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APPLICATION OF FRACTIONAL CALCULUS TO K-STARLIKE FUNCTIONS WITH POSITIVE COEFFICIENTS

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ABSTRACT

In the present paper we introduce and study a family of analytic and univalent functions related to K-starlike functions with positive coefficients by applying fractional derivative operator in the open unit disc. For this class coefficient estimates, distortion bounds, extreme points, radii of convexity and class preserving integral operator have been established. It is worthy to note that many of our results are either extensions or new approaches to those correspondence previously known results.

INTRODUCTION

Let $\mathcal{A}$ denote the class of functions f of the form

$$f\,(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

Which are analytic in the open unit disc $U = \{z : |z| < 1\}$ and S denote the subclass of $\mathcal{A}$ which are univalent in *U.*

In 1999, Kanas and Wisnlowska [5] see also Kanas and Srivastava [4] and Kanas and Wisniowska [6] studied the class of K-uniformly convex functions denoted by $K - UCV, 0 \le K \le \infty$ *so that* $f \in K - UCV$, if and only if

$$Re\left\{1 + \left(1 + Ke^{i\emptyset}\right)\frac{z f''(z)}{f'(z)}\right\} \ge 0$$

and *K – UCV (α)* denote the subclass of *S* if and only if

$$Re\left\{1 + \left(1 + Ke^{i\emptyset}\right)\frac{z f''(z)}{f'(z)}\right\} \ge \alpha, \qquad (0 \le \alpha < 1).$$

Further the class $K - S_p\,(\alpha)$ denoted the subclass of S if and only if

$$Re\left\{\left(1 + Ke^{i\emptyset}\right)\frac{z f''(z)}{f'(z)} - Ke^{i\emptyset}\right\} \ge \alpha \quad (0 \le \alpha < 1).$$

For $1 < \beta \le \frac{4}{3}$ *and* $z \in U$, *Let*

$$K - UCV^*\,(\beta) = \left\{f \in S : Re\left\{1 + \left(1 + Ke^{i\emptyset}\right)\frac{z f''(z)}{f'(z)}\right\} < \beta\right\},$$

and

$$K - S_p^*\,(\beta) = \left\{f \in S : Re\left\{\left(1 + Ke^{i\emptyset}\right)\frac{z f''(z)}{f'(z)} - Ke^{i\emptyset}\right\} < \beta\right\},$$

Further, Let *V* be subclass of consisting of function of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n\,, \qquad a_n \ge 0 \tag{1.2}$$

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Let $K - PUCV^* (\beta) \equiv K - UCV^* (\beta) \cap V$
$K - PS_p^*(\beta) \equiv K - S_p^*(\beta) \cap V$

In particular, When $K = 0$, We have $0 - UCV^* (\beta) \equiv L (\beta)$,
$$0 - S_p^*(\beta) \equiv M(\beta),\ 0 - PUCV^* \equiv U (\beta) \quad and \quad 0 - P S_p^*(\beta) \equiv V (\beta),$$

The classes $L (\beta)$, $M(\beta)$, $U (\beta)$ and $V (\beta)$ have been extensively studies by Uralegaddi *et.al* (13)

Porwal and Dixit [8] studied the classes of K- uniformly convex and K-uniformly starlike functions with positive coefficients only. We, also not that in [3], Dixit et al have also investigated and studied the univalent function with positive coefficients by the use of Salagean operator.

In this paper, we present our studies on a new subclass of univalent functions, using fractional calculus operator Ω^λ, which are motivated from the investigations of Srivastava and Owa [11], [12] Kumar et. Al [17] and Owa [9], A recent application of this operator $\Omega^\lambda$is given by Dixit and Pathak [1], [2] and Srivastava and Mishra [10]. We now define a new class $K - PS_\lambda^*(\beta)$.

