^2v^2}$

(vi) $E(\cos at) = v\int_0^\infty \cos at\; e^{-\frac{t}{v}} dt = \dfrac{v^2}{1+a^2v^2}$

2.1 Theorem: Let f(t) be the given function and $E\{f(t)\} = T(v)$ then $E\{f^1(t)\} = \dfrac{T(v)}{v} - vf(0)$

Proof: Given $E\{f(t)\} = T(v)$, By Definition of Elzaki Transform $E\{f(t)\} = v\int_0^\infty f(t) e^{-t/v} dt$

Now $E\{f^1(t)\} = v\int_0^\infty f^1(t) e^{\frac{-t}{v}} dt$

$$= v\left\{ \left(e^{-t/v} f(t)\right)_0^\infty - \int_0^\infty e^{-t/v}\left(\frac{-1}{v}\right) f(t)\, dt \right\} = v\left\{ -f(0) + \frac{1}{v}\int_0^\infty f(t) e^{-t/v} dt \right\}$$

$$= -vf(0) + \int_0^\infty f(t) e^{-t/v} dt = -vf(0) + \frac{T(v)}{v} \qquad \therefore E\{f^1(t)\} = \frac{T(v)}{v} - vf(0)$$

3. RL CIRCUITS

In a series circuit containing only a resistor and an inductor, Kirchhoff's second law states that the sum of the voltage drop across the inductor L and voltage drop across the resistor R is same as the impressed voltage E(t) on the circuit. Therefore, for the i(t), the differential equation is $L\dfrac{di}{dt} + Ri = E(t)$

3.1 Problem-1: A 12v battery is connected to a simple series circuit in which the inductance is ½ H and the resistance is 10 Ω. Determine the current I if i(0)=0.

Solution: From the given data we can draw the circuit diagram shown in Figure-(a).

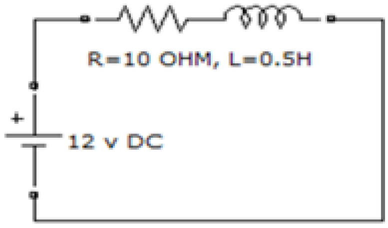

Figure-(a)

From Kirchhoff's second law we have $L\dfrac{di}{dt} + Ri = E(t)$

$$\Rightarrow \frac{1}{2}\frac{di}{dt} + 10i = 12 \Rightarrow \frac{di}{dt} + 20i = 24 \Rightarrow i^1(t) + 20i(t) = 24 \qquad (3.1.1)$$

Taking Elzaki Transform on both sides of (3.1.1), we get

$$E\left[i^1(t) + 20i\right] = E[24] \Rightarrow E\left[i^1(t)\right] + 20E[i(t)] = 24E[1] \Rightarrow \left[\frac{\bar{i}(v)}{v} - vi(0)\right] + 20\bar{i}(v) = 24v^2$$

Substituting the initial conditions, if t = 0 current i = 0, we get

$$\left(\frac{1}{v}+20\right)\bar{i}(v)=24v^2 \Rightarrow (1+20v)\bar{i}(v)=24v^3 \Rightarrow \bar{i}(v)=\frac{24v^3}{(1+20v)}$$

Taking both sides Inverse Elzaki Transform we get $i(t)=2\ 4\left[\frac{1}{20}-\frac{1}{20}e^{-20t}\right] \quad \Rightarrow i(t)=\frac{6}{5}-\frac{6}{5}e^{-20t}$. The graph is drawn between the time and current , which is shown in figure(b).

