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δωα- CLOSED FUNCTIONS AND Quasi-δωα CLOSED FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

O*pen and closed functions are most important concepts Mathematical Sciences. δωα-closed sets introduced by S.Chandrasekar, T.Rajesh Kannan et.al. In this paper we are introduced by δωα open functions, δωα closed functions, quasi δωα open functions, and quasi δωα closed functions in topological spaces.*

Key Words: *δωα-open functions, δωα-closed functions, quasi δωα-open functions and quasi δωα-closed functions*

AMS subject classification: *54C55, 54A05.*

1. INTRODUCTION

Velicko introduced δ-closed set in Topological spaces [11]. Using δ-closed set several results introduced by many researcher. ωα- closed set[1] introduced by S.S.Benchalli, *et al*., in the year 2009. Since the advent of these types of notions, several author have been introduced interesting results. δωα-closed sets [2] introduced by S.Chandrasekar, T.Rajesh Kannan *et.al.* In this paper we introduced δωα open functions, δωα closed functions, quasi δωα open functions, and quasi δωα closed functions in topological spaces and application properties are discussed detailed

2. PRELIMINARIES

Let us recall the following definition, which are useful in the sequel.

Definition 2.1: A subset A of a space (X, τ) is called
1) α- closed set [6] if cl(int(cl(A)))⊆ A
2) δ-closed [11] if A = $cl_\delta(A)$, where $cl_\delta(A) = \{x \in X: int(cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.
3) ω-closed set [10] if cl(A)⊆ U whenever A⊆ U, U is semi open in (X, τ).
4) ωα-closed set [1] if αcl(A) ⊆ U whenever A⊆ U, U is ω- open in (X, τ).
5) δωα-closed set[2] if $cl_\delta(A) \subseteq U$ whenever A⊆ U, U is ωα- open set in (X, τ).
   the complement of above mentioned closed sets is called respective open sets.

Definition 2.3: A function f: (X, τ) → (Y, σ) is called
(i) δωα-continuous if $f^{-1}(V)$ is δωα-closed in X for every closed subset V of Y;
(ii) δωα-irresolute if $f^{-1}(V)$ is δωα-closed in X for every δωα-closed subset V of Y;

3. δωα-OPEN FUNCTIONS

Definition 3.1: Let (X, τ) and (Y, σ) be topological spaces. A function f: X →Y is called δωα-open map if for every open set G in X, f (G) is a δωα-open set in Y.

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Theorem 3.2: Prove that a mapping f: (X, τ) → (Y, σ) is δωα-open if and only if for each x ∈ X, and U∈ τ such that x ∈ U, there exists a δωα-open set W ⊆Y containing f (x) such that W⊆ f(U) .

Proof: Follows immediately from Definition 3.1

Theorem 3.3: Let f: (X, τ) → (Y, σ) be δωα - open. If W ⊆ Y and F ⊆ X is a closed set containing f^{-1}(W), then there exists a δωα-closed H ⊆ Y containing W such that f^{-1}(H) ⊆ F.

Proof: Let H = Y- f (X- F) .from the definition of H, H is a δωα -closed .By our assumption f^{-1}(W) ⊆ F, we have f (X - F) ⊆ (Y- W) .Hence f^{-1}(H) = X-f^{-1}[f (X-F)]⊆X-(X -F) =F.

Theorem 3.4: Let f:(X, τ) → (Y, σ) be δωα-open and let B ⊆ Y. Then f^{-1}[δωα-Cl(δωα-Int(δωα- Cl(B)))] ⊆Cl[f^{-1} (B)]

Proof: Assume that B ⊆ Y. we know that f^{-1} (B) ⊆Cl [f^{-1} (B)] for any set. By Theorem 3.3, there exists a δωα-closed set B ⊆ H ⊆ Y, such that f^{-1}(H) ⊆Cl [f^{-1} (B)]. Thus, f^{-1}[δωα-Cl(δωα-Int(δωα-Cl(B)))]⊆f^{-1}[δωα-Cl(δωα-Int(δωα-Cl(H)))]⊆f^{-1}(H)⊆Cl[f^{-1}(B)] .

Theorem 3.5: Prove that a function f :(X, τ) → (Y, σ) is δωα-open if and only iff [Int (A)] ⊆ δωα-Int[f (A)], for all A⊆X.