Definition 1.1, Let $K - PS_p^\lambda (\beta)\left(0 \le \lambda \le 1; 1 < \beta \le \frac{4}{3}\right)$ be the class of functions f in V Satisfying the inequality
$$Re\left\{\left(1 + Ke^{i\emptyset}\right)\frac{z\left(\Omega^\lambda\right) f(z)}{\Omega^\lambda f(z)} - Ke^{i\emptyset}\right\} < \beta \qquad z \in U \tag{1.3}$$
Where $\Omega^\lambda f(z) = \Gamma (2 - \lambda) z^\lambda D_z^\lambda f(z) \quad (f \in V : 0 \le \lambda < 1)$

$D_z^\lambda f(z)$ denotes the fractional derivative of $f(z)$of order λ, as defined in [11] with
$$D_z^0 f(z) = f(z) \ and \ D_z^{'} f(z) = f^{'}(z)$$

It is Easily seen that
$$\Omega^o f(z) = f(z) and \ \Omega^{'} f(z) = z f^{'}(z)$$

For the class $K - P S_\lambda^*(\beta)$ of functions belonging to V, we prove number of sharp results including coefficient and distortion theorems.

2. FIRST WE OBTAIN COEFFICIENT CONDITION FOR FUNCTIONS IN $K - P S_\lambda^*(\beta)$.

Theorem 2.1: A function $f(z)$ belonging to V is in $K - PS_\lambda^*(\beta)$ *if and if*
$$\sum_{n=2}^{\infty}(n + nk - K - \beta)\ \emptyset(n)\ a_n \le \beta - 1 \tag{2.1}$$
Where, for Convenience
$$\emptyset (n) = \frac{\Gamma(n+1)\,\Gamma(2-\lambda)}{\Gamma(n+1-\lambda)}$$

The result (2.1) is sharp

Proof: We assume that the inequality (2.1) holds true and let $|z| = 1$. It suffices to show that
$$\left|\frac{\left(1 + Ke^{i\emptyset}\right)\dfrac{z\left(\Omega^\lambda f(z)\right)^{'}}{\Omega^\lambda f(z)} - Ke^{i\emptyset} - 1}{(1 + Ke^{i\emptyset})\,\dfrac{z\left(\Omega^\lambda f(z)\right)^{'}}{\Omega^\lambda f(z)} - Ke^{i\emptyset} - (2\beta - 1)}\right| < 1, \quad z \in U$$
$$\left|\frac{\left(1 + Ke^{i\emptyset}\right)\dfrac{z\left(\Omega^\lambda f(z)\right)^{'}}{\Omega^\lambda f(z)} - \left(Ke^{i\emptyset} + 1\right)}{(1 + Ke^{i\emptyset})\,\dfrac{z\left(\Omega^\lambda f(z)\right)^{'}}{\Omega^\lambda f(z)} - Ke^{i\emptyset} - (2\beta - 1)}\right| \le \frac{(K+1)\ \sum_{n=2}^{\infty}\emptyset\,(n)a_n|z|^{n-1}}{2(\beta-1) - \sum_{n=2}^{\infty}\{(K+1)\,(n-1) - (\beta-1)\,a_n\emptyset\,(n)\,|z|^{n-1}\}}$$
$$\le \frac{(K+1)\ \sum_{n=2}^{\infty}(n-1)\emptyset\,(n)a_n}{2(\beta-1) - \sum_{n=2}^{\infty}\{(K+1)\,(n-1) - 2\,(\beta-1)\}\,\emptyset\,(n)\,a_n}$$

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The last expression is bounded above by 1, if

$$\sum_{n=2}^{\infty}(n+n\,K-K\beta\,)\,\emptyset\,(n)\;a_n\;\le\;\beta-1 \tag{2.3}$$

But (2.3) is true by hypothesis. Hence we have $f\,(z)\in K-P\,S_\lambda^*\,(\beta)$

