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RELIABILITY OF TIME - DEPENDENT STRESS - STRENGTH SYSTEM WHICH FOLLOW DIFFERENT DISTRIBUTIONS WHEN THE NUMBER OF CYCLES FOLLOWS BINOMIAL DISTRIBUTION

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ABSTRACT

Reliability of time dependent stress strength system is carried out by considering stress/strength variables as deterministic or random fixed or random independent. The number of cycles in any period of time t is assumed to be random. Reliability is obtained for the random models when the numbers of cycles follow binomial distribution where stress and strength follow Pareto distribution and exponential distribution.Reliability computation are made for the above said model

Key words: *Binomial distribution, Pareto distribution, Exponential distribution, Deterministic, Random- fixed, Random- independent and Reliability.*

INTRODUCTION

Time dependent stress strength system is defined by Kapur, K.C. and Lamberson L.R [3].In stress-strength models component fails if the stress exceeds strength. The uncertainty about the stress and strength variables is classified into three categories. (1) Deterministic (2) Random fixed (3) Random independent. The components are assumed to be identical and the number of cycles for any time period t is assumed to be random. Expressions for system reliability have been attained when numbers of cycles follow binomial distribution and stress and strength both follow Pareto distribution andexponential distribution for random- fixed stress and random- independent strength, random – independent stress and random- fixed strength

NOTATIONS

f(x): The probability density functions of random variable X.
g(y): The probability density function of random variable Y.
R (t): Reliability at t time with n number of cycles.
R_i : Reliability after i cycles.
p: probability of success.
q: probability of failure.

RELIABILITY EVALUATION

Number of cycles occurring in a given time interval followsbinomial distribution, then

$$\pi_i(t) = P(X = i) = \binom{n}{i} p^i q^{n-i}, \quad i = 0,1,2, \ldots\ldots n.$$

Case-1: Random- fixed stress and Random- independent strength

Let $f(x)$ be the probability density function of random fixed stress X and $g(y)$ be the probability density function of random independent strength Y.

$$R_i = \int_0^\infty f(x)\left(\int_x^\infty g(y)dy\right)^i dx \,, i = 0,1,2, \ldots\ldots n$$

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$$R(t) = \sum_{i=0}^{n} \pi_i(t) R_i$$
$$= \sum_{i=o}^{n} \binom{n}{i} p^i q^{n-i} \int_0^\infty f(x) \left(\int_x^\infty g(y)dy\right)^i dx$$
$$= q^n \int_0^\infty f(x) \sum_{i=0}^{n} \binom{n}{i} \left(\frac{p}{q} \int_x^\infty g(y)dy\right)^i dx$$
$$= q^n \int_0^\infty f(x) \left(1 + \frac{p}{q} \int_x^\infty g(y)dy\right)^n dx$$

$$R(t) = q^n \int_0^\infty f(x) \left(1 + \frac{p}{q} G(x)\right)^n dx\ , where\ \ G(x) = \int_x^\infty g(y)dy$$

$$R(t) = \int_0^\infty f(x) \left(q + pG(x)\right)^n dx,\ \ where\ \ G(x) = \int_x^\infty g(y)dy \tag{1}$$

Case-2: Random- independent stress and Random- fixed strength

Let g(y) be the probability density function of random fixed strength Y and $f(x)$ be the probability density function of random independent stress$X$.

$$R(t) = \sum_{i=o}^{n} \left(\binom{n}{i} p^i q^{n-i} \int_0^\infty g(y) \left(\int_0^y f(x)dx\right)^i dy\right)$$
$$= q^n \int_0^\infty \left(g(y) \sum_{i=0}^{n} \binom{n}{i} \left(\frac{p}{q} \int_0^y f(x)dx\right)^i dy\right)$$
$$= q^n \int_0^\infty g(y) \left(1 + \frac{p}{q} \int_0^y f(x)dx\right)^n dy$$

$$R(t) = q^n \int_0^\infty g(y) \left(1 + \frac{p}{q} F(y)\right)^n dx,\ \ where\ \ F(y) = \int_0^y f(x)dx$$

$$R(t) = \int_0^\infty g(y) \left(q + pF(y)\right)^n dx\ \ , where\ \ F(y) = \int_0^y f(x)dx \tag{2}$$

