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NON EXISTENCE OF RELAXED SKOLEM MEAN LABELING FOR STAR GRAPHS

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ABSTRACT

In this paper we prove that the two star graph $K_{1,m} \cup K_{1,n}$ *is not a relaxed skolem mean graph if* $|m-n| > 5$ *and the three star graph* $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ *is not a relaxed skolem mean graph if* $|m-n| > \ell + 6$.

Keywords: *Relaxed Skolem mean graph and star.*

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1. INTRODUCTION

We proved that $K_{1,m}$ is not a relaxed skolem mean graph for $m \geq 5$. Also, we proved that the two star $K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq 5$. Next, we proved the existence of relaxed skolem mean graphs. In [3], the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a relaxed skolem mean graph if $|m-n| \leq \ell + 6$ for $\ell = 1, 2, 3, \ldots$; $m = 1, 2, 3, \ldots$; if $n = \ell + m + 6$ when $\ell \leq m < n$.

2. [4] RELAXED SKOLEM MEAN LABELING

Definition 2.1: A graph $G=(V, E)$ with p vertices and q edges is said to be a relaxed skolem mean graph if there exists a function f from the vertex set of G to $\{1, 2, 3, \ldots, p+1\}$ such that the induced map f* from the edge set of G to $\{2, 3, 4, \ldots, p+1\}$ defined by

$$f^*(e=uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd, then} \end{cases}$$

the resulting edges get distinct labels from the set $\{2, 3, 4, \ldots, p+1\}$.

Note 2.2: [4], In a Relaxed skolem mean graph, $p \geq q$.

Theorem 2.3: The two star $G = K_{1,m} \cup K_{1,n}$ is not a relaxed Skolem mean graph if $|m-n| > 5$.

Proof: Without loss of generality, let us consider $m \leq n$. Consider the primal graph under the condition $|m-n| > 5$ that is m = 1 and n = 7. That is $G = K_{1,1} \cup K_{1,7}$.

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Where $V(G) = \{v_{1,j} : 0 \le j \le 1\} \cup \{v_{2,j} : 0 \le i \le 7\}$

$E(G) = \{v_{1,0}v_{1,1}\} \cup \{v_{2,0}v_{2,j} : 1 \le j \le 7\}$

G has 10 vertices and 8 edges.

Suppose G is relaxed skolem mean graph.

Then there exists a function *f* from the vertex set of G to {1, 2, 3,….,11} such that the induced map *f** from the edge set of G to {2, 3,….,11} defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & if\ \ f(u)+f(v)\ is\ even \\ \dfrac{f(u)+f(v)+1}{2} & if\ \ f(u)+f(v)\ is\ odd \end{cases}$$

Then the resulting edges get distinct labels from the set {2, 3…..11}.

Then the vertex and edge mappings of G is given by
f : V(G)→{1,2,…..11}

f:E(G)→{2,3….11}*

Let, $t_{i,j} = f(v_{i,j})$ *and* $x_{i,j} = f^*(v_{i,0}\ v_{i,j})\ \forall\ i$ *and* j.

Now let us consider the following cases,

Case-(a): $t_{2,0} = 11$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{11+2n}{2}\right) = 6+n = \left(\frac{11+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10).These six labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 11,\ t_{2,1} = 1,\ t_{2,2} = 3\ , t_{2,7} = 2$.

Then the corresponding edge labels are $x_{2,1} = 6,\ x_{2,2} = 7,\ x_{2,7} = 7$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 11$.

Case-(b): $t_{2,0} = 10$

If $t_{2,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{10+2n-1}{2}\right) = 5+n = \left(\frac{10+2n}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), (7 or 8), 9 and 11. These six labels are not sufficient to label seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 10$, $t_{2,1} = 2$, $t_{2,2} = 4$, $t_{2,7} = 1$.
Then the corresponding edge labels are $x_{2,1} = 6$, $x_{2,2} = 7$, $x_{2,7} = 6$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 10$.