To prove that converse, we assume that $f(z)$ is defined by (1.2) and in the class $K-\;P\,S_\lambda^*(\beta)$, so that the condition (1.3) readily yields

$$Re\quad\left\{\frac{\beta-1-\;\sum_{n=2}^{\infty}(n-\beta)\emptyset\,(n)a_n\,z^{n-1}-\;Ke^{i\,\emptyset}\;\sum_{n=2}^{\infty}\emptyset\,(n)\,(n-1)\,z^{n-1}}{1+\;\sum_{n=2}^{\infty}\;\emptyset(n)\,a_n\,z^{n-1}}\right\}\ge 0$$

The above condition must hold for all values of z, $|z|=r<1$, Upon choosing the values of z on the positive real axis, where $0\le z=\;r\;<1$, we must have

$$Re\;\left\{\frac{\beta-1-\;\sum_{n=2}^{\infty}(n-\beta)\emptyset\,(n)a_n\,r^{n-1}}{1+\;\sum_{n=2}^{\infty}\emptyset\,(n)a_nr^{n-1}}-\frac{Ke^{i\,\emptyset}\sum_{n=2}^{\infty}\emptyset\,(n)(n-1)a_nr^{n-1}}{1+\;\sum_{n=2}^{\infty}\emptyset\,(n)a_nr^{n-1}}\right\}\ge 0$$

Since $Re\left(-\,Ke^{i\,\emptyset}\right)\;\ge\;-\left|Ke^{i\,\emptyset}\right|=\;-K, The\ above\ inequality\ reduces\ to$

$$\frac{\beta-1-\;\sum_{n=2}^{\infty}\emptyset\,(n)\,(n-\;\beta)a_n\,r^{n-1}-\;K\;\sum_{n=2}^{\infty}\emptyset\,(n)(n-1)a_nr^{n-1}}{1+\;\sum_{n=2}^{\infty}\emptyset\,(n)a_nr^{n-1}}\ge 0$$

Letting $r\;\to 1$ we have $\sum_{n=2}^{\infty}\emptyset(n)\;\;(n+nK-K-\;\beta)\,a_n\;\le\;\beta-1$, and the proof is complete.

Finally, we note that the assertion (2.1) of Theorem 2.1 is sharp, the external function being

$$f\,(z)=z+\;\frac{(\beta-1)}{(n-n\,K-K-\;\beta)\,\emptyset\,(n)}\;z^n$$

Remark 2.1: When $\lambda=0$, Theorem 2.1 reduces to the corresponding result due to Porwal and Dixit [8]

Remark 2.2: When $\lambda=1$, Theorem 2.1 reduce to the corresponding result due to Porwal and Dixit [8]

Remark 2.3: When = 0, Theorem 2.1 reduce to the corresponding result due to Dixit and Pathak [1]

We record in passing the following interesting consequence of Theorem 2.1.

Corollary 2.1: Let the Function *f(z)* defined by (1.2) belong to the class $K-PS_\lambda^*(\beta)$. Then

$$a_n\le\;\frac{(\beta-1)}{(n-n\,K-K-\;\beta)\,\emptyset\,(n)},\qquad(n\;\ge 2)$$

Theorem 2.2: Let the function *f(z)* defined by (1.2) be in the class $K-P\,S_\lambda^*(\beta)$, Then

$$|f(z)|\;\ge\;|z|-\;\frac{(\beta-1)\,(2-\;\lambda)}{2\,(2+K-\beta)}\;|z|^2 \tag{2.4}$$

and

$$|f(z)|\;\ge\;|z|+\;\frac{(\beta-1)\,(2-\;\lambda)}{2\,(2+K-\beta)}\;|z|^2, \tag{2.5}$$

