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More Functions Associated with Semi*δ-Open Sets

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ABSTRACT

*The aim of this paper is to introduce various functions associated with semi*δ-open sets. Here strongly semi*δ-continuous, perfectly semi*δ-continuous, totally semi*δ-continuous and slightly semi*δ-continuous functions are defined. Characterizations and some of their properties are investigated.*

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I. INTRODUCTION

In 1980, Jain [2] introduced the concept of totally continuous functions. Nour [6] defined totally semi-continuous and strongly semi-continuous functions. In 1995 T.M.Nour [7] introduced slightly semi-continuous functions. In 1997, Slightly continuity was introduced by Jain [3] .Quite recently, the authors [8, 9] introduced and studied some new concepts, namely semi*δ-open sets, semi*δ-closed sets. We have also defined semi*δ-continuous, semi*δ-irresolute functions and their contra versions [10]. In this paper we define the strongly semi*δ-continuous, perfectly semi*δ-continuous, totally semi*δ-continuous and slightly semi*δ-continuous functions and investigate their properties.

II. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) will always denote topological spaces on which no separation axioms are assumed, unless otherwise mentioned. When A is a subset of (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. We recall some known definitions needed in this paper.

Definition 2.1: A subset *A* of a topological space (*X*, τ) is **semi*δ-open** [8] if A⊆Cl*(δInt(A)).

Definition 2.2: A function f: (X, τ) ⟶ (Y, σ) is called a **strongly continuous** [4] if f^{-1}(V) is both open and closed in (X, τ) for each subset V in (Y, σ).

Definition 2.3: A function f: (X, τ) ⟶ (Y, σ) is called a **perfectly continuous** [5] if f^{-1}(V) is both open and closed in (X, τ) for every open set V in (Y, σ).

Definition 2.4: A function f : (X, τ) ⟶(Y, σ) is called **totally continuous** [2] if f^{-1}(V) is clopen set in X for each open set V of Y .

Definition 2.5: A function f: (X, τ) ⟶ (Y, σ) is called a **slightly continuous** [3] if f^{-1}(V) is open set in X for each clopen set V of Y .

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Definition 2.6: A function f: (X, τ) ⟶ (Y, σ) is called a **contra continuous** [1] if $f^{-1}(V)$ is closed in (X, τ) for every open set V in (Y, σ).

Definition 2.7: A function f: (X, τ)⟶ (Y, σ) is said to be **semi*δ-continuous** [10] if $f^{-1}(V)$ is semi*δ-open in (X, τ) for every open set V in (Y, σ).

Definition 2.8: A function F: X⟶Y is said to be **semi*δ-irresolute** [10] if $f^{-1}(V)$ is semi*δ-open in X for every semi*δ-open set V in Y.

Definition 2.9: A function f: (X, τ) ⟶ (Y, σ) is called **contra-semi*δ-continuous** [11] if $f^{-1}(V)$ is semi*δ- closed in X for every open set V in Y.

Definition 2.10: A function f: (X, τ) ⟶ (Y, σ) is said to be **contra-semi*δ-irresolute** [11] if $f^{-1}(V)$ is semi*δ-closed in (X, τ) for every semi*δ-open set V in (Y, σ).

Definition 2.11: A function f: X⟶Y is said to be **pre-semi*δ-open** [11**]** if f(V) is semi*δ-open in *Y* for every semi*δ-open set Vin *X*.

Definition 2.12: A topological space (X, τ) is said to be **$T_{S*\delta}$- space** [11], if every semi*δ-open set of X is open in X

Definition 2.13[12]: A space *X* is **locally indiscrete** if every open set in *X* is closed.

III. Strongly semi*δ-Continuous Function

Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is called **strongly semi*δ-continuous** if the inverse image of every semi*δ-open set in (Y, σ) is open in (X, τ).

Example 3.2: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{b},{c},{a, b},{b, c},{a, c},{a, b, c}, X} and σ = {ϕ, {a},{b}, {a, b},{a, b, c},Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = b, f(b) = a, f(c) = d, f(d) = c. The function f is strongly semi*δ-continuous.

Theorem 3.3: A map f: X ⟶ Y is strongly semi*δ-continuous if and only if the inverse image of every semi*δ-closed set in Y is closed in X.