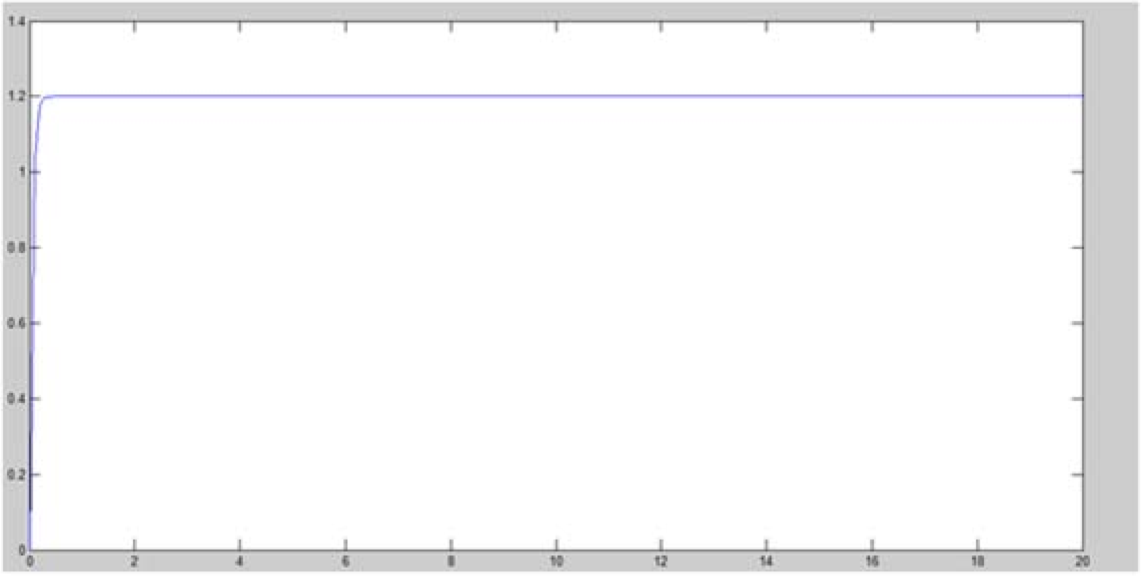

Figure-(b)

3.2 Problem-2: A generator having electromotive force 20cos5t volts are connected with a 10 Ω resistor and inductor 2H. If the switch k is closed at time t=0, obtain the differential equation for the current and determine the current at time t.

Solution: Given $E(t)$=20cos (5t) volts, R=10ohms, L=2 henrys, we can draw the circuit diagram shown in Figure-(c)

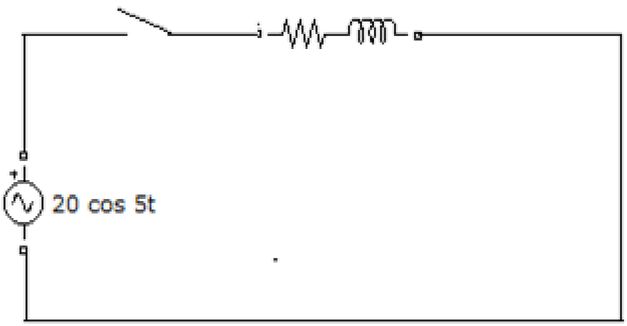

Figure-(c)

The differential equation to find the current i(t) in the given circuit is $L\frac{di}{dt}+Ri=E(t)$

$$\Rightarrow \frac{di}{dt}+\frac{R}{L}i=\frac{E(t)}{L} \Rightarrow \frac{di}{dt}+\frac{10}{2}i=\frac{20\cos(5t)}{2} \Rightarrow \frac{di}{dt}+5i=10\cos(5t) \Rightarrow i^1(t)+5i=10\cos(5t)$$

Taking both side Elzaki Transform

$$E\left\{i^1(t)+5i\right\}=10E\{\cos(5t)\} \Rightarrow E\left\{i^1(t)\right\}+5E\left\{i\right\}=10E\{\cos(5t)\} \Rightarrow \left[\frac{\bar{i}(v)}{v}-vi(0)\right]+5\bar{i}(v)=10\frac{v^2}{(1+25v^2)}$$

Substitute the initial conditions at t=0 current i = 0, we have

$$\frac{\bar{i}(v)}{v}+5\bar{i}(v)=10\frac{v^2}{1+25v^2}\Rightarrow\left(\frac{1}{v}+5\right)\bar{i}(v)=10\frac{v^2}{1+25v^2}\Rightarrow(1+5v)\bar{i}(v)=10\frac{v^3}{1+25v^2}$$

$$\Rightarrow\bar{i}(v)=10\frac{v^3}{\left(1+25v^2\right)\left(1+5v\right)}$$

After simplification, we get, $\bar{i}(v)=\frac{5v^3}{1+25v^2}+\frac{v^2}{1+25v^2}-\frac{v^2}{1+5v}$

Taking Inverse Elzaki Transform on both sides, We get i(t) = sin5t+cos5t-e $^{-5t}$. The graph is drawn to this problem, time vs current, which is shown in figure (d).