With equality for

$$f\,(z)=z+\frac{(\beta-1)\,(2-\;\lambda)}{2\,(2+K-\beta)}\;z^2$$

Further More

$$\left|D_Z^{\lambda}f(z)\right|\;\ge\;\frac{|z|^{\,(1-\,\lambda)}}{\Gamma\,(2-\lambda\,)}-\;\frac{(\beta-1)|z|^{(2-\,\lambda\,)}}{(n+n\,K-K-\;\beta)\,\Gamma\,(2-\;\lambda)} \tag{2.6}$$

$$\left|D_Z^{\lambda}f(z)\right|\;\le\;\frac{|z|^{\,(1-\,\lambda)}}{\Gamma\,(2-\lambda\,)}+\;\frac{(\beta-1)|z|^{(2-\,\lambda\,)}}{(n+n\,K-K-\;\beta)\,\Gamma\,(2-\;\lambda)} \tag{2.7}$$

Proof: Since $f(z)\;\in K-P\,S_\lambda^*\,(\beta)$, in view of theorem 2.1, we have

$$\frac{2\,(2+k-\,\beta)}{(2-\lambda\,)} \sum_{N=2}^{\infty} \leq \sum_{N=2}^{\infty} \emptyset\ (n)\ (n+n\,K-K-\,\beta)\,a_n\ \leq\ (\beta-1), \tag{2.8}$$

Which evidently yield

$$\sum_{N=2}^{\infty} a_n\ \leq\ \frac{(\beta-1)\,(2-\lambda\,)}{2\,(2+K-\,\beta)} \tag{2.9}$$

Consequently, we obtain

$$\begin{aligned} |f(z)|\ &\geq\ |z|-\ |z|^2 \sum_{n=2}^{\infty} a_n \\ &\geq\ |z|-\ \frac{(\beta-1)\,(2-\lambda\,)}{2\,(2+K-\,\beta)}\ |z|^2 \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} |f(z)|\ &\leq\ |z|+\ |z|^2 \sum_{n=2}^{\infty} a_n \\ &\leq\ |z|+\ \frac{(\beta-1)\,(2-\lambda\,)}{2\,(2+K-\,\beta)}\ |z|^2 \end{aligned} \tag{2.11}$$

Which prove the assertion (2.4) and (2.5). Next, by using the second inequality in (2.8), we observe that

$$\begin{aligned} \left|\Gamma\ (2-\lambda)\ z^{\lambda}\ D_z^{\lambda} f(z)\ \right|\ &\geq\ |z|-\sum_{n=2}^{\infty} \emptyset\ \ (n)a_n\ |z|^n \\ &\geq\ |z|-|z|^2 \sum_{n=2}^{\infty} \emptyset\ \ (n)a_n \\ &\geq\ |z|-|z|^2 \frac{(\beta-1)}{(n+nK-K-\,\beta)} \end{aligned}$$

and

$$\begin{aligned} \left|\Gamma\ (2-\lambda)\ z^{\lambda}\ D_z^{\lambda} f(z)\ \right|\ &\leq\ |z|+\sum_{n=2}^{\infty} \emptyset\ \ (n)a_n\ |z|^n \\ &\leq\ |z|+|z|^2 \sum_{n=2}^{\infty} \emptyset\ \ (n)a_n \\ &\leq\ |z|+|z|^2 \frac{(\beta-1)}{(n+nK-K-\,\beta)}, \end{aligned}$$

Which prove the assertions (2.6) and (2.7).

Remark 2.4: Putting $\lambda = 0$ in Theorem 2.2, we get the corresponding result given by Porwal and Dixit [8],

Remark 2.5: Putting $\lambda = 1$ in Theorem 2.2, we obtain the corresponding result given by Porwal and Dixit [8].

Remark 2.6: Putting $K = 0$ in Theorem 2.2, we obtain the corresponding result given by Dixit and Pathak [1].

The following covering result follows from assertion (2.4)

Corollary 2.2: Let f of the from (1.2) be so that $f\ \in K-P\ S_{\lambda}^{*}(\beta)$. Then

$$\left\{w:\ |w|<\ \frac{2\,(2+K-\,\beta)-\ (\beta-1)\,(2-\,\lambda)}{2\,(2+K-\,\beta)}\right\}<f\ (U).$$

The following properties are easy consequences of Theorem 2.1.