Proof: Necessity: Let A ⊆ X. Let x∈Int (A).Then there exists Ux∈τ such that x∈Ux⊆ A.So f (x)∈f (Ux)⊆f (A) and by hypothesis, f (Ux)∈δωα-σ. Hence f (x) ∈ δωα-Int [f (A)] .Thus f [Int(A)]⊆δωα-Int[f (A)] .

Sufficiency: Let U∈τ. Then by hypothesis, f [Int (U)] ⊆δωα-Int[f (U)] .Since Int(U) =U as U is open. Also δωα-Int[f (U)] ⊆f(U) . Hence f (U)=δωα-Int[f (U)] .Thus f (U) is δωα-open in Y. So f is δωα-open.

Theorem 3.6: Prove that a function f :(X, τ) → (Y, σ) is δωα-open if and only ifInt [f^{-1} (B)] ⊆f^{-1}[δωα-Int(B)], for all B⊆Y.

Proof:
Necessity: Let B⊆Y. Since Int [f^{-1}(B)] is open in X and f is δωα-open, f [Int (f^{-1} (B))] is δωα-open in Y. Also we have f [Int (f^{-1}(B))]⊆f [f^{-1}(B)]⊆B. Hence, f [Int (f^{-1}(B))] ⊆ δωα-Int (B). Therefore Int [f^{-1} (B)]⊆f^{-1}[δωα-Int(B)] .

Sufficiency: Let A ⊆ X. Then f (A)⊆Y. Hence by hypothesis, we obtain Int(A)⊆Int [f^{-1}(f (A))]⊆f^{-1}[δωα-Int(f (A))]. Thus f [Int(A)]⊆δωα-Int[f (A)] , for all A⊆X. Hence, by Theorem 3.5, f is δωα-open.

Theorem 3.7: Let f :(X, τ) → (Y, σ) be a mapping. Then a necessary and sufficientcondition for f to be δωα-open is that f^{-1}[δωα-Cl(B)]⊆Cl[f^{-1}(B)] for every subset B of Y.

Proof:
Necessity: Assume f is δωα-open. Let B⊆Y. Let x∈ f^{-1}[δωα-Cl(B)] .Then f(x)∈δωα-Cl (B).Let U∈τ such that x∈ U. Since f is δωα -open, then f (U) is a δωα-open setin Y. Therefore, B∩f (U)≠ ϕThen U ∩f^{-1} (B) ≠ ϕ. Hence x ∈Cl [f^{-1} (B)].We conclude that f^{-1}[δωα-Cl (B)]⊆Cl [f^{-1}(B)] .

Sufficiency: Let B ⊆Y.Then (Y - B) ⊆ Y. By hypothesis, f^{-1}[δωα-Cl (Y - B)]⊆Cl [f^{-1} (Y - B)]. This implies X -Cl [f^{-1} (Y - B)] ⊆ X - f^{-1}[δωα-Cl (Y - B)].Hence X-Cl [X-f^{-1} (B)] ⊆ f^{-1}[Y-δωα-Cl(Y - B)].Then Int [f^{-1} (B)] ⊆f^{-1} [δωα -Int(B)] . Now form Theorem 3.6, it follows that f is δωα-open.

4. δωα-CLOSED FUNCTIONS

In this section we introduce δωα-closed functions and study certain properties and characterizations of this type of functions.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) is called δωα-closed if the image of each closed set in X is a δωα-closed set in Y .

Theorem 4.2: Prove that a mapping f:(X, τ) → (Y, σ) is δωα-closed if and only δωα-Cl [f (A)] ⊆ f [Cl (A)] for each A ⊆ X.

Proof:
Necessity: Let f be δωα-closed and let A⊆ X. Then f (A) ⊆ f [Cl (A)] and f [Cl(A)] is a δωα-closed set in Y. Thus δωα-Cl [f (A)] ⊆ f [Cl (A)] .

Sufficiency: suppose that δωα-Cl [f (A)] ⊆ f [Cl (A)], for each A ⊆ X. Let A⊆ X be aclosed set. Then δωα-Cl[f (A)]⊆ f [Cl (A)] = f (A) . This shows that f (A) is a δωα-closed set. Hence f is δωα-closed.