Case-(c): $t_{2,0} = 9$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some *n,j and k* then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{9+2n}{2}\right) = 5+n = \left(\frac{9+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6 or 7), 8, (10 or 11).These six labels are not sufficient to label seven vertices, $t_{2,0}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 9$, $t_{2,1} = 1$, $t_{2,2} = 3$, $t_{2,7} = 2$.
Then the corresponding edge labels are $x_{2,1} = 5$, $x_{2,2} = 6$, $x_{2,7} = 6$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 9$.

Case- (d): $t_{2,0} = 8$.

If $t_{2,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some n, j and k then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{1+2n}{2}\right) = 1+n = \left(\frac{1+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), (7), (9 or 10), (11). These six labels are not sufficient to label seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 8$, $t_{2,1} = 2$, $t_{2,2} = 4$, $t_{2,7} = 1$.
Then the corresponding edge labels are $x_{2,1} = 5$, $x_{2,2} = 6$, $x_{2,7} = 5$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 8$.

Case-(e): $t_{2,0} = 7$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{7+2n}{2}\right) = 4+n = \left(\frac{7+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6), (8 or 9), (10 or 11). These six labels are not sufficient to label seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 7$, $t_{2,1} = 1$, $t_{2,2} = 3$, $t_{2,7} = 2$.

Then the corresponding edge labels are $x_{2,1} = 4$, $x_{2,2} = 5$, $x_{2,7} = 5$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 7$.

Case-(f): $t_{2,0} = 6$.

If $t_{2,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some n, j and k then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{6+2n-1}{2}\right) = 3+n = \left(\frac{6+2n}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5), (7 or 8), (9 or 10), (11). These six labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 6$, $t_{2,1} = 1$, $t_{2,2} = 3$, $t_{2,7} = 2$.

Then the corresponding edge labels are $x_{2,1} = 4$, $x_{2,2} = 5$, $x_{2,7} = 4$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 6$.

Case-(g): $t_{2,0} = 5$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{5+2n}{2}\right) = 3+n = \left(\frac{5+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4), (6 or 7), (8 or 9), (10 or 11).These six labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 5$, $t_{2,1} = 1$, $t_{2,2} = 3$, $t_{2,7} = 2$.

Then the corresponding edge labels are $x_{2,1} = 3$, $x_{2,2} = 4$, $x_{2,7} = 4$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 5$.

Case-(h): $t_{2,0} = 4$.

If $t_{2,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{4+2n-1}{2}\right) = 2+n = \left(\frac{4+2n}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3), (5 or 6), (7 or 8), (9 or 10), (11).These six labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 4,\ t_{2,1} = 2,\ t_{2,2} = 3,\ t_{2,7} = 1$.

Then the corresponding edge labels are $x_{2,1} = 3,\ x_{2,2} = 4,\ x_{2,7} = 3$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 4$.

Case-(i): $t_{2,0} = 3$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some *n, j and k* then

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{3+2n}{2}\right) = 4+n = \left(\frac{3+2n+1}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2), (4 or 5), (6 or 7), (8 or 9), (10 or 11). These six labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0}=3,\ t_{2,3} = 4, t_{2,4} = 6, t_{2,7} = 5$.

Then the corresponding edge labels are $x_{2,3} = 4$, $x_{2,4} = 5,\ x_{2,7} = 4$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 3$.

Case-(j): $t_{2,0} = 2$.

If $t_{2,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then,

$$x_{2,j} = f^*(v_{2,0}v_{2,j}) = \left(\frac{2+2n-1}{2}\right) = 1+n = \left(\frac{2+2n}{2}\right) = f^*(v_{2,0}v_{2,k}) = x_{2,k}$$

Therefore, the possibilities for the pendent vertices are (1), (3 or 4), (5 or 6), (7 or 8), (9 or 10), (11). These six labels are not sufficient to labels even vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 2,\ t_{2,2} = 3,\ t_{2,3} = 5, t_{2,7} = 4$.

Then the corresponding edge labels are $x_{2,2} = 3,\ x_{2,3} = 4, x_{2,7} = 3$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 2$.