If Stress follows Pareto Distribution and Strength follows Exponential Distribution

Case-1: Random - fixed stress and Random - independent strength

From equation(1)

$$R(t) = \int_o^\infty f(x) \left(q + pG(x)\right)^n dx\ \ , where\ \ G(x) = \int_x^\infty g(y)dy$$

$$G(x) = \int_x^\infty \mu e^{-\mu y} dy = \ e^{-\mu x}$$

$$R(t) = \int_0^\infty \frac{\lambda k^\lambda}{x^{\lambda+1}} (q + Pe^{-\mu x})^n dx$$
$$= \lambda k^\lambda \int_0^\infty x^{-(\lambda+1)} \left[n_{c_0} q^n + n_{c_1} q^{n-1} (pe^{-\mu x}) + n_{c_2} q^{n-2} (p^2 e^{-2\mu x}) + \cdots \ldots \ldots. + n_{c_n} q^0 (p^n e^{-n\mu x})\right].$$

$$R(t) = \lambda k^\lambda \Big[\int_0^\infty n_{c_0} q^n x^{-(\lambda+1)}\, dx + \int_0^\infty n_{c_1} q^{n-1} p e^{-\mu x} x^{-(\lambda+1)}\, dx + \int_0^\infty n_{c_2} q^{n-2} p^2 e^{-2\mu x} x^{-(\lambda+1)}\, dx$$
$$+ \ldots \ldots + \int_0^\infty n_{c_n} q^0 p^n e^{-n\mu x} x^{-(\lambda+1)}\, dx\Big] \tag{1}$$

$$I_0 = \int_0^\infty n_{c_0} q^n x^{-(\lambda+1)}\, dx = n_{c_0} q^n \int_0^\infty x^{-(\lambda+1)}\, dx$$
$$= n_{c_0} q^n \left[\frac{x^{-\lambda-1+1}}{-\lambda-1+1}\right]_0^\infty$$
$$= n_{c_0} q^n \left[\frac{x^{-\lambda}}{-\lambda}\right]_0^\infty = 0$$

$$I_0 = \ \int_0^\infty n_{c_0} q^n x^{-(\lambda+1)}\, dx = 0 \ldots \ldots (i)$$

let $\quad I_1 = \int_0^\infty n_{c_1} p q^{n-1} e^{-\mu x} x^{-(\lambda+1)}\, dx$

$$= n_{c_1} p q^{n-1} \int_0^\infty e^{-\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_1} p q^{n-1} \left[\left[e^{-\mu x} \frac{x^{-\lambda-1+1}}{-\lambda-1+1}\right]_0^\infty - \int_0^\infty e^{-\mu x} (-\mu) \frac{x^{-\lambda}}{-\lambda}\, dx\right]$$
$$= n_{c_1} p q^{n-1} \left[0 - \frac{\mu}{\lambda} \int_0^\infty e^{-\mu x} x^{-\lambda}\, dx\right]$$

$$= \frac{n_{c_1} p q^{n-1}}{\lambda}\left[-\frac{\mu}{\mu^{-\lambda+1}}\Gamma(-\lambda+1)\right]$$
$$= \frac{n_{c_1} p q^{n-1}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]$$

$$I_1 = \int_0^{\infty} n_{c_1} p q^{n-1} e^{-\mu x} x^{-(\lambda+1)}\, dx = \frac{n_{c_1} p q^{n-1}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] \dots\dots..(ii)$$

