

INFLUENCE OF AN INCLINED MAGNETIC FIELD ON THE MIXED CONVECTIVE FLOW OF A SECOND GRADE FLUID IN A VERTICAL CHANNEL WITH PERMEABLE WALLS

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ABSTRACT

In this paper, the effect of an inclined magnetic field on the fully developed mixed convection flow of a second grade fluid in a vertical channel with permeable walls is investigated. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are discussed in detail through graphs.

Keywords: *Second grade fluid, Inclined magnetic field, mixed convection.*

1. INTRODUCTION

The approach of heat transfer through convection has received considerable attention as of its applications in wide areas of thermal engineering field. Convection is the approach of heat transfer between a solid surface and the moving fluid in contact with it. The faster the fluid motion, greater the convective heat transfer. Convective heat transfer is three type's viz., forced convection if the fluid is forced to flow over the surface, natural or free convection if the fluid motion is caused by buoyancy forces that are induced by density differences due to variation of temperature in the fluid. The third type is known as the mixed convection heat transfer which takes place when forced and natural convection act together in a system. Under this process both pressure forces and buoyant forces act together. Joye *et al.* [8] analyzed the mixed convection heat transfer in nuclear reactors and some aspects of electronic cooling. Boulama and Galanis [4] have investigated the fully developed mixed convection between parallel vertical plates with heat and mass transfer. Barletta *et al.* [3] have studied the Dual mixed convection flows in a vertical channel.

Over the past few decades, interest in the flow and heat transfer of non-Newtonian fluids has increased significantly due to the occurrence of these fluids in industrial processes. The equations of motion of non-Newtonian fluids are highly non-linear and one order higher than the Navier Stokes equations. The extension of the theory of Newtonian fluid mechanics to non-Newtonian fluids has proven not to be so straightforward. That is, the shear dependent viscosity and/or the elasticity of such fluids can make the theory quite complicated. Due to the complexity of the governing equations, for non-Newtonian fluids, finding accurate solutions is not easy. Ariel [1] has studied the laminar forced convection of a second – grade fluid through two parallel porous walls. Hayat and Abbas [6] have investigated the boundary layer flow of a Maxwell fluid in a channel with chemical reaction, the walls of two channels being permeable. Sajid *et al.* [12] have studied analytical solution for the problem of fully developed mixed convention flow of viscoelastic fluid between two permeable parallel vertical walls.

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The general MHD flow problems are studied considering the imposed magnetic field in a direction perpendicular to the direction of the flow. But here we are interested to investigate the nature of a problem of electrically conducting fluid past between two vertical plates in the presence of a magnetic field placed at different angles θ (where θ varies from 0 to $\pi/2$) to the motion of the fluid. MHD Poiseuille flow between two infinite parallel porous plates in an inclined magnetic field was analyzed by Manyonge *et al.* [10]. Sandeep and Sugunamma [13] have studied the effect of an inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Joseph *et al.* [7] have investigated the unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with Heat Transfer. Olanrewaju and Abbas [11] studied the convection heat and mass transfer in a hydromagnetic flow of a second grade fluid in the presence of thermal radiation and thermal diffusion.

In view of these, the effect of an inclined magnetic field on the fully developed mixed convection flow of a second grade fluid in a vertical channel with permeable walls is investigated. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. The effects of various emerging parameters on the velocity field and temperature field are discussed in detail through graphs.

2. MATHEMATICAL FORMULATION

We consider the fully developed laminar mixed convection flow of a second grade fluid in a vertical permeable channel, the space between the plates being h , as illustrated in Fig. 1. A uniform magnetic field B_0 acts at an angle $\alpha \left(0 \leq \alpha \leq \frac{\pi}{2} \right)$, to the Y -axis. It is assumed that the rate of injection at one wall is equal to the rate of suction at the other wall. A rectangular coordinate system (x, y) is chosen such that the x - axis is parallel to the gravitational acceleration vector g , but with opposite direction and the y - axis is transverse to the channel walls. The left wall (i.e, at $y = 0$) is maintained at constant temperature T_1 and the right wall (i.e, at $y = h$) is maintained at constant temperature T_2 , where $T_1 > T_2$. The flow assumed steady and fully developed, i.e. the transverse velocity is zero. Then, the continuity equation drops to $\partial u / \partial x = 0$.