Case (k): $t_{2,0} = 1$.

If $t_{2,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{3+2n}{2}\right) = 2+n = \left(\frac{3+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11). These five labels are not sufficient to labels seven vertices, $t_{2,j}$ for $1 \le j \le 7$.

Suppose $t_{2,7}$ takes any of the remaining values.

Let $t_{2,0} = 1,\ t_{2,1} = 2,\ t_{2,2} = 4,\ t_{2,7} = 3$.
Then the corresponding edge labels are $x_{2,1} = 2,\ x_{2,2} = 3,\ x_{2,7} = 2$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{2,0} = 1$.
$\Rightarrow$ G is not a relaxed skolem mean graph for all values of $t_{2,0}$.

Therefore G is not a relaxed skolem mean graph.

In general, we see that G is not a relaxed skolem mean graph if $|m-n| = 6$.

Similar argument asserts that $|m-n| = 7$ is not a relaxed skolem mean graph.

Similarly, we can prove for all greater values of *m*.

Therefore, G is not a relaxed skolem mean graph if $|m-n| > 5$.

Definition 2.4: The three star is the disjoint union of $K_{1,\ell}, K_{1,m}$ *and* $K_{1,n}$. *It is denoted by* $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$.

Theorem 2.5: The three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a relaxed Skolem mean graph if $|m-n| > \ell + 6$.

Proof: Let us consider the primal graph under the condition $|m-n| > \ell + 6$ and m = 1; n = 9.

Therefore $G = K_{1,1} \cup K_{1,1} \cup K_{1,9} = 2K_{1,1} \cup K_{1,9}$
Where $V(G) = \{v_{i,j} : 1 \le i \le 2,\ 0 \le j \le 1\} \cup \{v_{3,j} : 0 \le j \le 9\}$

$E(G) = \{v_{i,0}v_{i,1} : 1 \le i \le 2\} \cup \{v_{3,0}v_{3,j} : 1 \le j \le 9\}$

G has 14 vertices and 11 edges.

Suppose G is relaxed skolem mean graph.
Then there exists a function *f* from the vertex set of G to {1,2,...,15} such that the induced map *f** from the edge set of G to {2, 3,…,15} defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

Then the resulting edges get distinct label from the set {2, 3…., 15}.

Then the vertex and edge mappings of G is given by
f : V(G)→{1,2,….15}

f:V(G)→{2,3,….15}*

Let $t_{i,j} = f(v_{i,j})$ *and* $x_{i,j} = f^*(v_{i,0}\ v_{i,j})\ \forall\ i$ *and* j.

Now let us consider the following cases,

Case (a): $t_{3,0} = 15$.

If $t_{3,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{15+2n}{2}\right) = 8+n = \left(\frac{15+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), 14.These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 15, t_{3,1} = 1, t_{3,2} = 3, t_{3,9} = 2$.
Then the corresponding edge labels are $x_{3,1} = 8, x_{3,2} = 9, x_{3,9} = 9$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 15$.

Case (b): $t_{3,0} = 14$.
If $t_{3,j} = n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{14+2n-1}{2}\right) = 7+n = \left(\frac{14+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), (7 or 8), (9 or 10), (11 or 12), (13), (15). These eight labels are not sufficient to label nine vertices $t_{3,9}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 14,\ t_{3,1} = 2,\ t_{3,2} 4, t_{3,9} = 3$.
Then the corresponding edge labels are $x_{3,1} = 8, x_{3,2} = 9, x_{3,9} = 9$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 14$.

Case-(c): $t_{3,0} = 13$.

If $t_{3,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{13+2n}{2}\right) = 7+n = \left(\frac{13+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), 14. These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 13,\ t_{3,1} = 1,\ t_{3,2} = 3,\ t_{3,9} = 2$.

Then the corresponding edge labels are $x_{3,1} = 7$, $x_{3,2} = 8$, $x_{3,9} = 8$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 13$.

Case (d): $t_{3,0} = 12$.