Proof: Assume that f is strongly semi*δ-continuous. Let V be any semi*δ- closed set in Y. Then V^c is semi*δ-open in Y. Since f is strongly semi*δ- continuous, $f^{-1}(V^c)$ is open in X. But $f^{-1}(V^c)$ = X/ $f^{-1}(V)$ and so $f^{-1}(V)$ is closed in X. Conversely, assume that the inverse image of every semi*δ- closed set in Y is closed in X. Let V be any semi*δ-openset in Y. Hence V^c is semi*δ-closed in Y. By assumption, $f^{-1}(V^c)$ is closed in X, but $f^{-1}(V^c)$ = X/ $f^{-1}(V)$ and so $f^{-1}(V)$ is open in X. Therefore, f is strongly semi*δ- continuous.

Theorem 3.4: If a map f: X ⟶ Y is strongly semi*δ-continuous and a map g: Y ⟶ Z is semi*δ-continuous then g ∘ f: X ⟶ Z is continuous.

Proof: Let V be any open set in Z. Since g is semi*δ- continuous, $g^{-1}(V)$ is semi*δ- open in Y. Since f is strongly semi*δ-continuous $f^{-1}(g^{-1}(V))$ is open in X. That is $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in X. Therefore, g ∘f is continuous.

Theorem 3.5: If a map f: X ⟶ Y is strongly semi*δ- continuous and a map g: Y ⟶ Z is semi*δ- irresolute, then g ∘ f: X ⟶ Z is strongly semi*δ- continuous.

Proof: Let V be any semi*δ-open set in Z. Since g is semi*δ- irresolute, $g^{-1}(V)$ is semi*δ- open in Y. Also since f is strongly semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is open in X. Hence, g ∘ f: X ⟶ Z is strongly semi*δ- continuous.

Theorem 3.6: If f: X⟶ Y is strongly semi*****δ-continuous and g: Y ⟶ Z is contra semi*****δ-continuous, then their composition g ∘ f: X ⟶Z is contra continuous.

Proof: Let V be any open set in Z. Since, g is contra semi*****δ-continuous, then $g^{-1}(V)$ is semi*****δ-closed in Y and since f is strongly semi*****δ-continuous, by theorem 3.2 $f^{-1}(g^{-1}(V))$ is closed in X. Therefore, g ∘ f is contra continuous.

Theorem 3.7: If a map f: X ⟶ Y is semi*δ-continuous and a map g: Y ⟶ Z is strongly semi*δ-continuous, then g ∘ f: X ⟶ Z is semi*δ- irresolute.

Proof: Let V be any semi*δ-open set in Z. Since g is strongly semi*δ- continuous, $g^{-1}(V)$ is open in Y. Also since f is semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Hence, $g \circ f: X \longrightarrow Z$ is semi*δ- irresolute.

Theorem 3.8: If a map $f: X \longrightarrow Y$ is contra semi*δ- continuous and a map $g: Y \longrightarrow Z$ is strongly semi*δ-continuous, then $g \circ f: X \longrightarrow Z$ is contra-semi*δ- irresolute.

Proof: Let V be any semi*δ-open set in Z. Since g is strongly semi*δ- continuous, $g^{-1}(V)$ is open in Y. Also since f is contra-semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-closed in X. Hence $(g{\cdot}f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi*δ-closed in X. Hence, $g \circ f: X \longrightarrow Z$ is contra-semi*δ- irresolute.

Theorem 3.9: If $f : X \longrightarrow Y$ is continuous and $g: Y \longrightarrow Z$ strongly semi*δ- continuous then their composition $g \circ f : X \longrightarrow Z$ is strongly semi*δ- continuous.

Proof: Let V be any semi*δ- open set in Z. Since g is strongly semi*δ- continuous, $g^{-1}(V)$ is open in Y. Since f is continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open in X. Hence, $(g \circ f)$ is strongly semi*δ- continuous.

Theorem 3.10: Let $f: X \longrightarrow Y$ be a continuous function and Y be a $T_{S*\delta}$- space. Then f is strongly semi*δ- continuous.

Proof: Let V be any semi*δ- open set in Y. Since Y is $T_{S*\delta}$- space, V is open in Y. Also since f is continuous, $f^{-1}(V)$ is open in X. Hence, f is strongly semi*δ-continuous.

Theorem 3.11: Let $f: X \longrightarrow Y$ be semi*δ-irresolute and Y be $T_{S*\delta}$- space. Then f is strongly semi*δ-continuous.