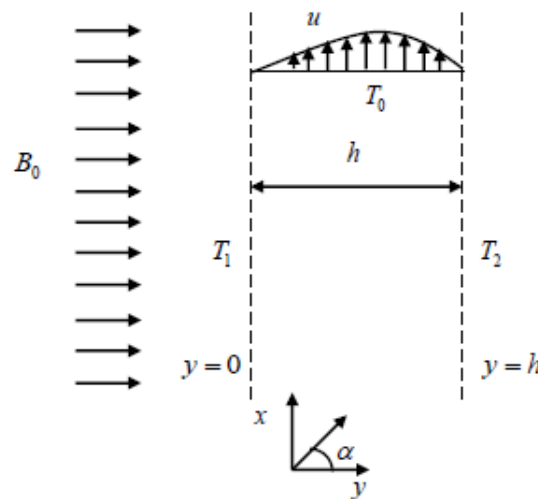


Figure-1: The physical model

Viscoelastic fluids can be modeled by Rivlin – Ericksen constitutive equation

$$\mathbf{S} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \quad (2.1)$$

where \mathbf{S} is the Cauchy stress tensor, p is the scalar pressure, μ, α_1 and α_2 are the material constants, customarily known as the coefficients of viscosity, elasticity and cross - viscosity, respectively. These material constants can be determined from viscometric flows for any real fluid. \mathbf{A}_1 and \mathbf{A}_2 are Rivlin-Ericksen tensors and they denote, respectively, the rate of strain and acceleration. \mathbf{A}_1 and \mathbf{A}_2 are defined by

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T \quad (2.2)$$

$$\text{and} \quad A_2 = \frac{dA_1}{dt} + A_1 (\nabla V) + (\nabla V)^T A_1 \quad (2.3)$$

where d/dt is the material time derivative and ∇ gradient operator and $(\)^T$ transpose operator. The viscoelastic fluids when modeled by Rivlin-Ericksen constitutive equation are termed as second-grade fluids. A detailed account of the characteristics of second - grade fluids is well documented by Dunn and Rajagopal [5]. From the thermodynamics consideration they assumed

$$\mu \geq 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \quad (2.4)$$

The basic equations of momentum and energy governing such a flow, subject to the Boussinesq approximation, are

$$\rho v_0 \frac{du}{dy} = -\frac{dp}{dx} + \mu \frac{d^2 u}{dy^2} + \alpha_1 v_0 \frac{d^3 u}{dy^3} - (\sigma B_0^2 \cos^2 \alpha) u + \rho g \beta (T - T_0) \quad (2.5)$$

$$v_0 \frac{dT}{dy} = \alpha \frac{d^2 T}{dy^2} \quad (2.6)$$

where p is the pressure, ρ is the density, μ is the dynamic viscosity of the fluid, g acceleration due to gravity, β coefficient of thermal expansion, α_1 is the viscoelastic parameter, σ is the electrical conductivity and v_0 is the transpiration cross flow velocity. Further, here dp/dx is a constant.

The boundary conditions are given by

$$u(0) = u(h) = 0, \quad T(0) = T_1, \quad T(h) = T_2 \quad (2.7)$$

Introducing the following non-dimensional variables

$$\bar{y} = \frac{y}{h}, \quad \bar{u} = \frac{u}{U}, \quad \theta = \frac{T - T_0}{T_2 - T_0}$$

into the equations (2.5) and (2.6), we obtain

$$kR \frac{d^3 u}{dy^3} + \frac{d^2 u}{dy^2} - R \frac{du}{dy} - (M^2 \cos^2 \alpha) u + \frac{Gr}{Re} \theta + A = 0 \quad (2.8)$$

$$\frac{d^2 \theta}{dy^2} - RPr \frac{d\theta}{dy} = 0 \quad (2.9)$$

where $k = \frac{\alpha_1}{\rho h^2}$ is the viscoelastic parameter, $M = hB_0 \sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $R = \frac{\rho v_0 h}{\mu}$ is the cross

flow Reynolds number, $Gr = \frac{g\beta(T_2 - T_1)h^3}{\nu^2}$ is the Grashof number, $Re = \frac{\rho U_0 h}{\mu}$ is the Reynolds number,

$Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $r_T = \frac{T_1 - T_0}{T_2 - T_0}$ is the wall temperature parameter and $A = -\left(\frac{dp}{dx}\right) \frac{U_0 \nu}{h^2}$ is the constant pressure gradient.

