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CRITICAL AND STABILITY CONCEPT FOR STRONG DOMINATING SET IN FUZZY GRAPHS

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ABSTRACT

In this paper we introduce the critical and stability concept to strong fuzzy dominating set in fuzzy graphs. The strong fuzzy dominating critical node is a node whose removal increases (or) decreases the strong fuzzy domination number. The stability of strong fuzzy dominating set is the minimum number of nodes whose removal increases (or) decreases the strong domination number.

Key words: *Fuzzy dominating set, strong fuzzy dominating set, strong fuzzy dominating critical node, strong fuzzy dominating stability, strong fuzzy domination number.*

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I. INTRODUCTION

Harary et al [1] explained an interesting application in voting situations using the concept of domination. Rosenfeld [5] introduced the notion of fuzzy graph and several fuzzy analogous of graph theoretic concepts such as paths, cycles, connectedness and etc. Somasundaram.A and Somasundaram.S [6] discussed domination in fuzzy graphs. Nagoor Gani.A and Basheer Ahamed.M [2] introduced strong and weak domination in fuzzy graphs. Ramachandra.S.R and Soner.N.D.[4] introduced strong domination critical and stability in graphs. In this paper we investigate the changes in the fuzzy cardinality of strong fuzzy dominating sets when we remove the nodes from G.

II. PRELIMINARIES

A *fuzzy graph* G=(σ,μ) is a non-empty set V together with a pair of functions $\sigma: V \rightarrow [0,1]$ and μ:VxV→[0,1] such that μ(u,v)≤ $\sigma(u) \wedge \sigma(v)$ for all u,v ∈ V, where $\sigma(u) \wedge \sigma(v)$is the minimum of $\sigma(u)$ *and* $\sigma(v)$.The *underlying crisp graph* of the fuzzy graph G=(σ,μ) is denoted as $G^*=(\sigma^*, \mu^*)$ where σ^*={ u∈ V/ $\sigma(u)$ >0} and μ^*= {(u,v)∈VxV / μ(u,v)>0}. A fuzzy graph G=(σ,μ) is a complete fuzzy graph if μ(u,v) = $\sigma(u) \wedge \sigma(v)$for all u,v ∈ σ^*. Two nodes u and v are said to be neighbours if μ(u,v) > 0.The *strong neighbourhood* of u is $N_S(u)$ = { v ∈ V : (u,v) is a strong arc}. $N_S[u]$ = $N_S(u)$ ∪ {u} is the *closed strong neighbourhood* of u. A *path* ρ in a fuzzy graph is a sequence of distinct nodes $u_0,u_1,u_2,...u_n$ such that $\mu(u_{i-1},u_i)$>0; 1≤ $i \leq n$ here n≥ 0 is called the *length* of the path ρ .The consecutive pairs (u_{i-1},u_i) are called the *arcs* of the path. A path ρ is called a *cycle* if $u_o=u_n$ and n≥3. An arc (u,v) is said to be a *strong arc* if μ(u,v) ≥ μ^∞ (u,v) and the node v is said to be a *strong neighbour* of u.

If μ(u,v) = 0 for every v∈V then u is called *isolated node*. A fuzzy graph G=(σ,μ) is *fuzzy bipartite* if it has a spanning fuzzy sub graph H= (τ,π) which is bipartite where for all edges (u,v) not in H, weight of (u,v) in G is strictly less than the strength of pair (u,v) in H.i.e μ(u,v)< π^∞(u,v). A fuzzy bipartite graph G with fuzzy bipartition (V_1,V_2) is said to be a complete fuzzy bipartite if for each node of V_1, every node of V_2 is a strong neighbor. Let G= (σ,μ) be a fuzzy graph and u be a node in G then there exist a node v such that (u,v) is a strong arc then we say that *u dominates v*. Let G=(σ,μ) be a fuzzy graph. A set D of V is said to be *fuzzy dominating set* of G if every v ∈ V-D there exist u ∈ D such that u dominates v. A fuzzy dominating set D of G is called a *minimal fuzzy dominating set* of G if no proper subset of D is a fuzzy dominating set. The *fuzzy domination number* γ_f(G) of the fuzzy graph G is the smallest number of nodes in any fuzzy dominating set of G. A fuzzy dominating set D of a fuzzy graph G such that $|D|$ = γ_f(G) is called minimum fuzzy dominating set.

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III. FUZZY DOMINATING CRITICAL NODES

Definition 3.1: Let G=(σ,μ) be a fuzzy graph. A node v of G is said to be fuzzy dominating critical node if its removal either increases (or) decreases the fuzzy domination number.

