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FUZZY BULK ARRIVAL QUEUES USING DSW ALGORITHM

N. SUBASHINI*[1], N. ANUSHEELA[2]

[1]Department of Mathematics,
United Institute of Technology, Coimbatore, Tamil Nadu, India.

[2]Department of Mathematics,
Government Arts College, Ooty, Tamil Nadu, India.

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ABSTRACT

In this study we propose a procedure to find the various performance measures in terms of crisp values for bulk arrival queues with varying batch sizes, in which the arrival rate and service rate are Fuzzy numbers. DSW algorithm is used to define membership functions of the performance measures of bulk arrival queuing system. The algorithm is based on the α – cut representation of fuzzy sets in a standard interval analysis. Numerical example is also illustrated to check the validity of the model.

Key Words: *Bulk arrival queues; α – cuts; Trapezoidal fuzzy number; DSW algorithm;*

1. INTRODUCTION:

Bulk arrival queuing models have been widely applied to many practical situations such as production / manufacturing systems, telecommunication systems, computer networks, congestion in road traffic, epidemic process in biology and inventory control. In production systems, the work will not begin until a specified number of raw materials are accumulated. Hence these systems are analyzed by a bulk arrival queuing model in which the system performance is evaluated.

Ahmed [1] considered a multi–channel bi–level heterogeneous servers bulk arrival queuing system with Erlangian service time. The study of bulk arrival queues may be said to have begun with Erlang's solution of the $M/E_k/1$ queue. (Brockmeyer [2]). Chaudhry, M.L., Lee, A. M., [3] gave the transient steady – state solution of a single channel queue with bulk arrival and intermittently available server. Chen [4] was able to conserve the fuzziness of input information when some information of bulk service queuing systems is ambiguous. Further in [5] he developed a non – linear programming approach to derive the membership functions of the steady – state performance measures in bulk arrival queuing systems with varying batch sizes, wherein the arrival rate and service rate are fuzzy numbers. Jeeva and Rathnakumari [6] studied bulk arrival single server Bernoulli feedback queue with fuzzy vacations and parameters. The membership functions of the system characteristics of a batch arrival queuing system with vacation policies were constructed by Ke *et al.* [7]. A bulk queue system in triangular fuzzy numbers using α – cut was modeled by Meenu Mittal *et al.* [8]. Mohammed Shapique. A., [9] worked on fuzzy queue with Erlang service model using DSW algorithm. Shanmugasundaram. S. andVenkatesh. B., [10] derived results for a multi-server fuzzy queuing model using DSW algorithm. R. Srinivasan [11] studied Fuzzy queueing model using DSW algorithm. S. Upadhyaya [12] investigated the system characteristics of batch arrival retrial queuing system with Bernoulli vacation schedule by assuming the arrival, service and vacation rates as fuzzy numbers.

In this paper, we develop an approach that provides system characteristics for bulk arrival queues. Through α -cuts and DSW algorithm, we transform the fuzzy queues to a family of crisp queues. The solutions completely yield the membership functions of the system characteristics, including the expected number of customers in the system and queue and the expected waiting time in the system and queue.

Corresponding Author: N. Subashini*[1],
[1]Department of Mathematics, United Institute of Technology, Coimbatore, Tamil Nadu, India.

2. PRELIMINARIES

2.1 Definition: Let Z denote a universe of discourse. A fuzzy set $\tilde{A}$ in Z is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval $[0,1]$.

(i.e.) $\tilde{A} = \{ (x,\ \eta_{\tilde{A}}(x));\ x \in Z \}$

Here $\eta_{\tilde{A}} : Z \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\eta_{\tilde{A}}(x)$ is called the membership value of $x \in Z$ of the fuzzy set $\tilde{A}$. Thus, the function value $\eta_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

2.2 Definition: A fuzzy set $\tilde{A}$ in the universe of discourse Z is a fuzzy number if and only if it satisfies the following conditions

$(i)\quad Z = \mathscr{R}$;

(ii) $\tilde{A}$ is normal;

(iii) $\tilde{A}$ is convex;

(iv) The membership function $\eta_{\tilde{A}}$ is piecewise continuous;

(v) There exists one and only one $x \in \mathscr{R}$, such that $\eta_{\tilde{A}}(x) = 1$.