The corresponding dimensionless boundary conditions are given by

$$u(0) = u(1) = 0, \quad \theta(0) = r_T, \quad \theta(1) = 1 \quad (2.10)$$

3. PERTURBATION SOLUTION

We consider the first - order perturbation solution of the boundary value problem (2.8) - (2.10) for small k . Since the constitute equation (2.1) has been derived up to only the first - order of smallness of k , therefore, the perturbation solution obtained by retaining the terms up to the same order of smallness of k must be quite logical and reasonable.

We write

$$u = u_0 + ku_1 \quad (3.1)$$

and $\theta = \theta_0 + k\theta_1 \quad (3.2)$

Substituting Eqs. (3.1) and (3.2) into the Eqs. (2.8) and (2.9) and boundary conditions (2.10) and then equating the like powers of k , we obtain

3.1 Zeroth-order system (k^0)

$$\frac{d^2 u_0}{dy^2} - R \frac{du_0}{dy} - (M^2 \cos^2 \alpha) u_0 = -\frac{Gr}{Re} \theta_0 - A \quad (3.3)$$

$$\frac{d^2 \theta_0}{dy^2} - RPr \frac{d\theta_0}{dy} = 0 \quad (3.4)$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0, \quad \theta_0(0) = r_T, \quad \theta_0(1) = 1 \quad (3.5)$$

3.2 First-order system (k^1)

$$\frac{d^2 u_1}{dy^2} - R \frac{du_1}{dy} - (M^2 \cos^2 \alpha) u_1 = -R \frac{d^3 u_0}{dy^3} - \frac{Gr}{Re} \theta_1 \quad (3.6)$$

$$\frac{d^2 \theta_1}{dy^2} - RPr \frac{d\theta_1}{dy} = 0 \quad (3.7)$$

Together with boundary conditions

$$u_1(0) = u_1(1) = 0, \quad \theta_1(0) = 0, \quad \theta_1(1) = 0 \quad (3.8)$$

3.3 Zeroth-order solution

Solving equations (3.3) and (3.4) using the boundary conditions (3.5), we get

$$\theta_0 = \frac{(1 - r_T e^{RPr}) + (r_T - 1) e^{RPr y}}{(1 - e^{RPr})} \quad (3.9)$$

$$u_0 = c_1 e^{ay} + c_2 e^{by} + \frac{Gr}{Re} (c_3 - c_4 e^{RPr y}) + \frac{A}{M^2 \cos^2 \alpha} \quad (3.10)$$

where $a = \frac{R + \sqrt{R^2 + 4M^2 \cos^2 \alpha}}{2},$

$$b = \frac{R - \sqrt{R^2 + 4M^2 \cos^2 \alpha}}{2}, \quad c_3 = \frac{(1 - r_T e^{RPr})}{(1 - e^{RPr}) M^2 \cos^2 \alpha},$$

$$c_4 = \frac{(r_T - 1)}{(1 - e^{RPr})(R^2 Pr^2 - R^2 Pr - M^2 \cos^2 \alpha)}, \quad c_5 = \frac{Gr}{Re} (c_3 - c_4) + \frac{A}{M^2 \cos^2 \alpha},$$

$$c_6 = \frac{Gr}{Re} (c_3 - c_4 e^{RPr}) + \frac{A}{M^2 \cos^2 \alpha}, \quad c_1 = \frac{c_6 - c_5 e^b}{e^b - e^a} \text{ and } c_2 = \frac{c_5 e^a - c_6}{e^b - e^a}.$$