We partition the nodes of G into three disjoint sets according to how their removal affects γ_f(G).Let V= $V_f^0 \cup V_f^+ \cup V_f^-$ for

V_f^0= {v∈V: γ_f (G-v) = γ_f (G)}
V_f^+ = {v∈V: γ_f (G-v) $>$ γ_f (G)}
V_f^- = {v∈ : γ_f (G-v) $<$ γ_f(G)}

RESULTS

1. If I is the set of all isolated nodes of G then I ⊆ V_f^-.
2. If D is a fuzzy dominating set , removing any node in V-D cannot increase the fuzzy domination number therefore removing any node in D increase the γ_f(G). In other words D ⊆ V_f^+.

Definition 3.2: Let G = (σ,μ) be a fuzzy graph. Two nodes u and v of G are *strong adjacent* if (u,v) is strong arc. Otherwise they are said to be weak.

The *strong degree* of a node v is the number of nodes that are strong adjacent to v. It is denoted by d_S(v).

Let G = (σ,μ) be a fuzzy graph. For any u, v ∈ V, *u strongly dominates* v (i) If u is strong adjacent to v and (ii) $d_S(u) \geq d_S(v)$.

A set D⊆V is a *strong fuzzy dominating set* of G if every node in V-D is strongly dominated by at least one node in D. A strong fuzzy dominating set D of G is called a minimal strong fuzzy dominating set if no proper subset of D is a strong fuzzy dominating set of G.

The smallest number of nodes in any strong fuzzy dominating set is called the *strong domination number* and is denoted by γ_{sf} (G). A strong fuzzy dominating set D of G such that $|D|$ = γ_{sf} (G) is called minimum strong fuzzy dominating set.

IV. STRONG AND WEAK FUZZY DOMINATING CRITICAL NODES

Definition 4.1: Let G=(σ,μ) be a fuzzy graph. A node v of G is said to be strong fuzzy dominating critical node if its removal either increase (or) decrease the fuzzy strong domination number.

We partition the nodes of G into three disjoint sets according to how their removal affects γ_{sf} (G).
Let V =$V_{Sf}^0 \cup V_{Sf}^+ \cup V_{Sf}^-$
For V_{Sf}^0= {v∈V: γ_{sf} (G-v) = γ_{sf}(G)}
V_{Sf}^+ = {v∈V: γ_{sf}(G-v) $>$ γ_{sf} (G)}
V_{Sf}^-= {v∈V: γ_{sf} (G-v) $<$ γ_{sf} (G)}

Definition 4.2:

γ_{sf} - Stability of fuzzy graph G written γ_{sf} is the minimum number of nodes whose removal changes γ_{sf} (G).
γ_{sf}^+- Stability of a fuzzy graph G written $\gamma_{sf}{}^+$ is the minimum number of nodes whose removal increases γ_{sf} (G).
γ_{sf}^- - Stability of a fuzzy graph G written $\gamma_{sf}{}^-$ is the minimum number of nodes whose removal decreases γ_{sf} (G).

V. STRONG FUZZY DOMINATING CRITICAL IN FUZZY GRAPHS

Proposition 5.1: If the removal of a node u from G increases γ_{sf} (G), then (i) u is not isolated node and end node and (ii) there is no strong fuzzy dominating set for G-N_s[u] having γ_{sf} (G) nodes which also dominates N_s[u] for some γ_{sf} - set D containing u.

Proof:

(i) Suppose γ_{sf}(G-v) $>$ γ_{sf} (G) and u∈D. Then clearly u is not an isolated, and also u is not end –node, sine for any fuzzy graph G if u is end-node then γ_{sf} (G-v) $\leq$ γ_{sf} (G), a contradiction.

(ii) Suppose there exists a weak fuzzy dominating set of G-N_s[u] with γ_{sf} (G) nodes. Then γ_{sf} (G-v) ≤ γ_{sf} (G), a contradiction.

VI. STRONG FUZZY DOMINATING STABILITY IN FUZZY PATHS

Proposition 6.1: For any fuzzy path ρ_n ,$\gamma_{sf}^{+}(\rho_n) = \begin{cases} 1 \ if \ n \equiv 0 \ (mod 3), \ n \geq 3 \\ 3 \ if \ n \equiv 1 (mod 3), \ n \geq 7 \\ 2 \ if \ n \equiv 2 (mod 3), \ n \geq 5 \end{cases}$

Proof: We consider the following three cases.