Theorem 4.3: Let f: (X, τ) → (Y, σ) be δωα-closed. If V ⊆ Y and E ⊆ X is anopen set containing f^{-1}(V), then there exists a δωα-open set G ⊆ Y containing V such thatf^{-1} (G) ⊆ E.

Proof: Let G = Y-f (X - E). Since f^{-1} (V) ⊆E, we have f (X - E) ⊆ Y -V. Since fis δωα-closed, then G is a δωα-open set and f^{-1}(G) =X-f^{-1} [f (X - E)] ⊆ X-(X - E) = E.

Theorem 4.4: Suppose that f: (X, τ) → (Y, σ) is a δωα-closed mapping.Then δωα-Int[δωα-Cl (f (A))] ⊆f[Cl(A)] for every subset A of X.

Proof: Suppose f is a δωα-closed mapping and A is an arbitrary subset of X. Then f [Cl(A)] is δωα-closed in Y. Then δωα-Int[δωα-Cl (f(Cl (A)))] ⊆ f [Cl (A)]. But also δωα-Int [δωα-Cl (f(A))] ⊆ δωα-Int[δωα-Cl (f (Cl (A)))]. Hence δωα-Int[δωα-Cl (f (A))] ⊆ f [Cl (A)] .

Theorem 4.5: Let f :(X, τ) → (Y, σ) be a δωα-closed function, and B,C⊆Y.

(i) If U is an open neighborhood of f^{-1}(B), then there exists a δωα-open neighborhood V of B such that f^{-1}(B) ⊆ f^{-1} (V) ⊆ U.
(ii) If f is also onto, then if f^{-1} (B) and f^{-1}(C) have disjoint open neighborhoods, so have B and C.

Proof:

(i) Let V = Y-f (X - U). Then V^C = Y - V = f (U^C). Since f is δωα-closed, so V is a δωα-open set. Since f^{-1}(B) ⊆ U, we have V^C = f (U^C) ⊆ f [f^{-1} (B^C)] ⊆ B^C. Hence, B⊆ V, and thus V is a δωα- open neighborhood of B. Further U^C⊆ f^{-1}[f (U^C)] = f^{-1} (V^C) =[f^{-1}(V)]^C. This proves that f^{-1}(V)⊆ U.
(ii) If f^{-1}(B) and f^{-1}(C) have disjoint open neighborhoods M and N, then by (i), we have δωα- open neighborhoods U and V of B and C respectively such that f^{-1} (B) ⊆f^{-1} (U) ⊆δωα-Int(M) and f^{-1} (C) ⊆f^{-1}(V) ⊆δωα-Int(N). Since M and N are disjoint, so are δωα-Int(M) and δωα-Int(N), and hence so f^{-1}(U) and f^{-1} (V) are disjoint as well. It follows that U and V are disjoint too as f is onto.

Theorem 4.6: Prove that a surjective mapping f :(X,τ) → (Y, σ) is δωα-closed if and only if for each subset B of Y and each open set U in X containing f^{-1}(B) , there exists a δωα-open set V in Y containing B such that f^{-1} (V) ⊆ U.

Proof:
Necessity: This follows from (1) of Theorem 4.5.

Sufficiency: Suppose F is an arbitrary closed set in X. Let y be an arbitrary point in Y - f (F). Then f^{-1} (y) ⊆X - f^{-1}[f (F)] ⊆ (X - F) and (X - F) is open in X. Hence by hypothesis, there existsa δωα-open set Vy containing y such that f^{-1} (Vy) ⊆ (X - F).This implies that y ∈ Vy⊆ [Y - f (F)]. Thus Y - f (F) = ∪{Vy: y ∈ Y - f (F)}. Hence Y - f (F), being a union of δωα-open sets, is δωα-open. Thus its complement f (F) is δωα- closed. This shows that f is δωα-closed.

Theorem 4.6: Let f: (X, τ) → (Y, σ) be a bijection. Then the following are equivalent:

(i) f is δωα-closed.
(ii) f is δωα-open.
(iii) f^{-1} is δωα-continuous.