3.4 First-order solution (or Solution for a second - grade fluid)

Solving Eq. (3.7) with corresponding boundary conditions, we obtain

$$\theta_1 = 0 \quad (3.11)$$

By substituting the equations (3.10) and (3.11) into the Eq. (3.6) and then solving the resulting equation with the corresponding conditions, we get

$$u_1 = c_7 e^{ay} + c_8 e^{by} - c_{10} y e^{ay} - c_{11} y e^{by} + c_9 e^{RPr y} \quad (3.12)$$

where
$$c_9 = \frac{Gr}{Re} c_4 \frac{R^4 Pr^3}{(R^2 Pr^2 - R^2 Pr - M^2 \cos^2 \alpha)}, \quad c_{10} = \frac{Rc_1 a^3}{2a - R}, \quad c_{11} = \frac{Rc_2 b^3}{2b - R},$$

$$c_{12} = c_9 e^{RPr} - c_{10} e^a - c_{11} e^b, \quad c_7 = \frac{c_{12} - c_9 e^b}{e^b - e^a}, \quad c_8 = \frac{e^a c_9 - c_{12}}{e^b - e^a}.$$

Finally, the perturbation solutions up to first order for θ and u are given by

$$\theta = \frac{(1 - r_T e^{RPr}) + (r_T - 1) e^{RPr y}}{(1 - e^{RPr})} \quad (3.13)$$

and

$$u = \left(\begin{aligned} &(c_1 + kc_7 - kc_{10} y) e^{ay} + (c_2 + kc_8 - kc_{11} y) e^{by} + \frac{Gr}{Re} (c_3 - c_4 e^{RPr y}) \\ &+ kc_9 e^{RPr y} + \frac{A}{M^2 \cos^2 \alpha} \end{aligned} \right) \quad (3.14)$$

The rate of heat transfer coefficient in terms of Nusselt number Nu at the plate $y = 0$ of the channel is given by

$$Nu = \left. \frac{\partial \theta}{\partial y} \right|_{y=0} = \frac{(r_T - 1) R Pr}{(1 - e^{RPr})} \quad (3.15)$$

Note that when $\alpha \rightarrow 0$ our results reduces to those given by Kavitha *et al.* (2012) and when $k = 0$, $R = 0, \alpha \rightarrow 0$ and $M \rightarrow 0$ our results reduces to those given by Aung and Worku (1986).

4. DISCUSSION OF THE RESULTS

In order to see the effects of M, k, R, Pr, Gr, Re and r_T on the velocity u , we have plotted Figs. 2-12.

Fig. 2 illustrates the effect of viscoelastic parameter k on u for $M = 1, r_T = 0.2, \alpha = \frac{\pi}{6}, R = 2, A = 1,$

$Gr = 1, Pr = 0.71$ and $Re = 1$. It is found that, the velocity u decreases with an increase in k . The point where the maximum velocity occurs is shifted away from the upper wall as the value of the viscoelastic parameter is increased. Further it is observed that, the velocity is more for Newtonian fluid ($k \rightarrow 0$) than that of second grade fluid.

The effect of Hartmann number M on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1, Pr = 0.71$

and $Re = 1$ is depicted in Fig. 3. It is observed that, the velocity u is decreases with increasing M . Further, it is found that, the velocity is more for non-conducting (magnetic) (i.e., $M \rightarrow 0$) second grade fluid than that of conducting second grade fluid.

Effect of α on u for $r_T = 0.2, k = 0.01, R = 2, A = 1, Gr = 1, Pr = 0.71$ and $Re = 1$ is depicted in Fig. 4. It is observed that the velocity u increases on increasing α .

Fig. 5 shows the Effect of cross flow Reynolds number R on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, A = 1,$

$Gr = 1, Pr = 0.71$ and $Re = 1$. It is found that, the velocity u decreases with increasing R .

Effect of Prandtl number Pr on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Re = 1$ is shown in Fig. 6. It is found that, the velocity u decreases on increasing Prandtl number Pr .

Fig. 7 illustrates the effect of Grashof number Gr on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Pr = 0.71$ and $Re = 1$. It is observed that, the velocity u increases with increasing Grashof number Gr .

Effect of Reynolds number Re on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Pr = 0.71$ is shown in Fig. 8. It is noted that, the velocity u decreases with increasing Reynolds number Re .

Fig. 9 shows the effect of wall temperature parameter r_T on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Pr = 0.71$. It is observed that, the velocity u increases with increasing r_T .

The effect of cross flow Reynolds number R on the temperature θ for $r_T = 0.2$ and $Pr = 0.71$ is presented in Fig. 10. It is noted that, the temperature θ decreases with increasing R .