Case-(i): Let n$\equiv 0(mod 3), n \geq 3$. Let $v_1,v_2,...v_n$ be the nodes of ρ_n then ρ_n-$\{v_2\}$ consists of an isolated node and a -fuzzy path of order n-2.Thus , γ_{sf} (ρ_n-$\{v_2\}$) = γ_{sf} (ρ_1) + γ_{sf} (ρ_{n-2}) $= 1 + \left\lceil \frac{(n-2)}{3} \right\rceil > \gamma_{sf}$ (ρ_n).Hence $\gamma_{sf}^{+}(\rho_n)$= 1.

Case-(ii): Let n$\equiv 1(mod 3)$, n$\geq$7. Let $v_1,v_2,...v_n$ be the nodes of ρ_n. Then ρ_n-$\{v_2,v_4,v_6\}$ consists of an 3 isolated nodes and a path of order n-6.Thus γ_{sf} (ρ_p-$\{v_2,v_4,v_6\}$)= 3(γ_{sf} (ρ_1))+ γ_{sf} (ρ_{n-6}) $= 3 + \left\lceil \frac{(n-6)}{3} \right\rceil > \gamma_{sf}$ (ρ_n). Hence $\gamma_{sf}^{+}(\rho_n)$= 3.

Case-(iii): Let n$\equiv$2 (mod3), n$\geq$5. Then ρ_n-$\{v_2,v_4\}$ consists of two isolated nodes and a path of order n-4. Thus γ_{sf} (ρ_n-$\{v_2, v_4,\}$) = 2(γ_{sf} (ρ_1)) + γ_{sf} (ρ_{n-4}) $= 2 + \left\lceil \frac{(n-4)}{3} \right\rceil > \gamma_{sf}$ (ρ_n). Hence $\gamma_{sf}^{+}(\rho_n)$= 2.

Proposition 6.2: For any fuzzy path ρ_n with n$\geq$4

$$\gamma_{sf}^{-}(\rho_n) = \begin{cases} 1 \ if \ n \equiv 1 \ (mod 3) \\ 2 \ if \ n \equiv 2 (mod 3) \\ 3 \ if \ n \equiv 0 (mod 3) \end{cases}$$

Proof:
Case-(i): Let n$\equiv$1(mod3). Let $v_1, v_2,....v_n$ be the nodes of ρ_n. Then ρ_n-$\{v_1\}$ consists of a path of order n-1. Then γ_{sf} (ρ_n-$\{v_1\}$) = γ_{sf} (ρ_{n-1}) = $\left\lceil \frac{(n-1)}{3} \right\rceil$ $since \ n \equiv 1 \ (mod 3) \ then \ n - 1 = 3K < \left\lceil \frac{n}{3} \right\rceil = \gamma_{sf}$ (ρ_n). Thus $\gamma_{sf}^{-}(\rho_n)$ = 1.

Case-(ii): Let n$\equiv$ 2(mod3) and Let $v_1,v_2,....v_n$ be the nodes of P_n. Then P_n-$\{v_1, v_2\}$ consists of a path of order n-2. Then γ_{sf} (ρ_n-$\{v_1,v_2\}$) = γ_{sf} (ρ_{n-2}) = $\left\lceil \frac{(n-2)}{3} \right\rceil < \left\lceil \frac{n}{3} \right\rceil = \gamma_{sf}$ (ρ_n). Hence $\gamma_{sf}^{-}(\rho_n)$ = 2.

Case-(iii): Let n$\equiv$ 0(mod3) and Let $v_1,v_2,....v_n$ be the nodes of ρ_n. Then ρ_n-$\{v_1, v_2, v_3\}$ consists of a path of order n-3. Then γ_{sf} (ρ_n-$\{v_1, v_2, v_3\}$) = γ_{sf} (ρ_{n-3}) = $\left\lceil \frac{(n-2)}{3} \right\rceil < \left\lceil \frac{n}{3} \right\rceil = \gamma_{sf}$ (ρ_n). Hence $\gamma_{sf}^{-}(\rho_n)$ = 3.

VII. STRONG FUZZY DOMINATING STABILITY IN FUZZY CYCLES

Proposition 7.1: For any fuzzy cycle C_p with n$\geq$6 nodes

$$\gamma_{sf}^{+}(C_n) = \begin{cases} 3 \ if \ n \equiv 0 \ (mod 3), \ n \geq 6 \\ 5 \ if \ n \equiv 1 \ (mod 3), \ n \geq 10 \\ 7 \ if \ n \equiv 2 \ (mod 3), \ n \geq 14 \end{cases}$$

Proof: Let C_n = $v_1, v_2, v_3,........v_n$. We consider the following three cases.