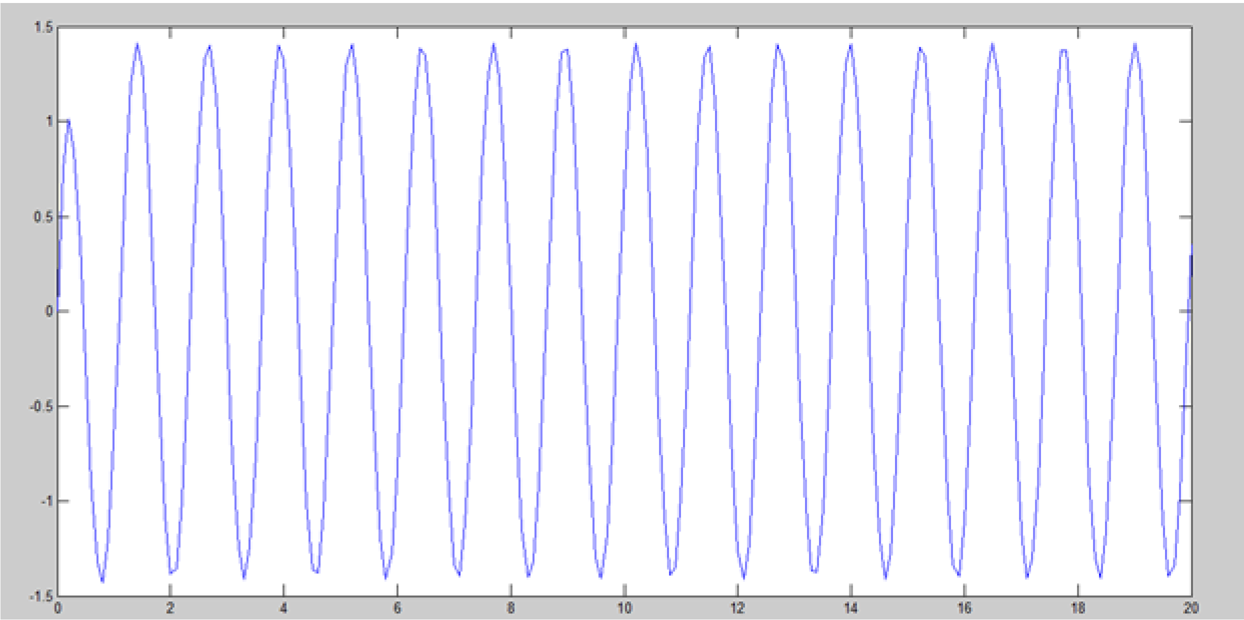

Figure-(d)

4. RC CIRCUITS

The basic differential equation governing the amount of charge ‘q’ in a simple RC circuit consisting of a resistance R , a capacitor C and an Electromotive force E is

$$\frac{dq}{dt}+\frac{1}{RC}q=\frac{E(t)}{R}$$

4.1 Problem-3: A decaying emf E=200e^{5t} is connected in a series with a 20 Ω resistor and 0.01F capacitor. Assuming q=0 at t=0. Find the charge q at any time t.

Solution: Given E(t)=200e^{5t}, R=20 Ω , C=0.01F. We can draw the circuit diagram shown in Figure-(e)

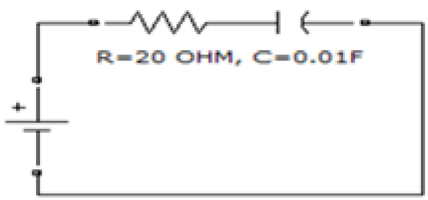

Figure-(e)

We have $\frac{dq}{dt}+\frac{1}{RC}q=\frac{E(t)}{R}\Rightarrow\frac{dq}{dt}+\frac{1}{20X\,0.01}q=\frac{200}{20}e^{5t}\Rightarrow\frac{dq}{dt}+5q=10e^{5t}\Rightarrow q^1(t)+5q(t)=10e^{5t}$

Taking Elzaki Transform on both sides of above equation, we get

$$E\{q^1(t)+5q(t)\}=10E\{e^{5t}\} \Rightarrow E\{q^1(t)\}+5E\{q(t)\}=10\frac{v^2}{1-25v} \Rightarrow \left[\frac{\bar{q}(v)}{v}-vq(0)\right]+5\bar{q}(v)=10\frac{v^2}{1-25v}$$

$$\Rightarrow \left(\frac{1}{v}+5\right)\bar{q}(v)=10\frac{v^2}{1-25v} \Rightarrow (1+5v)\bar{q}(v)=10\frac{v^3}{1-25v} \Rightarrow \bar{q}(v)=10\frac{v^3}{(1-25v)(1+5v)}$$

After simplifying, we have, $\bar{q}(v)=\frac{1}{3}\left[\frac{v^2}{1-25v}-\frac{v^2}{1+5v}\right]$ By taking Inverse Elzaki Transform, we get

q(t)= $\frac{1}{3}\left[e^{25t}-e^{-5t}\right]$. The plotted graph is shown in figure (f).