Fig. 11 illustrates the effect of Prandtl number Pr on the temperature θ for $r_T = 0.2$ and $R = 2$. It is found that, the temperature θ is decreases with increasing Prandtl number Pr .

The influence of r_T on temperature θ for $R = 2$ and $Pr = 0.71$ is shown in Fig. 12. It is noticed that, the temperature θ increases with an increase in r_T .

Table-1 shows the effect of R on Nusselt number Nu for $r_T = 0.2$ and $Pr = 0.71$. It is found that, the Nu decreases with increasing R .

The effect of Pr on Nusselt number Nu for $r_T = 0.2$ and $R = 2$ is depicted in Table-2. It is observed that, the Nu decreases with increasing Pr .

Table-3 shows the effect of r_T on Nusselt number Nu for $R = 2$ and $Pr = 0.71$. It is noted that, the Nu decreases with increasing r_T .

5. CONCLUSIONS

In this paper, the influence of an inclined magnetic field on mixed convective flow of a second grade fluid in a vertical channel with permeable walls is studied. The governing non – linear equations are solved for the velocity field and temperature field using the traditional perturbation technique. It is found that, the velocity u decreases with increasing k, M, R, Re and Pr , while it increases with increasing α, Gr and r_T . Also, it is observed that, the temperature θ decreases with increasing R and Pr , while it increases with increasing r_T . The Nusselt number Nu decreases with increasing R, Pr and r_T .

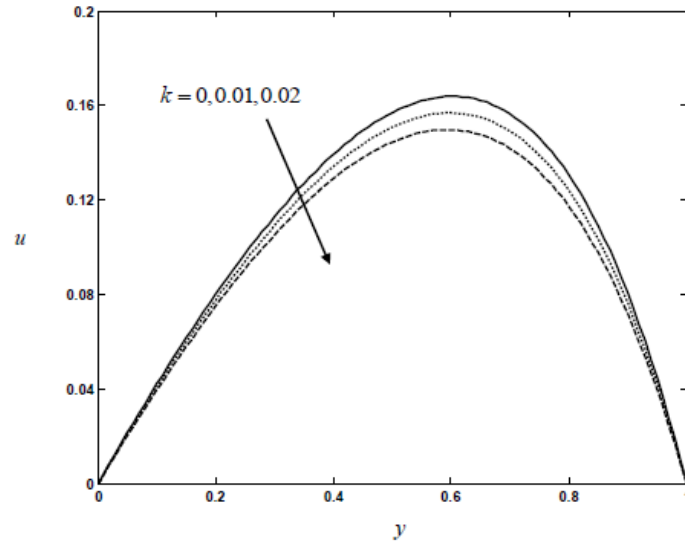


Figure-2: Effect of viscoelastic parameter k on u for $r_T = 0.2, M = 1, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1, Pr = 0.71$ and $Re = 1$.

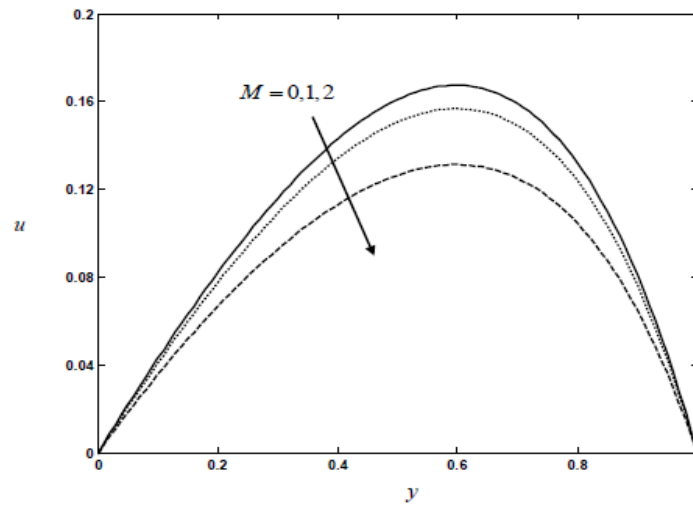


Figure-3: Effect of Hartmann number M on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1, Pr = 0.71$ and $Re = 1$.