let
$$I_2 = \int_0^{\infty} n_{c_2} p^2 q^{n-2} e^{-2\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_2} p^2 q^{n-2} \int_0^{\infty} e^{-2\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_2} p^2 q^{n-2}\left[\left[e^{-2\mu x}\frac{x^{-\lambda-1+1}}{-\lambda-1+1}\right]_0^{\infty} - \int_0^{\infty} e^{-2\mu x}(-2\mu)\frac{x^{-\lambda}}{-\lambda}\, dx\right]$$
$$= n_{c_2} p^2 q^{n-2}\left[0 - \frac{2\mu}{\lambda}\int_0^{\infty} e^{-2\mu x} x^{-\lambda}\, dx\right]$$
$$= \frac{n_{c_2} p^2 q^{n-2}}{\lambda}\left[-\frac{2\mu}{2\mu^{-\lambda+1}}\Gamma(-\lambda+1)\right]$$
$$= \frac{n_{c_2} p^2 q^{n-2}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]$$

$$I_2 = \int_0^{\infty} n_{c_2} p^2 q^{n-2} e^{-2\mu x} x^{-(\lambda+1)}\, dx = \frac{n_{c_2} p^2 q^{n-2}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] \dots\dots.(iii)$$

let
$$I_3 = \int_0^{\infty} n_{c_3} p^3 q^{n-3} e^{-3\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_3} p^3 q^{n-3} \int_0^{\infty} e^{-3\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_3} p^3 q^{n-3}\left[\left[e^{-3\mu x}\frac{x^{-\lambda-1+1}}{-\lambda-1+1}\right]_0^{\infty} - \int_0^{\infty} e^{-3\mu x}(-3\mu)\frac{x^{-\lambda}}{-\lambda}\, dx\right]$$
$$= n_{c_3} p^3 q^{n-3}\left[0 - \frac{3\mu}{\lambda}\int_0^{\infty} e^{-3\mu x} x^{-\lambda}\, dx\right]$$
$$= \frac{n_{c_3} p^3 q^{n-3}}{\lambda}\left[-\frac{3\mu}{3\mu^{-\lambda+1}}\Gamma(-\lambda+1)\right]$$
$$= \frac{n_{c_3} p^3 q^{n-3}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]$$

$$I_3 = \int_0^{\infty} n_{c_3} p^3 q^{n-3} e^{-3\mu x} x^{-(\lambda+1)}\, dx = \frac{n_{c_3} p^3 q^{n-3}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] \dots\dots.(iv)$$

Continuing this process up to n times more, we get

let
$$I_n = \int_0^{\infty} n_{c_n} p^n q^{n-n} e^{-n\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_n} p^n q^{n-n} \int_0^{\infty} e^{-n\mu x} x^{-(\lambda+1)}\, dx$$
$$= n_{c_n} p^n q^{n-n}\left[\left[e^{-n\mu x}\frac{x^{-\lambda-1+1}}{-\lambda-1+1}\right]_0^{\infty} - \int_0^{\infty} e^{-n\mu x}(-n\mu)\frac{x^{-\lambda}}{-\lambda}\, dx\right]$$
$$= n_{c_n} p^n q^{n-n}\left[0 - \frac{n\mu}{\lambda}\int_0^{\infty} e^{-n\mu x} x^{-\lambda}\, dx\right]$$
$$= \frac{n_{c_n} p^n q^{n-n}}{\lambda}\left[-\frac{n\mu}{n\mu^{-\lambda+1}}\Gamma(-\lambda+1)\right]$$
$$= \frac{n_{c_3} p^n q^{n-n}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]$$

$$I_n = \int_0^{\infty} n_{c_n} p^n q^{n-n} e^{-n\mu x} x^{-(\lambda+1)}\, dx = \frac{n_{c_n} p^n q^{n-n}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] \dots\dots.(v)$$

Substituting all these values in equation (1)

$$R(t) = \lambda k^{\lambda}\Bigg[0 + \frac{n_{c_1} p q^{n-1}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] + \frac{n_{c_2} p^2 q^{n-2}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] + \frac{n_{c_3} p^3 q^{n-3}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right] +$$
………+ncnpnqn−nλ−μλΓ(−λ+1)

$$R(t) = \frac{\lambda k^{\lambda}}{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]\left[n_{c_1} p q^{n-1} + n_{c_2} p^2 q^{n-2} + n_{c_2} p^3 q^{n-3} + \cdots \dots + n_{c_n} p^n q^{n-n}\right]$$

$$R(t) = k^{\lambda}\left[-\mu^{\lambda}\Gamma(-\lambda+1)\right]\sum_{k=1}^{n} n_{c_k} p^k\, q^{n-k}$$