Proof: Let V be any semi*δ-open set in Y. Since f is semi*δ-irresolute, $f^{-1}(V)$ is semi*δ-open in X. Also since X is $T_{S*\delta}$- space, $f^{-1}(V)$ is open in X. Hence f is strongly semi*δ-continuous.

Theorem 3.12: If $g: Y \longrightarrow Z$ is strongly semi*δ-continuous and injective, and $g \circ f : X \longrightarrow Z$ is semi*δ-closed then $f: X \longrightarrow Y$ is semi*δ-closed.

Proof: Let F be any closed set in X. Since, $g \circ f$ is semi*δ-closed which implies, $g \circ f(F)$ is semi*δ-closed in Z. Also since g is strongly semi*δ-continuous, $g^{-1}(g \circ f(F))$ is closed in Y. Since g is injective, f(F) is closed in Y. Hence f is semi*δ-closed.

IV. Perfectly semi*δ-Continuous Function

Definition 4.1: A map $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be **perfectly semi*δ- continuous** if the inverse image of every semi*δ-open set in (Y, σ) is both open and closed in (X, τ).

Theorem 4.2: If a map $f: X \longrightarrow Y$ is perfectly semi*δ- continuous then it is strongly semi*δ- continuous.

Proof: Assume that f is perfectly semi*δ-continuous. Let V be any semi*δ-open set in Y. Since f is perfectly semi*δ-continuous, $f^{-1}(V)$ is open in X. Therefore, f is strongly semi*δ- continuous.

Remark 4.3: The converse of the above theorem need not be true.

Example 4.4: Let X = Y= {a, b, c, d}, τ = {ϕ,{a},{b},{c},{a, b},{b, c},{a, c}{a, b, c}, X} and σ = {ϕ, {a},{b}, {a, b},{a, b, c},Y}. Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be defined by f(a) = b, f(b) = a, f(c) = d, f(d) = c. Clearly, f is strongly semi*δ-continuous. But $f^{-1}(\{a\}) = \{b\}$ is open in X, but not closed in X. Therefore f is not perfectly semi*δ-continuous.

Theorem 4.5: A map $f: X \longrightarrow Y$ is perfectly semi*δ- continuous if and only if $f^{-1}(V)$ is both open and closed in X for every semi*δ-closed set V in Y.

Proof: Let V be any semi*δ-closed set in Y. Then V^c is semi*δ-open in Y. Since, f is perfectly semi*δ-continuous, $f^{-1}(V^c)$ is both open and closed in X. But $f^{-1}(V^c) = X/ f^{-1}(V)$ and so $f^{-1}(V)$ is both open and closed in (X, τ). Conversely, assume that the inverse image of every semi*δ-closed set in Y is both open and closed in X. Let V be any semi*δ-open in Y. Then V^c is semi*δ-closed in Y. By assumption $f^{-1}(V^c)$ is both open and closed in X. But $f^{-1}(V^c) = X/ f^{-1}(V)$ and so $f^{-1}(V)$ is both open and closed in X. Therefore, f is perfectly semi*δ- continuous.

Theorem 4.6: Let f: X ⟶Y be a mapping from a discrete topological space X into any topological space Y, then the following statements are equivalent.

1) f is strongly semi*δ-continuous
2) f is perfectly semi*δ-continuous

Proof:
(1) ⇒ (2): Let V be any semi*δ-open set in Y. By hypothesis, $f^{-1}(V)$ is open in X. Since X is a discrete space, $f^{-1}(V)$ is closed in X. $f^{-1}(V)$ is both open and closed in X. Hence, f is perfectly semi*δ- continuous.

(2) ⇒ (1): Let V be any semi*δ-open set in Y. Then, $f^{-1}(V)$ is both open and closed in X. Hence, f is strongly semi*δ-continuous.

Theorem 4.7: If f: X ⟶ Y is continuous and g: Y⟶Z is perfectly semi*δ- continuous then their composition g ∘ f : X ⟶ Z is strongly semi*δ- continuous.

Proof: Let V be any semi*δ-open set in Z. Since g is perfectly semi*δ- continuous, $g^{-1}(V)$ is open and closed in Y. Since f is continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open in X. Hence, g ∘ f is strongly semi*δ-continuous.

V. Totally semi*δ-continuous functions

Definition 5.1: A function f: (X, τ) ⟶(Y, σ) is called **totally semi*δ-continuous function** if the inverse image of every open set in (Y, σ) is both semi*δ-open and semi*δ-closed subset in (X, τ).