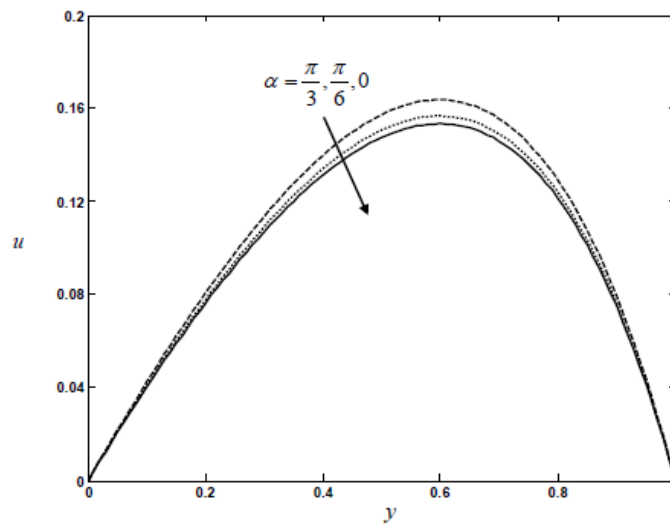


Figure-4: Effect of α on u for $r_T = 0.2, k = 0.01, R = 2, A = 1, Gr = 1, Pr = 0.71$ and $Re = 1$.

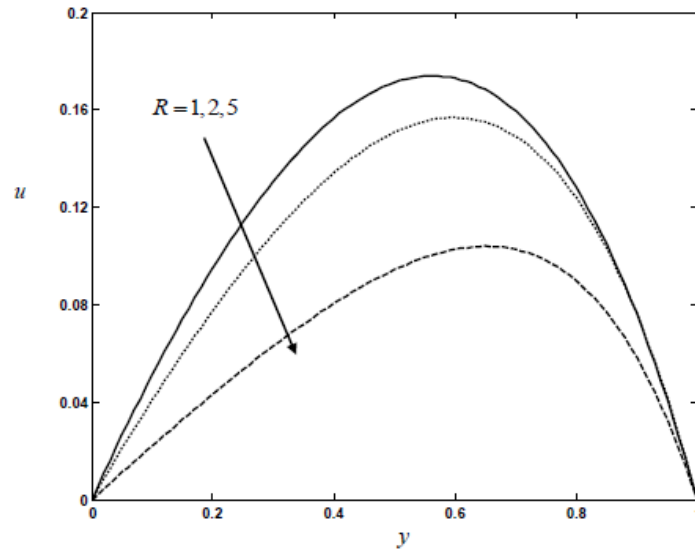


Figure-5: Effect of R on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, A = 1, Gr = 1, Pr = 0.71$ and $Re = 1$.

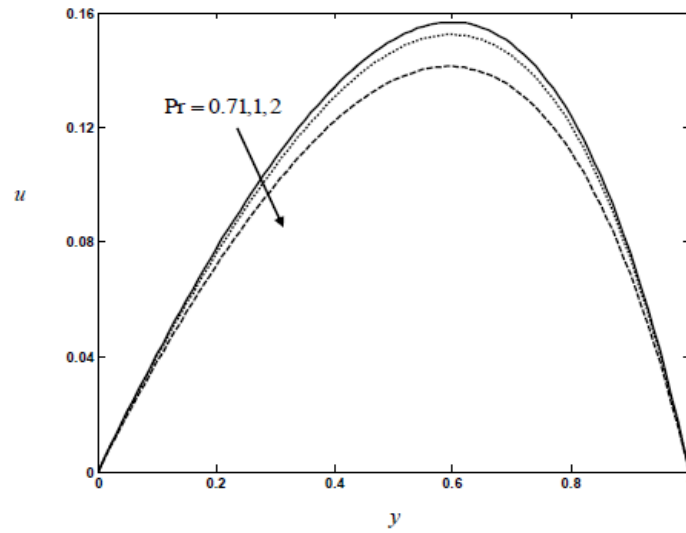


Figure-6: Effect of Prandtl number Pr on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Re = 1$