2.3 Definition: A trapezoidal fuzzy number $\tilde{A}(x)$ can be represented by $\tilde{A}(a_1, a_2, a_3, a_4; 1)$ with membership function $\eta_{\tilde{A}}(x)$ given by

$$\eta_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4 \\ 0, & otherwise. \end{cases}$$

2.4 Definition: The α - cut of a fuzzy number $\tilde{A}$ is defined as

$$\tilde{A}_\alpha = \{ x : \eta_{\tilde{A}}(x) \ge \alpha, x \in Z \text{ and } \alpha \in [0,1] \}$$

3. SOLUTION PROCEDURE

Consider a bulk arrival FM[K]/FM/1 queuing system. Suppose the group arrival rate and service rate are approximately known and can be represented by convex fuzzy sets. Let $\eta_{\tilde{\lambda}}(x)$ and $\eta_{\tilde{\mu}}(y)$ denote the membership functions of the group arrival rate and service rate respectively.

$$A = \{ (x, \eta_{\tilde{\lambda}}(x)) | \, x \in X \}$$
$$S = \{ (y, \eta_{\tilde{\mu}}(y)) | \, y \in Y \}$$

where X and Y are the crisp universal sets of the arrival and service rates respectively.

The α – cut of inter arrival time and service time are represented as

$$A(\alpha) = \{ a \in X / \eta_A(a) \ge \alpha \}$$
$$S(\alpha) = \{ s \in Y / \eta_S(s) \ge \alpha \}$$

The trapezoidal membership function P(A, S)

$$\eta_{P(A,S)} = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2, \\ 1, & a_2 \le x \le a_3, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4. \end{cases}$$

where $a_1 < a_2 < a_3 < a_4$. An approximate method of extension is propagating fuzziness for continuous valued mapping which determines the membership functions for the output variables.

4. FUZZY BULK ARRIVAL QUEUES WITH VARYING FUZZY BATCH SIZES

Consider a queuing system in which customers arrive at a single – server facility as a Poisson process with group arrival rate of all batches, where $\tilde{\lambda}$ is a fuzzy number and all service times are independently and identically distributed according to an exponential distribution with fuzzy service rate $\tilde{\mu}$. The actual number of customers in any arriving module is stochastically equivalent to a generic random variable K, which may take on any positive integer with probability f(K). Customers are served according to a first-come-first-served (FCFS) discipline and both the size of calling population and the system capacity are infinite. This model will hereafter be denoted by $FM^{[K]}$ / FM / 1.

5. INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by order of real numbers with lower and upper bounds. Then, $I_1 = [a, b]$, $a \le b$ and $I_2 = [c, d]$, $c \le d$. Define a general arithmetic property with the symbol $* = [+, -, \times, \div]$ symbolically the operation $I_1 * I_2 = [a, b] * [c, d]$ represents another interval. The interval calculation depends on the magnitudes and signs of the elements a, b, c, d.

$[a, b] + [c, d] = [a + c, b + d]$

$[a, b] - [c, d] = [a - d, b - c]$

$[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]$

$[a, b] \div [c, d] = [a, b] \times [1/d, 1/c]$, provided that $0 \notin [c, d]$.

$$\alpha[a,b] = \begin{cases} [\alpha a, \alpha b] \text{ for } \alpha > 0 \\ [\alpha b, \alpha a] \text{ for } \alpha < 0 \end{cases}$$

6. DSW ALGORITHM

The DSW algorithm consists of the following steps:

1) Select a α – cut value where $0 \le \alpha \le 1$.
2) Find the intervals in the input membership functions that correspond to this α.
3) Using standard binary interval operations compute the interval for the output membership function for the selected α – cut level.
4) Repeat steps 1-3 for different values of α to compute α – cut representation of the solution.