Proof:
(i) = (ii): Let U∈ τ. Then X -U is closed in X. By (i), f (X-U) is δωα-closed in Y.But f (X-U) = f(X)-f (U)=Y-f(U). Thus f (U) is δωα -open in Y. This shows that f is δωα-open.
(ii) = (iii): Let U ⊆ X is an open set. Since f is δωα-open. So f (U) = $(f^{-1})^{-1}$ (U) is δωα- open in Y. Hence $f^{-1}$is δωα-continuous.
(iii) = (i): Let A be an arbitrary closed set in X. Then X-A is open in X. Since f^{-1} is δωα- continuous, $(f^{-1})^{-1}$(X -A) is δωα-open in Y. But $(f^{-1})^{-1}$(X - A) = f (X - A) =Y - f (A) .Thus f (A) is δωα- closed in Y. This shows that f is δωα-closed.

Remark 4.7: A bijection f :(X,τ) → (Y,σ) may be open and closed but neither δωα- open nor δωα-closed.

5. QUASI δωα-OPEN FUNCTIONS

We introduce a new definition as follows:

Definition 5.1: A function f: X →Y is said to be quasi δωα-open if the image of every δωα-open set in X is open in Y.

It is evident that, the concepts of quasi δωα-openness and δωα-continuity coincide if the function is a bijection.

Theorem 5.2: A function f: X →Y is quasi δωα-open if and only if for every subset U of X, f(δωα-int(U)) ⊂ int(f(U)).

Proof: Let f be a quasi δωα-open function and U be a subset of X. Now, we have δωα-int(U) ⊂ U and δωα-int(U) is a δωα-open set. Hence, we obtain that f(δωα-int(U)) ⊂ f(U). As f(δωα-int(U)) is open, f(δωα-int(U)) ⊂int(f (U)). Conversely, assume that U is a δωα-open set in X. Then, f(U)=f(δωα-int(U))⊂int(f(U)) but int(f(U))⊂f(U). Consequently, f(U) = int(f(U)) and f(U) is open in Y. Hence f is quasi δωα-open.

Lemma 5.3: If a function f: X → Y is quasi δωα-open, then δωα-int(f^{-1}(G)) ⊂f^{-1}(int(G)) for every subset G of Y.

Proof: Let G be any arbitrary subset of Y. Then, δωα-int(f^{-1}(G)) is a δωα-open set in X and since f is quasi δωα-open, then f (δωα-int(f^{-1}(G))) ⊂int(f(f^{-1}(G))) ⊂int(G). Thus, δωα-int(f^{-1}(G)) ⊂f^{-1}(int(G)).

Recall that a subset S is called a δωα-neighbourhood of a point x of X if there exists a δωα-open set U such that x ∈ U ⊂ S.

Theorem 5.4: For a function f: X → Y, the following are equivalent:
(i) f is quasi δωα-open;
(ii) For each subset U of X, f (δωα-int(U)) ⊂int(f (U));
(iii) For each x ∈ X and each δωα-neighbourhood U of x in X, there exists a neighbourhood f(U)of f(x) in Y such that f(V) ⊂ f(U).

Proof
(i) ⇒ (ii): It follows from Theorem 5.2.
(ii) ⇒ (iii): Let x ∈ X and U be an arbitrary δωα-neighbourhood of x in X.
Then there exists a δωα-open set V in X such that x ∈ V ⊂ U. Then by (ii), we have f(V) = f(δωα-int(V)) ⊂int(f(V)) and hence f(V) = int(f(V)). Therefore, it follows that f(V) is open in Y such that f(x) ∈ f(V) ⊂ f(U).
(iii) ⇒ (i): Let U be an arbitrary δωα-open set in X such that x ∈ U. Then for each f(x) = y ∈ f (U), by (iii) there exists a neighbourhood Vy of y in Y such that Vy⊂f (U). As Vy is a neighbourhood of y, there exists an open set Wy in Y such that y ∈Wy⊂Vy. Thus f (U) = ∪{Wy: y ∈f (U)} which is an open set in Y. This implies that f is quasi δωα-open function.

Theorem 5.5: A function f: X → Y is quasi δωα-open if and only if for any subset B of Y and for any δωα-closed set F of X containing f^{-1}(B), there exists a closed set G of Y containing B such that f^{-1}(G) ⊂ F.