If $t_{3,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{12+2n-1}{2}\right) = 6+n = \left(\frac{12+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), (7 or 8), (9 or 10), (11), (13or 14), (15).These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 12,\ t_{3,8} = 15,\ t_{3,7} = 13,\ t_{3,9} = 14$.

Then the corresponding edge labels are $x_{3,8} = 14,\ x_{3,7} = 13,\ x_{3,9} = 13$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 12$.

Case-(e): $t_{3,0} = 11$.

If $t_{3,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{11+2n}{2}\right) = 6+n = \left(\frac{11+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}.$$

Therefore, the possibilities for the pendent vertices (1), (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10), (12 or 13), (14 or15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 11,\ t_{3,1} = 1,\ t_{3,2} = 3,\ t_{3,9} = 2$.

Then the corresponding edge labels are $x_{3,1} = 6,\ x_{3,2} = 7$, $x_{3,9} = 7$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 11$.

Case-(f): $t_{3,0} = 10$.

If $t_{3,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n, j and k* then

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{10+2n-1}{2}\right) = 5+n = \left(\frac{10+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), (7 or 8), 9, (11 or 12), (13 or 14), (15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 10,\ t_{3,1} = 2,\ t_{3,2} = 4,\ t_{3,9} = 3$.

Then the corresponding edge labels are $x_{3,1} = 6,\ x_{3,2} = 7\ x_{3,9} = 7$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 10$.

Case-(g): $t_{3,0} = 9$

If $t_{3,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{9+2n}{2}\right) = 5+n = \left(\frac{9+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), (6 or 7), (8), (10 or 11), (12 or 13), (14 or15).These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 9,\ t_{3,1} = 1,\ t_{3,2} = 3,\ t_{3,9} = 2$.

Then the corresponding edge labels are $x_{3,1} = 5,\ x_{3,2} = 6,\ x_{3,9} = 6$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 9$.

Case-(h): $t_{3,0} = 8$.

If $t_{3,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some *n , j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{8+2n-1}{2}\right) = 4+n = \left(\frac{8+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5 or 6), 7, (9 or 10), (11 or 12), (13 or 14), (15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 8,\ t_{3,1} = 2,\ t_{3,2} = 4,\ t_{3,9} = 3$.

Then the corresponding edge labels are $x_{3,1} = 5,\ x_{3,2} = 6,\ x_{3,9} = 6$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 8$.

Case-(i): $t_{3,0} = 7$.

If $t_{3,j} = 2n \; and \; t_{2,k} = 2n+1$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{7+2n}{2}\right) = 4+n = \left(\frac{7+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), (4 or 5), 6,(8 or 9), (10 or 11), (12 or 13), (14 or15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 7$, $t_{3,1} = 1$, $t_{3,2} = 3$, $t_{3,9} = 2$.

Then the corresponding edge labels are $x_{3,1} = 4$, $x_{3,2} = 5$, $x_{3,9} = 5$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 7$.

Case-(j): $t_{3,0} = 6$.

If $t_{3,j} = 2n-1 \; and \; t_{2,k} = 2n$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{6+2n-1}{2}\right) = 3+n = \left(\frac{6+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3 or 4), (5), (7 or 8), (9 or 10), (11 or 12), (13 or 14), (15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 6$, $t_{3,2} = 3$, $t_{3,3} = 5$, $t_{3,9} = 4$.

Then the corresponding edge labels are $x_{3,2} = 5$, $x_{3,3} = 6$, $x_{3,9} = 5$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 6$.

Case-(k): $t_{3,0} = 5$.

If $t_{3,j} = 2n \; and \; t_{2,k} = 2n+1$ for some *n, j and k* then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{5+2n}{2}\right) = 3+n = \left(\frac{5+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2 or 3), 4, (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or15). These eight labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,o} = 5$, $t_{3,1} = 1$, $t_{3,2} = 3$, $t_{3,9} = 2$.

Then the corresponding edge labels are $x_{3,1} = 3,\ x_{3,2} = 4,\ x_{3,9} = 4$.In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0}$= 5.

Case-(ℓ): $t_{3,0} = 4$.