Theorem 2.3: Let $0\ \leq\ \lambda\ \leq 1,\ 1<\ \beta_1<\ \beta_2\ \leq\ \frac{4}{3}$, Then

$$K-P\ S_{\lambda}^{*}(\beta_1)\ \subset K-P\ \ S_{\lambda}^{*}(\beta_2) \tag{2.12}$$

Theorem 2.4: Let $0\ \leq\ \lambda\ \ \leq\ \mu\ \leq 1, 1<\ \beta \leq\ \frac{4}{3}$, Then

$$K-P\ S_{\lambda}^{*}(\beta)\ \supset\ K-P\ \ S_{\mu}^{*}(\beta) \tag{2.13}$$

Theorem 2.5: Let function $f_1(z)$, $f_2(z)$, …………………… $f_m(z)$ defined by

$$f_j(z) = z + \sum_{n=2}^{\infty} c_{n,j} z^n, \left(c_{n,j} \geq 0\right) \ be\ in\ the\ class\ K - P\ S_\lambda^*(\beta) \tag{2.14}$$

Then the function $h(z)$ given by

$$h(z) = \frac{1}{m} \sum_{j=1}^{m} f_j\,(z)\ \ is\ also\ in\ the\ class\ K - P\ S_\lambda^*(\beta) \tag{2.15}$$

Theorem 2.6: Let $f_1(z) = z$ and

$$f_n(z) = z + \frac{(\beta - 1)\, z^n}{(n + nK - K\ -\ \beta)\, \emptyset\, (n)},$$

Then $\quad f \in K - P\ S_\lambda^*(\beta)$

if only if it can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \lambda_n\, f_n(z) \tag{2.16}$$

Remark: The extreme points of $K - P\ \ S_\lambda^*(\beta)$ are function $f_n(z),\ n = 1,\ 2,\ 3$ .... defined in Theorem 2.6.

Definition, Let $f(z)$ be defined by (1.2) and let

$$g\ (z) = z + \sum_{n=2}^{\infty} b_n z^n, \qquad (b_n\ \geq 0) \tag{2.17}$$

The Hadamard Product of $f(z)$ *and* $g(z)$ is defined here by

$$(f^* g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \tag{2.18}$$

An interesting property of Hadamard product of several functions is described in following way.

Theorem 2.7: Let the function $f_1(z)$, $f_2(z)$ ………………..$f_m(z)$ defined by

$$f_1(z) = z + \sum_{n=2}^{\infty} c_{n,j} z^n, \left(c_{n,j} \geq 0\right) \tag{2.19}$$

be in the classes $K - P\ \ S_\lambda^*\left(\beta_j\right)$ j = *1, 2, 3, 4, ………………, m* respectively, Also let

$$\lambda\ \geq 2\begin{pmatrix} max\, \beta_j\ -\ 1 - K \\ 1 \leq j \leq m \end{pmatrix} \tag{2.20}$$

Then

$$(f_1^*\ f_2^*\ ---------*\ f_m)\ (z) \in K -\ S_\lambda^* \left(\prod_{j=1}^{m} \beta_j \right) \tag{2.21}$$

Proof: Since $f_1(z) \in K - P\ \ S_\lambda^*\left(\beta_j\right)$ j= *1, 2, 3, 4………, m*, by using Theorem 2.1,

We have

$$\sum_{n=2}^{\infty} (n + nK - K\, \beta)\, \emptyset\ (n)\, C_{n,j}\ \leq\ \beta - 1 \quad and \tag{2.22}$$

$$\sum_{n=2}^{\infty} c_{n,j}\ \leq\ \frac{(\beta_1 -\ 1)\ (2 -\ \lambda)}{2\ (2 + K -\ \beta)}, for\ each\ j = 1, 2, 3, \ldots \ldots \ldots .., m \tag{2.23}$$