Theorem 5.2: Every totally semi*δ-continuous function is semi*δ-continuous.

Proof: Let V be any open set in Y. Since, f is totally semi*δ-continuous function, $f^{-1}(V)$ is both semi*δ-open and semi*δ-closed in X. Therefore, f is semi*δ-continuous.

Remark 5.3: The converse of above theorem need not be true.

Example 5.4: Let X =Y ={a, b, c, d}, τ = {ϕ, {a},{b},{a, b},{a, b, c}, X}, σ ={ϕ,{a},{a, b}, Y}. Let f: (X, τ) ⟶(Y, σ) be defined by f (a) = c, f(b) = a, f(c) = d, f(d) = b. S*δO(X, τ) = {ϕ, {a},{b},{a, b},{a, d},{b, d},{a, b, d},X} and S*δ C(X, τ)={ϕ,{c},{a, c},{b, c},{c, d},{a, c, d},{b, c, d}, X}. Clearly f is semi*δ-continuous. But, since $f^{-1}(\{a\}) = \{b\}$ is semi*δ-open in X but not semi*δ-closed in X, f is not totally semi*δ-continuous.

Theorem 5.5: If f: X → Y is semi*δ-irresolute and g: Y → Z is totally semi*δ-continuous then g ∘ f: X → Z is totally semi*δ-continuous.

Proof: Let V be any open set in Z. Since g is totally semi*δ-continuous, $g^{-1}(V)$ is semi*δ-open and semi*δ-closed in Y. Since f is semi*δ-irresolute, $f^{-1}(g^{-1}(V))$ is semi*δ-open and semi*δ-closed in X. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, g∘f is totally semi*δ-continuous.

Theorem 5.6: If f: X → Y is totally semi*δ-continuous and g: Y → Z is continuous then g∘ f: X → Z is totally semi*δ-continuous.

Proof: Let V be any open set in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Since, f is totally semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-clopen in X. Hence, g ∘ f is totally semi*δ-continuous.

Theorem 5.7: Every totally semi*δ-continuous function is totally semi-continuous.

Proof: Suppose f: X⟶Y is totally semi*δ-continuous. Let V be an open set in Y. Since f is totally semi*δ-continuous, $f^{-1}(V)$ is semi*δ-regular in X and hence semi-regular in X. Therefore f is totally semi-continuous.

Theorem 5.8: Every totally semi*δ-continuous function is totally semi*-continuous.

Proof: Suppose f: X⟶Y is totally semi*δ-continuous. Let V be an open set in Y. Since f is totally semi*δ-continuous, $f^{-1}(V)$ is semi*δ-regular in X and hence semi*-regular in X. Therefore f is totally semi*-continuous.

Theorem 5.9: Every totally semi*δ-continuous function is totally semi*α-continuous.

Proof: Suppose f: X⟶Y is totally semi*δ-continuous. Let V be an open set in Y. Since f is totally semi*δ-continuous, $f^{-1}(V)$ is semi*δ-regular in X and hence semi*α-regular in X. Therefore f is totally semi*α-continuous.

Theorem 5.10: A function f: X⟶Y is totally semi*δ-continuous if and only if f is both semi*δ-continuous and contra-semi*δ-continuous.

Proof: Let V be any open set in Y. Since f is totally semi*δ-continuous, $f^{-1}(V)$ is both semi*****δ-open and semi*****δ-closed in X. Hence f is both semi*δ-continuous and contra-semi*δ-continuous.

Conversely, Let V be any open set in Y. Since f is both semi*δ-continuous and contra-semi*δ-continuous, $f^{-1}(V)$ is both semi*****δ-open and semi*****δ-closed in X. Hence f is totally semi*δ-continuous.

VI. Slightly semi*δ-continuous functions

Definition 6.1: A function (X, τ) ⟶(Y, σ) is called **slightly semi*δ-continuous** at a point $x \in X$ if for each clopen subset V of Y containing f(x), there exists a semi*****δ-open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly semi*δ-continuous if f is slightly semi*δ-continuous at each of its points.

Definition 6.2: A function (X, τ) ⟶(Y, σ) is said to be **slightly semi*δ-continuous** if the inverse image of every clopen set in Y is semi*****δ-open in X.