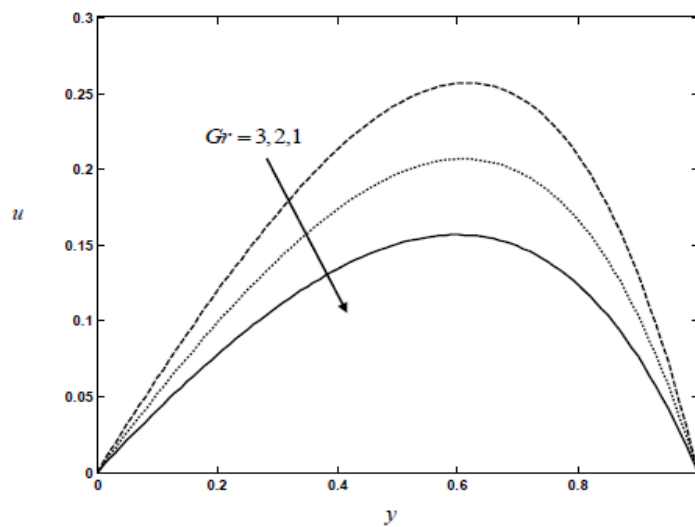


Figure-7: Effect of Grashof number Gr on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Pr = 0.71$ and $Re = 1$.

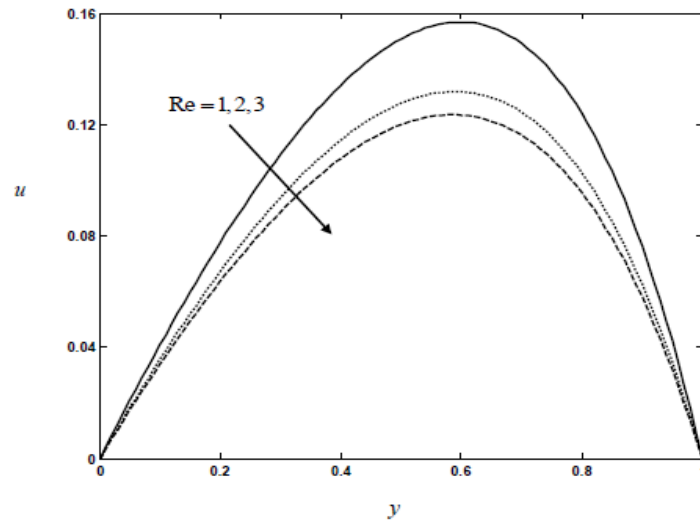


Figure-8: Effect of Reynolds number Re on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Pr = 0.71$.

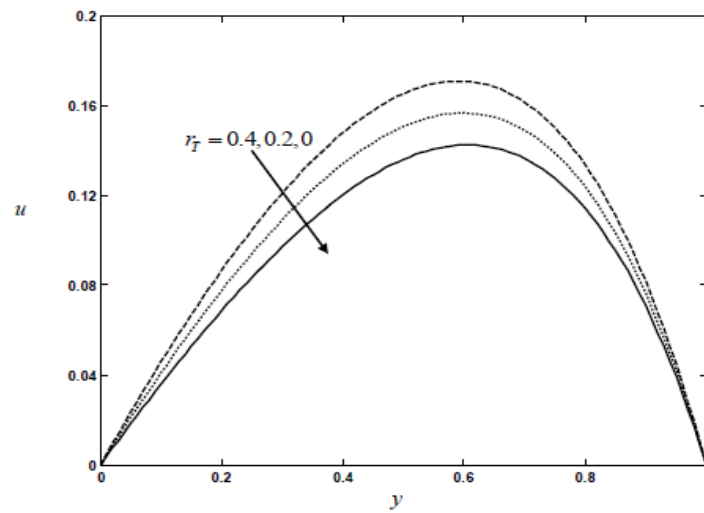


Figure-9: Effect of wall temperature parameter r_T on u for $r_T = 0.2, k = 0.01, \alpha = \frac{\pi}{6}, R = 2, A = 1, Gr = 1$ and $Pr = 0.71$.