7. NUMERICAL EXAMPLE

Consider a centralized parallel processing system in which jobs arrive in batches. The probability mass function of the batch size random variable K is a geometric distribution, which is often studied in crisp bulk arrival queues [1, 2, 5] with expected value of 2; i.e., the probability mass function is Pr $(A = k) = 0.5\,(1-0.5)^{k-1}, \quad k = 1, 2, 3, \ldots\ldots\ldots\ldots\infty$

Jobs arrive at this system in accordance with a Poisson process and the service times follow an exponential distribution. Both the group arrival rate and service rate are trapezoidal fuzzy numbers represented by $\tilde{\lambda} = [2,3,4,5]$ and $\tilde{\mu} = [13,14,15,16]$ per minute respectively. The system manager wants to evaluate the performance measures of the system such as the expected number of customers in the queue. We have E [K] = 2 and it is easy to find

$E[K^2] = Var[K] + \{E[K]\}^2 = 6.$

Therefore, we have,

$$L_q = \frac{x\{yE[K^2] + 2x(E[K])^2 - yE[K]\}}{2y\{y - xE\|K\|\}} = \frac{2x(2x+y)}{y(y-2x)}.$$

Using Little's formula,

$$L_s = L_q + \frac{\lambda}{\mu}$$

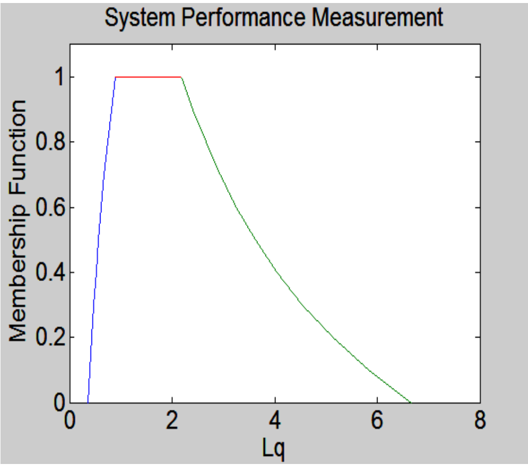
$$W_q = \frac{L_q}{\lambda}$$

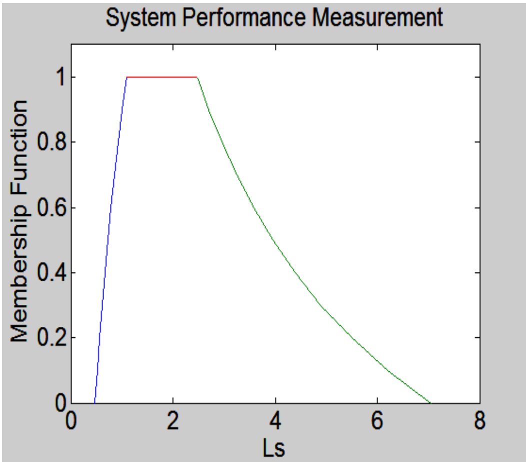
$$W_s = \frac{L_s}{\lambda}$$

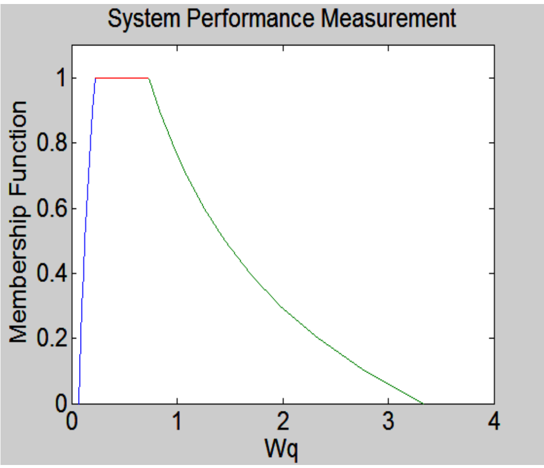
where x = [2+α, 5-α] and y = [13+α,16-α] .