Example 6.3: Let X = Y={ a, b, c, d} and τ = {ϕ, {a},{b},{c},{a, b}, {a, c}, {b, c}, {a, b, c}, X} , σ = {ϕ, {a}, {b, c, d},Y} and S*δO(X, τ) = {ϕ, {a},{b}, {c},{a, b},{a, c}, {a, d}{b, c},{b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d},{b, c, d},X}. Let *g:* (X, τ) ⟶(Y, σ) be defined by g(a)=d , g(b)=c g(c)=a, g(d)=b. The function f is slightly semi*δ-continuous.

Theorem 6.4: The definition 6.1 and 6.2 are equivalent.

Proof: Suppose the definition 6.1 holds. Let V be any clopen set in Y and $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists a semi***δ**-open set U_x such that $x \in U_x$ and $f(U_x) \subseteq V$. Now $x \in U_x \subseteq f^{-1}(V)$. And $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Since, arbitrary union of semi*δ-open set is semi***δ**-open, $f^{-1}(V)$ is semi***δ**-open in X and therefore, f is slightly semi*δ-continuous.

Suppose, the definition 6.2 holds. Let $f(x) \in V$ where, V is any clopen set in Y. Since, f is slightly semi*δ-continuous, $x \in f^{-1}(V)$ where $f^{-1}(V)$ is semi***δ**-open in X. Let $U = f^{-1}(V)$. Then U is semi***δ**-open in X, $x \in U$ and $f(U) \subseteq V$.

Theorem 6.5: For a function f: (X, τ) ⟶(Y, σ), the following statements are equivalent.

(i) f is slightly semi*δ-continuous.
(ii) The inverse image of every clopen set V of Y is semi*δ-open in X.
(iii) The inverse image of every clopen set V of Y is semi*δ-closed in X.
(iv) The inverse image of every clopen set V of Y is semi*δ-regular in X.

Proof:
(i)⇒ (ii): Follows from the theorem 6.4

(ii)⇒ (iii): Let V be a clopen set in Y which implies V^c is clopen in Y. By (ii), $f^{-1}(V^c) = (f^{-1}(V))^c$ is semi*δ-open in X. Therefore, $f^{-1}(V)$ is semi*δ-closed in X.

(iii) ⇒ (iv): By (ii) and (iii), $f^{-1}(V)$ is semi*δ-regularin X.

(iv) ⇒ (i): Let V be any clopen set in Y containing f(x), by (iv) $f^{-1}(V)$ is semi*δ-regular in X. Take $U = f^{-1}(V)$, then $f(U) \subseteq V$. Hence, f is slightly semi*δ-continuous.

Theorem 6.6: Every semi*δ-continuous function is slightly semi*δ-continuous.

Proof: Let f: X ⟶Y be a semi*δ-continuous function. Let V be any clopen set in Y. Since f is semi*δ-continuous, $f^{-1}(V)$ is semi*****δ-open in X and semi*****δ-closed in X. Hence by theorem 6.5, f is slightly semi*δ-continuous.

Remark 6.7: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.8: Let X =Y={a, b, c}, τ = {ϕ,{a},{b},{a, b},X}, σ = {ϕ, {a},{c},{a, b}, {a, c}, Y}, and σ^c = {ϕ, {b},{c},{a, b}, {b, c}, Y}, then S*δO(X, τ) = {ϕ, {a},{b},{a, b},{b, c},{a, c}, X}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a)= b , f(b)=c and f(c)=a .The function f is slightly semi*δ-continuous. But f^{-1}({a}) = {c} is not semi*δ-open in X. Hence f is not semi*δ-continuous.

Theorem 6.9: Every contra semi*δ-continuous function is slightly semi*δ-continuous.

Proof: Let f: (X, τ) ⟶(Y, σ) be a contra semi*δ-continuous function. Let V be any clopen set in Y. Then by the definition of contra semi*δ-continuous and by theorem 3.11[11], f^{-1} (V) is semi*δ-regular in X. Hence by theorem 6.5, f is slightly semi*δ-continuous.

Remark 6.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 6.11: Let X=Y={a, b, c}, τ = {ϕ, {a},{b},{a, b}, X}, σ = {ϕ, {a},{c},{a, b}, {a, c}, Y}, and σ^c = {ϕ, {b},{c},{a, b}, {b, c}, Y}, then S*δO(X, τ) = {ϕ, {a},{b},{a, b},{b, c}, {a, c}, X}. Let f: (X, τ) ⟶(Y, σ) be defined by f(a)= c, f(b)= a, f(c)=b. The function f is slightly semi*δ-continuous. But f^{-1} ({a, c}) = {a, b} is not semi*δ-closed in X. Hence f is not contra semi*δ-continuous.