Using (2.22) for any j_0 and (2.23) for the rest, we obtain

$$\sum_{n=2}^{\infty} \emptyset\ (n) \left[n + nK - K \prod_{j=1}^{m} \beta_j \right] \prod_{j=1}^{m} c_{n,j} \leq \prod_{j=1}^{m} \frac{(\beta_1 -\ 1)\ (2 -\ \lambda)^{m-1}}{2^{m-1}\ \prod_{j=1}^{m} \left(2 - K -\ \beta_j\right)}$$

$$\leq \prod_{j=1}^{m} \frac{(\beta_1 - 1)\,(1-\lambda)^{m-1}}{2^{m-1}\left[2+K-\begin{matrix} max(\beta_j) \\ 1 \leq j \leq m \end{matrix}\right]^{m-1}}$$
$$\leq \left(\prod_{j=1}^{m} \beta_j - 1\right)$$

Since, by (2.20)

$$\left[\frac{(2-\lambda)}{2\left(2+K-max(\beta_j)\right)}\right] \leq 1 \tag{2.24}$$

Consequently, we have the assertion (2.21) with the help of Theorem 2.1.

For $\beta_j = \beta$ *j= 1, 2, 3,………………., m.* Theorem 2.7 yields

Corollary 2.3: Let each of the function *$f_1(z)$, $f_2(z)$ ……………$f_m(z)$* defined by (2.19) be in the same class $K - P\ S_\lambda^*(\beta)$. *Also let* $\lambda \geq 2\,(\beta - 1 - K)$. *Then*

$$(f_1^*, f_2^* \ldots\ldots\ldots\ldots\ldots\ldots\ldots * f_m)\,(z) \in K - P\ S_\lambda^*(\beta^m) \tag{2.25}$$

Next, we prove.

Theorem 2.8: Let the function f(z) defined by (1.2) and g(z) defined by (2.17) be in the classes $K - P\ S_\lambda^*(\beta_1)$ and $K - P\ S_\lambda^*(\beta_2)$ respectively. Then the Hadamard product *(f*g) (z)* belong to the class $K - P\ S_\lambda^*(\beta^2 - 2\beta + 2)$,
Where $\beta = max\{\beta_1\beta_2\}$ (2.26)

Proof: Since $f(z) = K - P\ S_\lambda^*(\beta_1)$ and $g(z) = K - P\ S_\lambda^*(\beta_2)$, in view of Theorem 2.1, we have

$$\sum_{n=2}^{\infty}(n + nK - K - \beta_1\beta_2)\,\emptyset\,(n)\,a_n b_n \leq \sum_{n=2}^{\infty} \emptyset\,(n)\,(n + nK - K - \beta_1)\,a_n b_n$$
$$\leq \frac{(\beta_1 - 1)\,(\beta_2 - 1)\,(2-\lambda)}{2\,(2+K-\beta_2)}$$
$$\leq (\beta - 1)^2 = \beta^2 - 2\beta + 2 - 1$$
$$= (\beta^2 - 2\beta + 2) - 1$$

Moreover, $1 < \beta^2 - 2\beta + 2 \leq \frac{4}{3}$, *for* $1 < \beta \leq \frac{4}{3}$

Hence, by Theorem 2.1, the Hadamard product *(f*g) (z)* is in the class $K - P\ S_\lambda^*\,(\beta^2 - 2\beta + 2)$ with β given by (2.26)

Corollary 2.4: Under the hypothesis of above Theorem. The Hadamard product *(f*g) (z)* **belong to the class** $K - P\ S_\lambda^*\,(\beta)$.

Proof: In view of Theorem 2.3, we have

$$K - P\ S_\lambda^*\,(\beta) \supset KP\ S_\lambda^*\,(\beta^2 - 2\beta + 2) \tag{2.27}$$

Which in conjunction with Theorem 2.1 Show that *(f*z) (Z)* $K - K - P\ S_\lambda^*$, where β is given by (2.26)

Finally, an interesting Theorem on Hadmard product with extremal functions is given by.