Case-(i): Let $n \equiv 0 \ (mod 3), n \geq 6$. Then the removal of the set of nodes $\{v_1,v_3,v_5\}$ leaves two isolated nodes and a path of order n-5.Thus γ_{sf} (C_n-$\{v_1,v_3,v_5\}$) = 2(γ_{sf} (ρ_1)) + γ_{sf} (ρ_{n-5}) $= 2 + \left\lceil \frac{(n-5)}{3} \right\rceil > \gamma_{sf}$ (C_n). Hence $\gamma_{sf}^{+}(C_n)$=3.

Case-(ii): Let $n \equiv 1 \ (mod 3), \ n \geq 10$. Then the removal of the set of nodes $\{v_1,v_3,v_5,v_7,v_9\}$ leaves four isolated nodes and a path of order n-9.Thus γ_{sf} (C_n-$\{v_1,v_3,v_5,v_7,v_9\}$) = 4(γ_{sf} (ρ_1)) + γ_{sf} (ρ_{n-9}) = 4 +$\left\lceil \frac{(n-9)}{3} \right\rceil > \gamma_{sf}$($C_n$). Hence $\gamma_{sf}^{+}(C_n)$=5.

Case-(iii): Let $n \equiv 2 \ (mod 3), n \geq 14$. Then the removal of the set of nodes $\{v_1,v_3,v_5,v_7,v_9,v_{11},v_{13}\}$ leaves six isolated nodes and a path of order n-13.Thus γ_{sf}(C_n-$\{v_1,v_3,v_5,v_7,v_9,v_{11},v_{13}\}$)=6 ($\gamma_{sf}$($\rho_1$))+$\gamma_{sf}$($\rho_{n-13}$) $= 6 + \left\lceil \frac{(n-13)}{3} \right\rceil > \gamma_{sf}$($C_n$). Hence $\gamma_{sf}^{+}(C_n)$=7.

Proposition 7.2: For any fuzzy cycle C_n with n$\geq$6 nodes

$$\gamma_{sf}^{-}(C_n) = \begin{cases} 1 \ if \ n \equiv 1 \ (mod 3) \\ 2 \ if \ n \equiv 2 (mod 3) \\ 3 \ if \ n \equiv 0 (mod 3) \end{cases}$$

Proof:
Case-(i): Let n≡1(mod3). Let v_1, v_2,....v_p be the nodes of C_n. Then C_n-{v_1} consists of a path of order n-1. Then $\gamma_{sf}(C_n\text{-}\{v_1\}) = \gamma_{sf}(\rho_{n-1}) = \left\lceil\frac{(n-1)}{3}\right\rceil < \left\lceil\frac{n}{3}\right\rceil = \gamma_{sf}(C_n)$. Hence $\gamma_{sf}^{-}(C_n) = 1$.

Case-(ii): Let n≡ 2(mod3). Let v_1, v_2,....v_n be the nodes of C_n. Then C_n-{v_1,v_2} consists of a path of order n-2. Then $\gamma_{sf}(C_n\text{-}\{v_1,v_2\}) = \gamma_{sf}(\rho_{n-2}) = \left\lceil\frac{(n-2)}{3}\right\rceil < \left\lceil\frac{n}{3}\right\rceil = \gamma_{sf}(C_n)$. Hence $\gamma_{sf}^{-}(C_n) = 2$.

Case-(iii): Let n≡ 0(mod3) .Let v_1, v_2,....v_n be the nodes of C_n. Then C_n-{v_1, v_2, v_3} consists of a path of order n-3. Then $\gamma_{sf}(C_n\text{-}\{v_1, v_2, v_3\}) = \gamma_{sf}(\rho_{n-3}) = \left\lceil\frac{(n-3)}{3}\right\rceil < \left\lceil\frac{n}{3}\right\rceil = \gamma_{sf}(C_n)$. Hence $\gamma_{sf}^{-}(C_n) = 3$.

VIII. STRONG AND WEAK FUZZY DOMINATING STABILITY IN COMPLETE FUZZY GRAPHS AND COMPLETE FUZZY BIPARTITE GRAPHS

Proposition 8.1: Let $K_{m,n}$ be a complete fuzzy bipartite graph, which is neither K_2 nor $K_{2,2}$ then

$$\gamma_{sf}^{+}(K_{m,n}) = \begin{cases} 1 \; if \; 1 = m < n \\ 1 \; if \; m = n \; \geq 4 \\ \;\; m \; if \; m < n \end{cases}$$

Proof: Suppose a complete fuzzy bipartite graph is star. Then $\gamma_{sf}(K_{1,n}) = 1$. Let v be a node of strong degree n. Then $K_{1,n}$ – v has atleast two components. Hence $\gamma_{sf}(K_{1,n}\text{- } v) \geq 2 > 1 = \gamma_{sf}(K_{1,n})$. Therefore $\gamma_{sf}^{+}(K_{1,n}) = 1$.