Theorem 6.12: If f: X → Y is semi*δ-irresolute and g: Y → Z is slightly semi*δ-continuous then g∘f : X → Z is slightly semi*δ-continuous.

Proof: Let V be any clopen set in Z. Since g is slightly semi*δ-continuous, g^{-1}(V) is semi*δ-open in Y. Since f is semi*δ-irresolute, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, g ∘ f is slightly semi*δ-continuous.

Theorem 6.13: If f: X → Y is semi*δ-irresolute and g: Y → Z is semi*δ-continuous then g∘f: X → Z is slightly semi*δ-continuous.

Proof: Let V be any clopen set in Z. Since g is semi*δ-continuous, g^{-1}(V) is semi*δ-open in Y. Since f is semi*δ-irresolute,$f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, g ∘ f is slightly semi*δ-continuous.

Theorem 6.14: If f: X → Y is semi*δ-continuous and g: Y → Z is slightly-continuous then g∘f: X → Z is slightly semi*δ-continuous.

Proof: Let V be any clopen set in Z. Since, g is slightly-continuous, g^{-1}(V) is open in Y. Since, f is semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Since $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$, g ∘ f is slightly semi*δ-continuous.

Theorem 6.15: If f: X → Y is strongly semi*δ-continuous and g: Y → Z is slightly semi*δ- continuous then g ∘f: X → Z is slightly-continuous.

Proof: Let V be any clopen set in Z. Since g is slightly semi*δ-continuous, g^{-1}(V) is semi*δ-open in Y. Since f is strongly semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is open in X. Therefore, g ∘ f is slightly continuous.

Theorem 6.16: If f: X → Y is slightly semi*δ-continuous and g: Y → Z is perfectly semi*δ- continuous then g ∘f: X → Z is semi*δ-irresolute.

Proof: Let V be any semi*δ-open set in Z. Since g is perfectly semi*δ-continuous, g^{-1}(V) is both open and closed in Y. Since f is slightly semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Hence, g ∘ f is semi*δ-irresolute.

Theorem 6.17: If f: X → Y is slightly semi*δ-continuous and g: Y → Z is contra continuous then g∘f: X → Z is slightly semi*δ- continuous.

Proof: Let V be any clopen set in Z. Since g is contra continuous, g^{-1}(V) is open and closed in Y. Since, f is slightly semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Hence g ∘ f is slightly semi*δ-continuous.

Theorem 6.18: If f: X → Y is semi*δ- irresolute and g: Y → Z is contra semi*δ-continuous then g∘f: X → Z is slightly semi*δ- continuous.

Proof: Let V be any clopen set in Z. Since g is contra semi*δ-continuous, g^{-1}(V) is semi*δ-open and semi*δ-closed in Y. Since f is semi*δ-irresolute, $f^{-1}(g^{-1}(V))$ is semi*δ-open and semi*δ-closed in X. Hence g∘ f is slightly semi*δ-continuous.

Theorem 6.19: If the function f: X ⟶Y is slightly semi*δ-continuous and (X, τ) is $T_{S*\delta}$- space, then f is slightly continuous.

Proof: Let V be any clopen set in Y. Since g is slightly semi*δ-continuous, $f^{-1}(V)$ is semi*δ-open in X. Since X is $T_{S*\delta}$- space, $f^{-1}(V)$ is open in X. Hence f is slightly continuous.

Theorem 6.20: Let f: X ⟶Y and g: Y ⟶Z be functions. If f is surjective and pre-semi*δ-open and g∘ f: X ⟶ Z is slightly semi*δ-continuous, then g is slightly semi*δ-continuous.

Proof: Let V be any clopen set in Z. Since g ∘ f: X ⟶ Z is slightly semi*δ-continuous, $f^{-1}(g^{-1}(V))$ is semi*δ-open in X. Since f is surjective and pre-semi*δ-open $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semi*δ-open in Y. Hence, g is slightly semi*δ-continuous.

Theorem 6.21: If f: X⟶Y is a slightly semi*δ-continuous and Y is a locally indiscrete space then f is semi*δ-continuous.

Proof: Let V be any open subset in Y. Since Y is a locally indiscrete space, V is closed in Y. Since f is slightly semi*δ-continuous, $f^{-1}(V)$ is semi*δ-open in X. Hence, f is semi*δ-continuous.

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