Proof: Suppose f is quasi δωα-open. Let B ⊂ Y and F be a δωα-closed set of X containing f^{-1}(B). Now, put G = Y – f (X – F). It is clear that f^{-1}(B) ⊂F implies B ⊂ G. Since f is quasi δωα-open, we obtain G is a closed set of Y. Moreover, we have f^{-1}(G) ⊂ F.

Conversely, let U be a δωα-open set of X and put B = Y-f (U). Then X-U is a δωα-closed set in X containing f^{-1}(B). By hypothesis, there exists a closed set F of Y such that B ⊂ F and f^{-1}(F) ⊂ X-U. Hence, we obtain f(U) ⊂ Y-F. On the other hand, it follows that B ⊂ F, Y-F ⊂ Y-B = f(U). Thus, we obtain f (U) = Y-F which is open and hence f is a quasi δωα-open function.

Theorem 5.6: A function f: X → Y is quasi δωα-open if and only if f^{-1}(cl(B)) ⊂δωα-cl(f^{-1}(B)) for every subset B of Y.

Proof: Suppose that f is quasi δωα-open. For any subset B of Y, f^{-1}(B)⊂δωα-cl(f^{-1}(B)). Therefore by Theorem 5.5, there exists a closed set F in Y such that B ⊂ F and f^{-1}(F) ⊂δωα-cl(f^{-1}(B)). Therefore, we obtain
f^{-1}(cl(B)) ⊂ f^{-1}(F) ⊂δωα-cl(f^{-1}(B)).

Conversely, let B ⊂ Y and F be a δωα-closed of X containing f^{-1}(B). Put W = clY (B), then we have B ⊂ W and W is closed in Y and f^{-1}(W) ⊂δωα-cl(f^{-1}(B)) ⊂ F. Then, by Theorem 5.5, f is quasi δωα-open.

Lemma 5.7: Let f: X → Y and g : Y → Z be two functions and g o f: X → Z is quasi δωα-open. If g is continuous and injective, then f is quasi δωα-open.

Proof: Let U be a δωα-open set in X. Then (g o f)(U) is open in Z since g o f is quasi δωα-open. Again g is an injective continuous function, f(U) = g^{-1}(g o f (U))) is open in Y. This shows that f is quasi δωα-open.

6. QUASI δωα-CLOSED FUNCTIONS

Definition 6.1: A function f: X → Y is said to be quasi δωα-closed if the image of each δωα-closed set in X is closed in Y.

Remark 6.2: Quasi δωα-closed function is independent to closed map.

Example 6.3: Let X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a}, Y}. Define a function f: (X, τ) → (Y, σ) be identity map. Then f is quasi δωα-closed but not closed.

Example 6.4: Let X = Y = {a, b, c}, τ = σ = {ϕ, {a}, {b, c}, X} and closed set {ϕ, {a}, {b, c}, X} and δωα-closedset {all power set in X}. Define a function f :(X, τ)→ (Y, σ) be identity map. Cleary closed but not quasi δωα-closed function

Remark6.5: Quasi δωα-closed function is independent to δωα-closed map as shown by the following example.

Example 6.6: Let X = Y = {a, b, c}, τ = σ = {ϕ, {a}, {a, b}, {a, c}, X} and closed set {ϕ, {b}, {c}, {b, c}, X} and δωα –closed set {ϕ, {b, c}, X}.

Example 6.7: Let function f: (X, τ) →(Y, σ) be identity map. Cleary quasi δωα-closed function but not δωα-closed map let X = Y = {a, b, c}, τ = σ = {ϕ, {a}, {b, c}, X} and closed set {ϕ, {a}, {b, c}, X} and δωα –closed set {all power set in X}.

Define a function f: (X, τ) →(Y, σ) be identity map. Cleary δωα closed but not quasi δωα-closed function

Lemma 6.8: If a function f: X → Y is quasi δωα-closed, then $f^{-1}(cl(B)) \subset$δωα-cl(f^{-1}(B)) for every subset B of Y.

Proof: This proof is similar to the proof of Lemma 5.3.

Theorem 6.9: A function f : X→ Y is quasi δωα -closed if and only if for any subset B of Y and for any δωα-open set G of X containing $f^{-1}(B)$, there exists an open set U of Y containing B such that $f^{-1}(U) \subset G$.

Proof: This proof is similar to that of Theorem 5.5.

