International Journal of Mathematical Archive-2(9), 2011, Page: 1569-1571
IJMA Available online through www.ijma.info ISSN 2229 – 5046

Symmetric and Permutational Generating Sets of S_{10k+r} and A_{10k+r} Using the Mathieu Group M_{10}

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(Received on: 14-07-11; Accepted on: 01-08-11)

ABSTRACT

In this paper, we show how to generate S_{10k+r} and A_{10k+r} using the Mathieu Group M_{10} and an element of order $k+r$ in S_{10k+r} and A_{10k+r} for all positive integers $k, r > 1$. We also show how to generate S_{10k+r} and A_{10k+r} symmetrically using a symmetric generating set.

Key words: Symmetric generator set M_{10}.

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1. INTRODUCTION:

The Mathieu group M_{10}, of order 720, is one of the well known non-simple groups. Eassa [3] showed that

$$M_{10} = \left\langle X, Y \mid X^5 = Y^4 = [X, Y]^3 = (XYXYX)^5 = (XY^2)^2 = 1 \right\rangle.$$

Al-Amri [1], Hammas [2] and Al-Amri and Eassa [3] studied symmetric and permutational generating sets of S_{10k+1} and A_{10k+1} using some progenitors.

In this paper, we will show that S_{10k+r} and A_{10k+r} can by generated using the Mathieu group M_{10} and an element of order $k+r$ in S_{10k+r} and A_{10k+r} respectively for all integers $k, r > 1$. We will also show that S_{10k+r} and A_{10k+r} can by symmetrically generated using a symmetric generating set.

2. PRELIMINARY RESULT:

Lemma: 2.1 [3] The Mathieu group M_{10} of order 720 can be generated as follows;

$$M_{10} = \langle (1,\ 2,\ 3,\ 4,\ 5)(6,\ 7,\ 8,\ 9,\ 10), (1,\ 7,\ 4,\ 9)(2,\ 10,\ 3,\ 6) \rangle.$$

3. PERMUTATIONAL GENERATING SET OF S_{10k+r} AND A_{10k+r}

Theorem: 3.1 S_{10k+r} and A_{10k+r} can be generated using the Mathieu group M_{10} and an element of order $k+r$ in S_{10k+r} and A_{10k+r} respectively.

Proof: Let

X = (1, …,5) (6, …, 10) … (10(k-1) +1, …, 10(k-1) +5) (10(k-1) +6 … , 10(k-1) +10), Y =(1,7,4,9)(2,10, 3, 6) … (10(k-1)+1, 10(k-1)+7, 10(k-1)+4, 10(k-1)+9) (10(k-1)+2, 10(k-1)+10, 10(k-1)+3, 10(k-1)+6) and Z = (5, 15, …, 10 (k-1) +5, 10k+1, …, 10k+r), be three permutations, the first is of order 5, the second is of order 4 and the third is of order $k+r$. Let $H = \langle X, Y \rangle$. By Lemma 2.1, $H \cong M_{10}$. Let $G = \langle X, Y, Z \rangle$

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Let $\lambda = ([Z_1, Z_2] * [Z_1, Z_2]^z)^2 = (10k+1, 10k+2, 10k+3)$. We have the following two cases;

Case (1): If *r* is an odd integer. Let $\beta = X(XY)^2 ZYZ_1$. It is not difficult to show that β = (1, 9,2,6,7 …, 10 (*k*-2)+8,10*k*+2, ….,10*k*+*r* ,…, 10*k*+1, 10*k*+3) which is a cycle of order 10*k*+*r* .Now, if f *k* is an odd integers,

Then $G = \langle \beta, \lambda \rangle \cong S_{10k+r}$. Otherwise, $G = \langle \beta, \lambda \rangle \cong A_{10k+r}$.

Case (2): If *r* is an even integer. Let $\sigma = X(XY)^2 ZYZ_1 Z$. It is not difficult to show that σ = (1,9,2,6,7 …, 10(*k*-2)+8,10*k*+1, 10*k*+3, ...,10*k*+*r* , 5,…, 10*k*+(*r*-1)) which is a cycle of order 10*k*+*r*. Now, if *k* is an even integers, then $G = \langle \sigma, \lambda \rangle \cong S_{10k+r}$ .Otherwise, $G = \langle \sigma, \lambda \rangle \cong A_{10k+r}$. ◊

Corollary 3.2: Let $G = \langle Y, Z \rangle$, where *Y* and *Z* are the elements described in the previous theorem.
Then G$\cong C_4 \times C_{k+r}$, for all integers $k, r > 1$.