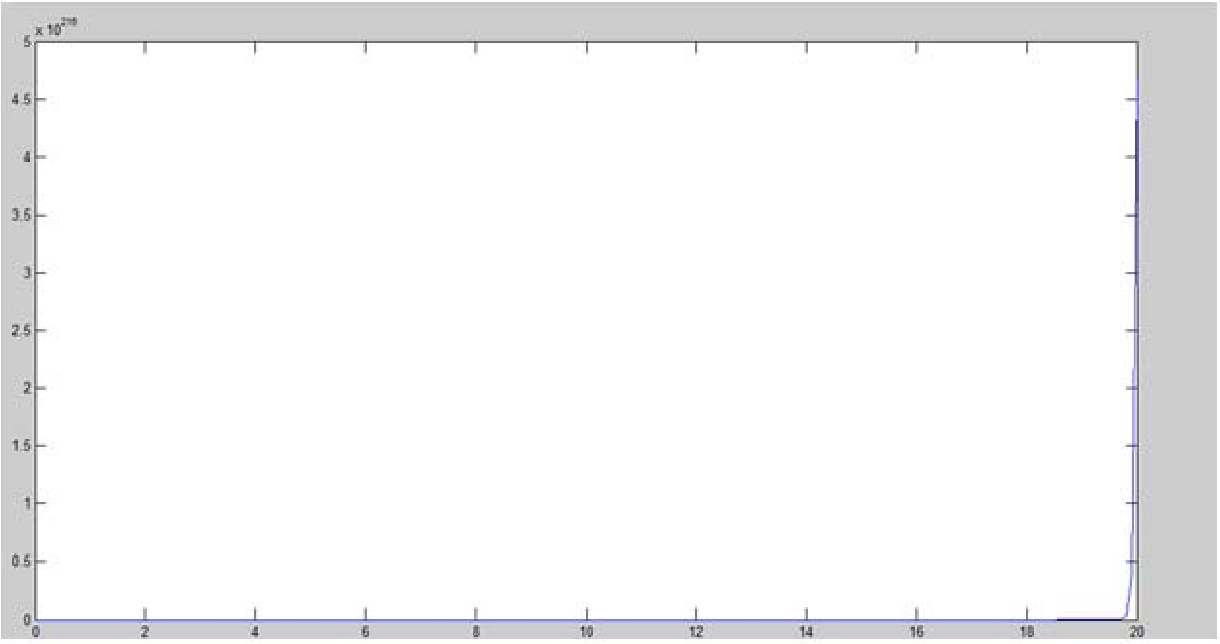

Figure-(f)

4.2 Problem - 4: A RC circuit has an emf of 300 cos (2t) volts, a resistance of 150ohms and a capacitance of $\frac{1}{600}$ farads and an initial charge on the capacitor of 5 coulombs. Find the charge on the capacitor at any time t.

Solution: Given E(t)=300cos(2t) volts, R= 150 Ohms, C= $\frac{1}{600}$ farads. With the help of this data a, circuit diagram is drawn and it shown as Figure-(g).

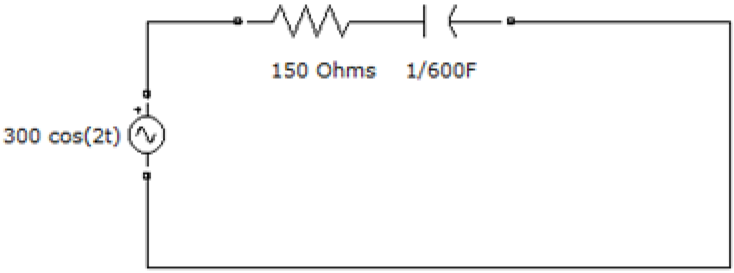

Figure-(g)