If $t_{3,j} = 2n-1$ *and* $t_{2,k} = 2n$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{4+2n-1}{2}\right) = 2+n = \left(\frac{4+2n}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1 or 2), (3), (5 or 6), (7 or 8), (9 or 10), (11 or 12), (13 or 14), (15). These eight lables are not sufficient to lable nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,o} = 4,\ t_{3,2} = 3,\ t_{3,3} = 5, t_{3,9} = 6$.

Then the corresponding edge labels are $x_{3,2} = 4$, $x_{3,3} = 5, x_{3,9} = 5$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0} = 4$.

Case-(m): $t_{3,0} = 3$.

If $t_{3,j} = 2n$ *and* $t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{3+2n}{2}\right) = 2+n = \left(\frac{3+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (2), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15). These eight lables are not sufficient to lable nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 3,\ t_{3,3} = 4,\ t_{3,4} = 6,\ t_{3,9} = 5$.

Then the corresponding edge labels are $x_{3,3} = 4,\ x_{3,4} = 5,\ x_{3,9} = 4$.In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.
Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 3$.

Case-(n): $t_{3,0} = 2$.

If $t_{3,j} = 2n+1$ *and* $t_{2,k} = 2n+2$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{2+2n+1}{2}\right) = 2+n = \left(\frac{2+2n+2}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (1), (3 or 4), (5 or 6), (7 or 8), (9 or 10), (11 or 12), (13 or 14), (15). These eight labels are not sufficient to label nine vertices $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 2,\ t_{3,2} = 3,\ t_{3,3} = 5,\ t_{3,9} = 4$.
Then the corresponding edge labels are $x_{3,2} = 3,\ x_{3,3} = 4,\ x_{3,9} = 3$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore, G is not a relaxed skolem mean graph when $t_{3,0} = 2$.

Case-(o): $t_{3,0} = 1$.

If $t_{3,j} = 2n \ \ and \ \ t_{2,k} = 2n+1$ for some n, j and k then,

$$x_{3,j} = f^*(v_{3,0}v_{3,j}) = \left(\frac{1+2n}{2}\right) = 1+n = \left(\frac{1+2n+1}{2}\right) = f^*(v_{3,0}v_{3,k}) = x_{3,k}$$

Therefore, the possibilities for the pendent vertices are (2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15). These seven labels are not sufficient to label nine vertices, $t_{3,j}$ for $1 \le j \le 9$.

Suppose $t_{3,9}$ takes any of the remaining values.

Let $t_{3,0} = 1,\ t_{3,1} = 3,\ t_{3,2} = 5,\ t_{3,8} = 2,\ t_{3,9} = 4$.
Then the corresponding edge labels are $x_{3,1} = 2,\ x_{3,2} = 3,\ x_{3,8} = 4,\ x_{3,9} = 3$. In this case it is not possible to label the vertices without labeling three of them as consecutive integers. If there are three consecutive integers then two of them will induce the same edge label.

Therefore G is not a relaxed skolem mean graph when $t_{3,0}$ = 1.

G is not a relaxed skolem mean graph for all values of $t_{3,0}$.
Therefore, G is not a relaxed skolem mean graph.

Similarly $G = K_{1,2} \cup K_{1,2} \cup K_{1,11}$ is not a relaxed skolem mean graph.

In general we see that G is not a relaxed skolem mean graph if $|m-n| = \ell + 7$. With similar argument we can prove for all greater values
Therefore, G is not a relaxed skolem mean graph if $|m-n| > \ell + 6$.

3. APPLICATION OF GRAPH LABELING

The skolem mean labeling is applied on a graph (network), such as bus topology, mesh topology and star topology in order to solve the problems in establishing fastness, efficient communication and various issues in that area, in which the following will be taken into account.

1. A protocol, with secured communication can be achieved, provided the graph (network) is sufficiently connected.
2. To find an efficient way for safer transmissions in areas such as Cellular telephony, Wi – Fi, Security systems and many more.
3. Channel labeling can be used to determine the time at which sensor communicate.

CONCLUSION

Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

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