Theorem 2.9: Let the function $f_j(z)$ $(j = 1, 2)$ defined by (2.14) be in the class $K - P\ S_\lambda^*(\beta)$
Then $(f_1 * f_2)\,(z) \in K - KP\ S_\lambda^*(\gamma)$ (2.28)
Where

$$\gamma\,(\beta_1, \lambda, K) = \frac{(2-\lambda)\,(\beta-1)^2\,(2+K) + 2\,(2+K-\beta)^2}{(2-\lambda)\,(\beta-1)^2 + 2\,(2+K-\beta)^2} \tag{2.29}$$

The result is sharp, the extermal function being

$$f_j\,(z) = z + \frac{(\beta-1)\,(2-\lambda)}{2\,(2+K-\beta)} z^2 \quad (j = 1, 2)$$

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Proof: If Suffices to prove that

$$\sum_{n=2}^{\infty} \emptyset\,(n)\,\frac{(n+nK-K-\gamma)}{(\gamma-1)}\ c_{n,1}c_{n,2}\ \leq 1 \tag{2.30}$$

For $\gamma \leq\ \gamma\,(\beta,\ \lambda, K)$. By virtue of the Cauchy-Schwarz inequality, it follows from (2.1) that

$$\sum_{n=2}^{\infty} \emptyset\,(n)\,\frac{(n+nK-K-\gamma)}{(\beta-1)}\ \sqrt{c_{n,1}\,c_{n,2}}\ \leq 1 \tag{2.31}$$

Hence, we need to find the largest γ such that

$$\sum_{n=2}^{\infty} \emptyset\,(n)\,\frac{(n+nK-K-\gamma)}{(\gamma-1)}\ c_{n,1}c_{n,2}\ \leq \sum_{n=2}^{\infty} \emptyset\,(n)\,\frac{(n+nK-K-\gamma)}{(\beta-1)}\ \sqrt{c_{n,1}\,c_{n,2}}$$

Or, equivalently,

$$\sqrt{c_{n,1}\,c_{n,2}}\ \leq \sum_{n=2}^{\infty} \emptyset\,(n)\,\frac{|(\gamma-1)|(n+nK-K-\beta)}{(n+nK-K-\gamma)(\beta-1)}\,, (n\ \geq 2) \tag{2.32}$$

In view of (2.31), it is sufficient to find the largest γ such that

$$\frac{(\beta-1)}{\emptyset\,(n)(n+nK-K-\beta)} \leq \frac{(\gamma-1)(n+nK-K-\beta)}{(n+nK-K-\gamma)(\beta-1)} \tag{2.33}$$

The inequality (2.33) yields

$$\varphi\,(n)\frac{(\beta-1)^2}{(n+nK-K-\beta)} \leq \frac{(\gamma-1)}{(n+nK-K-\gamma)} \qquad Where\ \varphi\,(n) = \frac{1}{\varphi\,(n)}$$

or

$$\frac{\varphi\,(n)|(\beta-1)^2|(n+nK-K) +\ (n+nK-K-\beta)^2}{\varphi\,(n)(\beta-1)^2 +\ (n+nK-K-\beta)^2}\ \leq\ \gamma$$

Since $\varphi\,(n)$ is a decreasing function of $n\,(n\ \geq 2)\ for\ fixed\ \lambda$, we have

$$\gamma\ \geq \frac{(2-\gamma)\,(\beta-1)^2\,(2-K1) +\ 2\,(2+K-\beta)^2}{(2-\gamma)(\beta-1)^2 +\ 2(2+K-\beta)^2}$$

Which prove the assertion (2.28) under the constraint (2.29),

Finally, by taking the function

$$f_j(z) = z + \frac{(\beta-1)\,(2-\gamma)}{2(2+K-\beta)}\,z^2, \quad (j = 1, 2),$$

We can prove that the result is sharp.

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