We have,

$$\frac{dq}{dt}+\frac{1}{RC}q=\frac{E(t)}{R} \Rightarrow \frac{dq}{dt}+\frac{600}{150}q=\frac{300\cos(2t)}{150} \Rightarrow \frac{dq}{dt}+4q=2\cos(2t) \Rightarrow q^1(t)+4q(t)=2\cos(2t)$$

Taking both sides Elzaki Transform

$$E\{q^1(t)+4q(t)\}=2E\{\cos(2t)\}\Rightarrow E\{q^1(t)\}+4E\{q(t)\}=2\frac{v^2}{1+4v^2}\Rightarrow\left[\frac{\overline{q}(v)}{v}-vq(0)\right]+4\overline{q}(v)=2\frac{v^2}{1+4v^2}$$

$$\Rightarrow\left(\frac{1}{v}+4\right)\overline{q}(v)=2\frac{v^2}{1+4v^2}+5v\Rightarrow(1+4v)\overline{q}(v)=2\frac{v^3}{1+4v^2}+5v^2\Rightarrow\overline{q}(v)=2\frac{v^3}{(1+4v^2)(1+4v)}+\frac{5v^2}{(1+4v)}$$

after simplifying, we have, $\overline{q}(v)=\frac{23}{5}\left(\frac{v^2}{1+4v}\right)+\frac{2}{5}\left(\frac{v^3}{1+4v^2}\right)+\frac{2}{5}\left(\frac{v^2}{1+4v^2}\right)$

By taking Inverse Elzaki Transform, We get q (t) = $\frac{23}{5}e^{-4t}+\frac{1}{5}\sin 2t+\frac{2}{5}\cos 2t$. The graph is shown in Figure-(h).

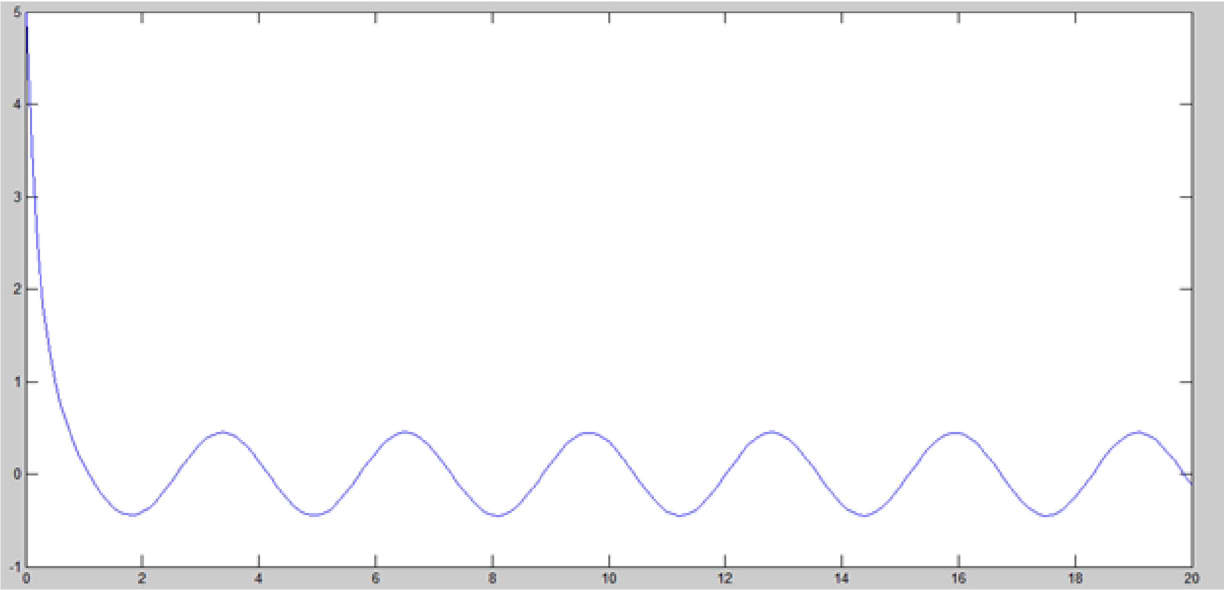

Figure-(h)

5. CONCLUSION

In the presented work, we have successfully applied Elzaki Transform for solving the problems on RL and RC Electrical circuits.

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