If Stress Follows Exponential Distribution and Strength FollowsPareto Distribution

Case-2: Random- independent stress and Random- fixed strength
From equation (2)

$$R(t) = \int_0^\infty g(y)\left(q + pF(y)\right)^n dy \ \ , where\ \ F(y) = \int_0^y f(x)dx$$

$$F(y) = \int_0^y \lambda e^{-\lambda x} dx = \left(1 - e^{-\lambda y}\right)$$

$$R(t) = \int_0^\infty \frac{\mu k^\mu}{y^{\mu+1}}\left(q + p\left(1 - e^{-\lambda y}\right)\right)^n dy = \int_0^\infty \frac{\mu k^\mu}{y^{\mu+1}}\left(1 - pe^{-\lambda y}\right)^n dy$$

$$R(t) = \int_0^\infty \frac{\mu k^\mu}{y^{\mu+1}}\left(1 - pe^{-\lambda y}\right)^n dy$$
$$= \mu k^\mu \int_0^\infty y^{-(\mu+1)}\,[n_{c_0} - n_{c_1} pe^{-\lambda y} + n_{c_2} p^2 e^{-2\lambda y} - n_{c_3} p^3 e^{-3\lambda y} + \cdots \ldots \ldots . + (-1)^n n_{c_n} p^n e^{-n\lambda y}.$$

$$R(t) = \mu k^\mu [\int_0^\infty n_{c_0} y^{-(\mu+1)}\, dy + \int_0^\infty n_{c_1} pe^{-\lambda y} x^{-(\mu+1)}\, dy + \int_0^\infty n_{c_2} p^2 e^{-2\lambda y} y^{-(\mu+1)}\, dy + \cdots \ldots \ldots \ldots +$$
$$(-1)^n \int_0^\infty n_{c_n} p^n e^{-n\lambda y} x^{-(\lambda+1)}\, dy] \ldots \ldots \ldots (1)$$

$$I_0 = \int_0^\infty n_{c_0} y^{-(\mu+1)}\, dy = n_{c_0} \int_0^\infty y^{-(\mu+1)}\, dy$$
$$= n_{c_0}\left[\frac{y^{-\mu-1+1}}{-\mu-1+1}\right]_0^\infty$$
$$= n_{c_0}\left[\frac{y^{-\lambda}}{-\mu}\right]_0^\infty = 0$$

$$I_0 = \int_0^\infty n_{c_0} y^{-(\mu+1)}\, dy = 0 \ldots \ldots . . (i)$$

let

$$I_1 = \int_0^\infty n_{c_1} pe^{-\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_1} p \int_0^\infty e^{-\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_1} p\left[\left[e^{-\lambda y}\frac{y^{-\mu-1+1}}{-\mu-1+1}\right]_0^\infty - \int_0^\infty e^{-\lambda y}(-\lambda)\frac{y^{-\mu}}{-\mu}\, dy\right]$$
$$= n_{c_1} p\left[0 - \frac{\lambda}{\mu}\int_0^\infty e^{-\lambda y} y^{-\mu}\, dy\right]$$
$$= \frac{n_{c_1} p}{\mu}\left[-\frac{\lambda}{\lambda^{-\mu+1}}\Gamma(-\mu+1)\right]$$
$$= \frac{n_{c_1} p}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right]$$

$$I_1 = \int_0^\infty n_{c_1} pe^{-\lambda y} y^{-(\mu+1)}\, dy = \frac{n_{c_1} p}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \ldots \ldots . . (ii)$$

let

$$I_2 = \int_0^\infty n_{c_2} p^2 e^{-2\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_2} p^2 \int_0^\infty e^{-2\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_2} p^2\left[\left[e^{-2\lambda y}\frac{y^{-\mu-1+1}}{-\mu-1+1}\right]_0^\infty - \int_0^\infty e^{-2\lambda y}(-2\lambda)\frac{y^{-\mu}}{-\mu}\, dy\right]$$
$$= n_{c_2} p^2\left[0 - \frac{2\lambda}{\mu}\int_0^\infty e^{-\lambda y} y^{-\mu}\, dy\right]$$
$$= \frac{n_{c_2} p^2}{\mu}\left[-\frac{2\lambda}{2\lambda^{-\mu+1}}\Gamma(-\mu+1)\right]$$
$$= \frac{n_{c_2} p^2}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right]$$