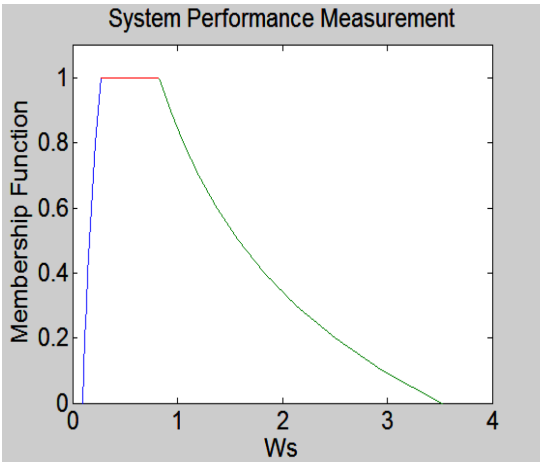
Table: The α – cuts of L_q , L_s , W_q , W_s at α values

α	x_α^L	x_α^U	y_α^L	y_α^U	$(L_q)_\alpha^L$	$(L_q)_\alpha^U$	$(L_s)_\alpha^L$	$(L_s)_\alpha^U$	$(W_q)_\alpha^L$	$(W_q)_\alpha^U$	$(W_s)_\alpha^L$	$(W_s)_\alpha^U$
0	2	5	13	16	0.3542	6.6667	0.4792	7.0513	0.0708	3.3333	0.0958	3.5256
0.1	2.1	4.9	13.1	15.9	0.3906	5.8260	0.5227	6.2001	0.0797	2.7743	0.1067	2.9524
0.2	2.2	4.8	13.2	15.8	0.4299	5.1313	0.5692	5.4950	0.0896	2.3324	0.1186	2.4977
0.3	2.3	4.7	13.3	15.7	0.4725	4.5487	0.6190	4.9021	0.1005	1.9777	0.1317	2.1313
0.4	2.4	4.6	13.4	15.6	0.5185	4.0540	0.6724	4.3973	0.1127	1.6892	0.1462	1.8322
0.5	2.5	4.5	13.5	15.5	0.5684	3.6296	0.7297	3.9630	0.1263	1.4519	0.1621	1.5852
0.6	2.6	4.4	13.6	15.4	0.6224	3.2623	0.7912	3.5858	0.1415	1.2547	0.1798	1.3792
0.7	2.7	4.3	13.7	15.3	0.6809	2.9418	0.8574	3.2556	0.1584	1.0895	0.1994	1.2058
0.8	2.8	4.2	13.8	15.2	0.7445	2.6602	0.9287	2.9646	0.1773	0.9501	0.2211	1.0588
0.9	2.9	4.1	13.9	15.1	0.8136	2.4115	1.0057	2.7064	0.1985	0.8315	0.2453	0.9333
1.0	3	4	14	15	0.8889	2.1905	1.0889	2.4762	0.2222	0.7302	0.2722	0.8254









With the help of MATLAB software we perform α – cuts of arrival , service rate and fuzzy expected number of jobs in queue at eleven distinct levels of α namely 0, 0.1, 0.2, 0.3, …1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in above Table. Similarly the performance measures such as L_s, L_q, W_q and W_s were also presented in the Table. The α – cut represents the possibility that these four performance measure will lie in the associated range. Specially, when α = 0 the range, the performance measures could appear and for α = 1 the range, the performance measures are likely to be. For example, while these four performance measures are fuzzy, the most likely value of the expected queue length L_q falls between 0.8889 and 2.1905 and its value is impossible to fall outside the range of 0.3542 and 6.6667. Similarly the expected length of the system L_s falls between 1.0889 and 2.4762 and won't fall outside the range of 0.4792 and 7.0513. The expected waiting time in the queue falls between 0.2222 min and 0.7302 min approximately and it will never fall below 0.0708 min or exceed 3.3333 min approximately. Also it is definitely possible that the expected system waiting time falls between 0.2722 min and 0.8254 min approximately and it will never fall below 0.0958 min or exceed 3.5256 min approximately. The above data will be very suitable for designing a queuing system.

8. CONCLUSION

In this paper, we have used the α – cut approach to analyze a fuzzy bulk arrival queuing model. The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables such as fuzzy numbers defined on the real line. A numerical example was given to illustrate the effectiveness of the proposed technique. Also, the proposed method can be employed to analyze the other fuzzy queuing systems.

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