Suppose $K_{m,n}$ is not a star and m = n≥ 4. We know that $\gamma_{sf}(K_{m,n}) = 2$ (if m=n). Let $V=V_1 \cup V_2$ be a vertex set of $K_{m,n}$ such that $|V_1| \leq$ m, $|V_2| \leq$ n. Let v ∈ V_1 so $\gamma_{sf}(K_{m,n}\text{- } v)$ = m-1 = 3 > $\gamma_{sf}(K_{m,n})$ =2.

Proposition 8.2: Let $K_{m,n}$ be a complete fuzzy bipartite graph which is neither K_2 nor $K_{2,2}$ then

$$\gamma_{sf}^{-}(K_{m,n}) = \begin{cases} m + n - 1 \quad if \; m = n \\ \;\; 1 \quad if \; 3 \leq m \leq n \end{cases}$$

Proof: Suppose m=n and $V=V_1 \cup V_2$ be the vertex set of $K_{m,n}$ such that $|V_1|$ =m, $|V_2| =$ n. Then $\gamma_{sf}(K_{m,n}\text{- } V_1\text{-}V_2\text{+}1)$= 1 < 2 = $\gamma_{sf}(K_{m,n})$. Hence $\gamma_{sf}^{-}(K_{m,n})$ = m+n-1.

Suppose $3 \leq m \leq n$ $\gamma_{sf}(K_{m,n})$ = m if m<n. Let v ∈ V_1 so $\gamma_{sf}(K_{m,n}\text{- } v)$ = m-1 < m = $\gamma_{sf}(K_{m,n})$. Hence $\gamma_{sf}^{-}(K_{m,n}) = 1$.

Theorem 8.3: A vertex v of V(G) is in V_{Sf}^{0} if and only if G = $\bigcup_{i=2}^{t} K_i$ where t≥2 and K_i, i= 2,3, ...t are complete fuzzy graphs.

Proof: Let D be an γ_{sf}-set of G. Clearly $|D|$ = t-1. Since there is no path between K_i, i= 2,3,...t. Let V = $V_2 \cup V_3 \cup ... \cup V_t$ where V_i are vertex sets of K_i respectively. Suppose u∈V_r, $2 \leq r \leq t$. As K_r-u is a complete fuzzy graph of order at least r-1 and $\gamma_{sf}(K_r\text{- } u) = \gamma_{sf}(K_r)$ then D will still γ_{sf} - set of G if such a node is removed thus for any u ∈ V. We have u ∈ V_{Sf}^{0}.Conversely let V(G) = V_{Sf}^{0}(G) and D be an γ_{sf} - set of G. Note that every node of strong degree at least one must be in D. Also note that all nodes of V-D must be in <N_S[D]>. Furthermore there can be no path in G, connecting two nodes of D. A graph with these properties is only union of complete fuzzy graphs of order at least two.

Theorem 8.4: A vertex v of V(G) is in V_{Sf}^{+} if and only if G = $\bigcup_{r=2}^{t} K_{r,r}$

Proof: Let D be an γ_{sf}-set of G. Clearly $|D|$ = 2(t-3). Since there is no path between $K_{i,i}$, i= 4,5,...t. Let V = $V_4 \cup V_5 \cup ... \cup V_t$ where V_i are vertex sets of $K_{i,i}$ respectively. Suppose u∈V_r, $4 \leq r \leq t$. As $K_{r,r}$-u is a complete fuzzy graph of order at least r-1 and $\gamma_{sf}(K_{r,r}\text{- } u) = \gamma_{sf}(K_{r,r})$ then D will still γ_{sf} - set of G if such a vertex is removed thus for any u ∈ V. We have u ∈ V_{Sf}^{+}. Conversely let V(G) = V_{Sf}^{+}(G) and D be an γ_{sf} - set of G. Note that every node of strong degree at least one must be in D. Also note that all nodes of V-D must be in <N_S[D]>. Furthermore there can be no path in G, connecting two nodes of D. A graph with these properties is only union of complete fuzzy graphs of order at least four.

CONCLUSION

In this paper we discussed critical and stability concept of strong fuzzy dominating set for several classes of fuzzy graphs and obtained some results. Here we considered all the complete fuzzy bipartite graphs $K_{m,n}$ except K_2 and $K_{2,2}$.

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