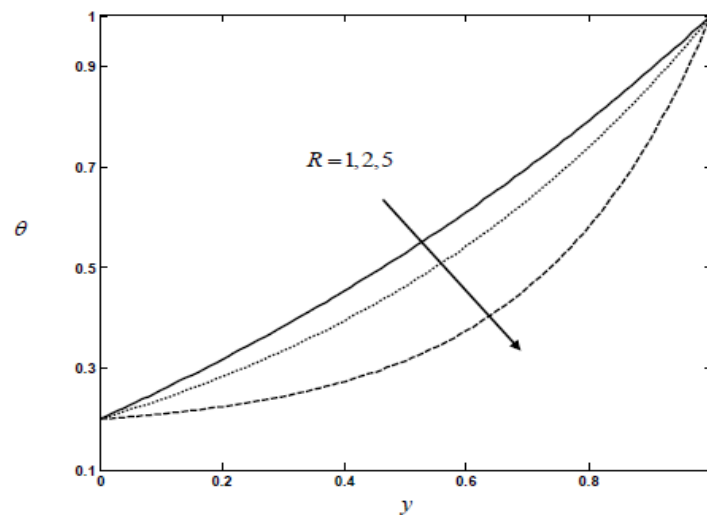


Figure-11: Effect of R on θ for $r_T = 0.2$ and $Pr = 0.71$.

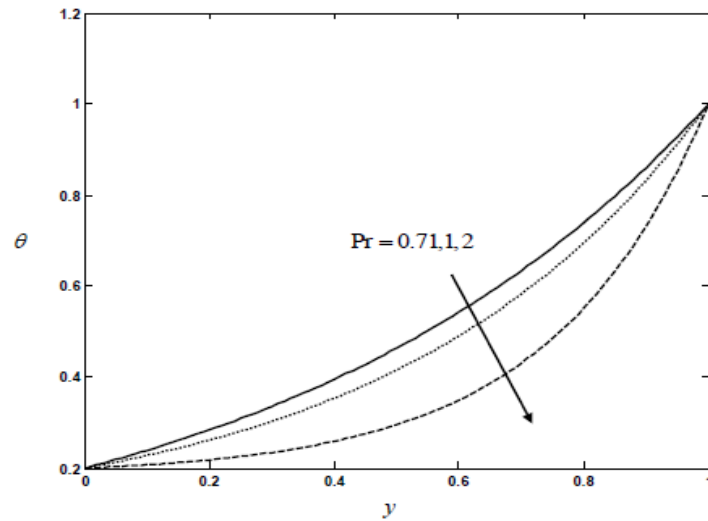


Figure-11: Effect of Pr on θ for $r_T = 0.2$ and $R = 2$.

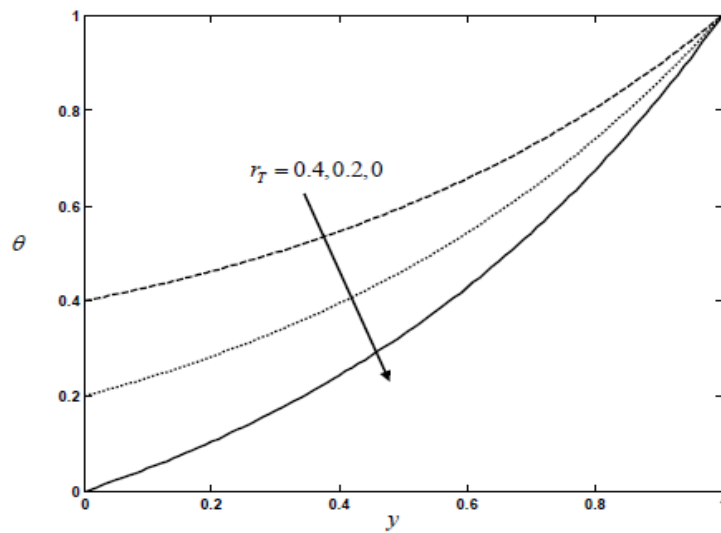


Figure-11: Effect of r_T on θ for $R = 2$ and $Pr = 0.71$.

Table-1: Effect of R on Nusselt number Nu for $r_T = 0.2$ and $Pr = 0.71$.

R	Nu
1	0.5493
2	0.3621
5	0.0840

Table-2: Effect of Pr on Nusselt number Nu for $r_T = 0.2$ and $R = 2$.

Pr	Nu
0.71	0.3621
1	0.2504
2	0.0597

Table-3: Effect of r_T on Nusselt number Nu for $R = 2$ and $Pr = 0.71$.

r_T	Nu
0	0.4526
0.2	0.3621
0.4	0.2716

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