Definition 6.10: A function f: X → Y is called δωα*-closed if the image of every δωα-closed subset of X is δωα-closed in Y.

Theorem 6.11: If f: X →Y and g: Y → Z are two quasi δωα-closed functions, then f: X → Z is a need not be quasi δωα-closed function.

Proof: Obvious. Furthermore, we have the above example.

Theorem 6.12: Let f : X → Y and g : Y → Z be any two functions. Then
(i) if f is δωα-closed and g is quasi δωα-closed, then g o f is closed;
(ii) if f is quasi δωα-closed and g is δωα-closed, then g o f is δωα*-closed;
(iii) if f is δωα* -closed and g is quasi δωα-closed, then g o f is quasi δωα-closed.

Proof: Obvious.

Theorem 6.13: Let f : X → Y and g : Y → Z be two functions such that g o f : X → Z is quasi δωα-closed. Then
(i) if f is δωα-irresolute surjective, then g is δωα-closed .
(ii) if g is δωα-continuous injective, then f is δωα*-closed.

Proof:
(i) Suppose that F is an arbitrary δωα-closed in Y. As f is δωα-irresolute, $f^{-1}(F)$ is δωα-closed in X. Since g o f is quasi δωα-closed and f is surjective, g o f (f^{-1}(F))) = g (F), which is closed in Z. This implies that g is a δωα-closed function.
(ii) Suppose that F is any δωα-closed set in X. Since g o f is quasi δωα-closed, (g o f)(F) is closed in Z. Again g is a δωα-continuous injective function, g^{-1}(g o f (F)) = f (F), which is δωα-closed in Y. This shows that f is δωα*-closed.

Theorem 6.14: Let X and Y be topological spaces. Then the function g : X→ Y is a quasi δωα -closed if and only if g(X) is closed in Y and g(V) - g(X-V) is open in g(X) whenever V is δωα-open in X.

Proof: Necessity: Suppose g: X → Y is a quasi δωα-closed function. Since X is δωα-closed, g(X) is closed in Y and g(V) - g(X-V) = g(V)∩g(X) - g(X-V) is open in g(X) when V is δωα-open in X.

Sufficiency: Suppose g(X) is closed in Y, g(V)- g(X-V) is open in g(X) when V is δωα-open in X, and let C be closed in X. Then g(C) = g(X) - (g(X-C) - g(C)) is closed in g(X) and hence, closed in Y .

Corollary 6.15: Let X and Y be topological spaces. Then a surjective function g : X→ Y is quasi δωα -closed if and only if g(V) - g(X-V) is open in Y whenever V is δωα-open in X.

Proof: Obvious.

Corollary 6.16: Let X and Y be topological spaces and let g : X → Y be a δωα-continuous quasi δωα-closed surjective function. Then the topology on Y is {g(V) - g(X-V) : V is δωα-open in X}.

Proof: Let W be open in Y. Then g^{-1} (W) is δωα-open in X, and g(g^{-1}(W)) - g(X- g^{-1}(W)) = W. Hence, all open sets in Y are of the form g(V) - g(X-V), V is δωα-open in X. On the other hand, all sets of the form g(V) - g(X-V), V is δωα-open in X, are open in Y from Corollary 6.15.

Definition 6.17: A topological space (X, τ) is said to be δωα*-normal if for any pair of disjoint δωα-closed subsets F1 and F2 of X, there exist disjoint open sets U and V such that F1 ⊂ U and F2 ⊂ V.

Theorem 6.18: Let X and Y be topological spaces with X is δωα*-normal. If g: X → Y is a δωα-continuous quasi δωα-closed surjective function, then Y is normal.

Proof: Let K and M be disjoint closed subsets of Y. Then g^{-1}(K) and g^{-1}(M) are disjoint δωα-closed subsets of X. Since X is δωα*-normal, there exist disjoint open sets V and W such that g^{-1}(K) ⊂ V and g^{-1}(M) ⊂ W. Then K ⊂(g(V)-g(X-V)) and M ⊂(g(W)-g(X-W)). Further by Corollary 6.15, (g(V)-g(X-V)) and (g(W)-g(X-W)) are open sets in Y and clearly (g(V) - g(X-V)) ∩ (g (W) - g(X-W)) = ϕ. This shows that Y is normal.

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