Proof: Since *Y, Z* are disjoint permutations of orders 4 and *k* + *r* respectively, then, it is clear that,

$G = \langle Y, Z \rangle \cong C_4 \times C_{k+r}$. ◊

Note: Since T = (5, 15… 10(*k*-1) +5, 10*k*+1… 10*k*+*r*), is in S_{10k+r} or A_{10k+r} depending on *r* then this element is going to be used in the following theorem.

4. SYMMETRIC GENERATING SET OF S_{10k+r} AND A_{10k+r} .

Theorem: 4.1 Let *T* be the element described above. For all integers $k, r > 1$, S_{10k+r} and A_{10k+r} can be symmetrically generated using the symmetric generating set $\Gamma = \{T_0, T_1, T_2, ..., T_5\}$, where $T_0 = T$ and $T_i = T^{X^i}$ for all $1 \le i \le 5$, where X be the element described in Theorem 3.1.

Proof: Let $G = \langle \Gamma \rangle$, let $\sigma = ([T, T_1] * [T_1, T_2]) * ([T_2, T_3] * [T_3, T_4])^{-1} = (1,2,3,4,5)$ Conjugating by $T_i = T^{X^i}$ for all $1 \le i \le 5$ .We can get the element τ = (1, …, 5)(6, …, 10)(10(*k*-1)+1, …, 10(*k*-1)+5).

Hence $\zeta = \tau T_1 = (1, 2, 3, ..., 10k+r)$. Let $\lambda = ([T_1, T_2] * [T_2, T_3])^{-1} = (1, 2, 3)$.

Now, if both of *k* and *r* are either even or odd integers, then $G = \langle \zeta, \lambda \rangle \cong S_{10k+r}$. Otherwise, $G = \langle \zeta, \lambda \rangle \cong A_{10k+r}$.

Corollary: 4.2 Let $\Gamma = \{T_0, T_1, T_2, \ldots, T_4\}$ be the symmetric generating set which has been described in the previous theorem, If we remove *m* elements of the set Γ for all $1 \le i \le 3$ then the resulting set generates $S_{[10-(2-m)]k+r}$ or $A_{[10-(2-m)]k+r}$,depending on *k* and *r*. If we remove 4 elements then the resulting set generates C_{2k+r} .

Proof: Let $\Gamma_1 = \{T, T_1\}$, let $\alpha_1 = [T, T_1] = (1, 5)(10k+1, 10k+2)$,

$\beta_1 = (T^{-1}[T, T_1]^{\alpha_1})T^2 = (5, 10, 15)$. Let $\beta_i = (\beta_i - 1)^{T^2}$ for all $2 \le i \le k$

Let $\xi = \beta_k \times \beta_{k-1} \times \ldots \times \beta_1$, then;

$\xi = (5, 10, 15, ... 10(k-2)+5, 10(k-1)+5, 10k+1)$, of order $2k+1$.

$\tau = T_1\xi = (1,\ 6,\ ...,\ 10(k-1)+1, ..., 10k+1,\ ...,\ 10k+r)$. Let: $m_1 = [T_1,\ T_2]$ and $m_2 = m_1^{T_1}$. Let $\delta = m_2 m_1$ and $\zeta = (\delta^{T_3}\delta)^2$, then $\sigma = (\zeta)^{\tau^{-1}} = (5, 10(k-1)+1, 10k+r)$

Now, if both of *k* and *r* are either even or odd integers, then $\langle \Gamma_1 \rangle = \langle \tau, \sigma \rangle \cong S_{4k+r}$. Otherwise, then $\langle \Gamma_1 \rangle = \langle \tau, \sigma \rangle \cong A_{4k+r}$ .Let $\Gamma_2 = \{T, T_1, T_2\}$,let $\lambda_1 = TT_1^{-1} = (5,\ 10, \ldots, 10(k-1)+5,\ ...,\ 1,\ ...\ ,\ 10k+r)$,

which is a cycle of order $4k+r$ and $\lambda_2 = \lambda_1 T_2 = (5,\ 10,\ ...,\ 10(k-1)+5,\ ...,\ 1,\ ...\ , 10k+1,\ ...,\ 10k+r)$,

which is a cycle of order $6k+r$. Now as in the previous the case , $\langle \Gamma_2 \rangle \cong S_{6k+r}$ or A_{6k+r} depending whither *r* is an odd or even integers. The rest of the proof goes the same.

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