$$I_2 = \int_0^\infty n_{c_2} p^2 e^{-2\lambda y} y^{-(\mu+1)}\, dy = \frac{n_{c_2} p^2}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \ldots \ldots . . (iii)$$

let

$$I_3 = \int_0^\infty n_{c_3} p^3 e^{-3\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_3} p^3 \int_0^\infty e^{-3\lambda y} y^{-(\mu+1)}\, dy$$
$$= n_{c_3} p^3\left[\left[e^{-3\lambda y}\frac{y^{-\mu-1+1}}{-\mu-1+1}\right]_0^\infty - \int_0^\infty e^{-3\lambda y}(-3\lambda)\frac{y^{-\mu}}{-\mu}\, dy\right]$$
$$= n_{c_3} p^3\left[0 - \frac{3\lambda}{\mu}\int_0^\infty e^{-3\lambda y} y^{-\mu}\, dy\right]$$
$$= \frac{n_{c_3} p^3}{\mu}\left[-\frac{3\lambda}{3\lambda^{-\mu+1}}\Gamma(-\mu+1)\right]$$
$$= \frac{n_{c_3} p^3}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right]$$

$$I_3 = \int_0^\infty n_{c_3} p^3 e^{-3\lambda y} y^{-(\mu+1)}\, dy = \frac{n_{c_3} p^3}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \ldots\ldots (iv)$$

Continuing this process up to n times more, we get

$$\begin{aligned}
let \qquad I_n &= \int_0^\infty n_{c_n} p^n e^{-n\lambda y} y^{-(\mu+1)}\, dy \\
&= n_{c_n} p^n \int_0^\infty e^{-n\lambda y} y^{-(\mu+1)}\, dy \\
&= n_{c_n} p^n \left[\left[e^{-n\lambda y} \frac{y^{-\mu-1+1}}{-\mu-1+1}\right]_0^\infty - \int_0^\infty e^{-n\lambda y}(-n\lambda)\frac{y^{-\mu}}{-\mu}\, dy\right] \\
&= n_{c_n} p^n \left[0 - \frac{n\lambda}{\mu}\int_0^\infty e^{-n\lambda y} y^{-\mu}\, dy\right] \\
&= \frac{n_{c_n} p^n}{\mu}\left[-\frac{n\lambda}{n\lambda^{-\mu+1}}\Gamma(-\mu+1)\right] \\
&= \frac{n_{c_n} n}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right]
\end{aligned}$$

$$I_2 = \int_0^\infty n_{c_n} p^n e^{-n\lambda y} y^{-(\mu+1)}\, dy = \frac{n_{c_n} p^n}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \ldots\ldots (v)$$

Substituting all these values in equation (1)

$$= \mu k^\mu \begin{bmatrix} 0 - \frac{n_{c_1} p}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] + \frac{n_{c_2} p^2}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] + \frac{n_{c_3} p^3}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \\ + \cdots \ldots\ldots - \frac{n_{c_n} p^n}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right] \end{bmatrix}$$

$$R(t) = \frac{\mu k^\mu}{\mu}\left[-\lambda^\mu \Gamma(-\mu+1)\right]\left[n_{c_1} p - n_{c_2} p^2 + n_{c_3} p^3 - \cdots \ldots + (-1)^{n-1} p^n\right]$$

$$R(t) = k^\mu\left[-\lambda^\mu \Gamma(-\mu+1)\right] \sum_{k=1}^n (-1)^{k-1}\, n_{c_k} p^k$$

CONCLUSION

General reliability formula for n cycles is derived when number of cycles follows binomial distribution for the following two cases: (1) random- fixed Stress and random- independent Strength. (2) random- independent Stress and random- fixed Strength.

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