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DYNAMICAL PROPERTIES OF PIEZO-ELECTRIC MICROSTRETCH SOLID SUBJECTED TO LASER AND RAMP TYPE HEATING SOURCE

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ABSTRACT

This research paper is concerned with the thermomechanical interactions of ultra-laser heat source with the piezo-electric microstretch thermoelastic material. The medium is subjected to mechanical and thermal boundary conditions. Integral transform technique has been applied to the basis equations to solve the problem.

Keywords: *Piezo-electric, ultra-laser heat source, Microstretch thermoelastic, normal source, thermal source.*

1. INTRODUCTION

Now a days, the smart materials have a great importance in engineering, technology and sciences. The important features of smart materials are due to their internal molecular structures known as smart structure for example sensors, actuators etc. One of the smart materials currently under research applications are piezo-electric materials. The piezo-electric substance are those which generate electricity (known as piezo-electricity) in response to mechanical stress. Such type of materials are used in actuators and sensors due to their direct and converse piezo-electric effects. To ensure that the piezo-electric appliances are functional in extreme temperature conditions, the thermal effects are to be considered in mathematical model development. So, a result of these electrical-thermal-mechanical coupling thermoelastic theories of piezo-electric materials have been developed. First of all a theory of piezo-electricity was developed by Mindlin (1961). Mindlin (1974) write the basic governing relations for piezo-electric thermoelastic solid. Later Nowacki (1978) deduced the physical laws and theorems for thermo piezo-electric substances. Later Chandrasekharaiah (1984) extended this theory also including the finite speed of thermal disturbances. The thermoelastic theory of piezo-electric materials was applied to composite plate by Tauchert (1992). Recently some more problems related to piezo-electric thermoelastic materials have been investigated by Othman & Ahmed (2016) and Vashishth & Sukhija (2017).

Eringen (1999) presented micropolar piezoelectricity and magnetoelasticity. Eringen (2003) introduced the electromagnetic theory of microstretch thermoelasticity. The various applications of this theory are in porous elastic bodies, animal bones, synthetic materials having microscopic components etc. The special cases of this theory are the theory of piezoelectricity and the theory of magnetoelasticity. The materials having linear coupling between mechanical and electric field are known as piezoelectric materials. There are wide use of these materials in intelligent structure systems, ultrasonic transducers, piezoelectric composite structures, loudspeakers etc. Iesan (2006) developed the linear theory of microstretch piezoelectricity and established uniqueness theorem and reciprocity relation.

Eringen (1999) developed the theory and basic equations of microstretch thermoelastic solids. Microstretch continuum is a model for Bravais lattice having basis on the atomic level and two phase dipolar substance having core on macroscopic level. Examples of microstretch thermoelastic materials are composite materials filled with chopped elastic fibers, porous elastic fluids whose pores have gases or inviscid liquids, or other elastic inclusions and liquid-solid crystal.

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Laser technologies have a lot of utilities in medical science, industries, metallurgies and nondestructive testing and evolution. High rated thermal processes are interesting in the development of theories of thermoelasticity, due to thermal- mechanical coupling. The thermal shock creates very fast movements in the internal molecular particles, which causes rise in very significant inertial forces, and in vibration. The ultra-short lasers have pulse durations ranging from nanoseconds to femto seconds. In irradiation of ultra-short pulsed laser, the high intensity energy flux and ultra-short duration lead to a very large thermal gradients. So, in these cases, Fourier law of heating is no longer valid. Rose (1984) developed an analytical mathematical basis for point laser source. Scruby *et al.* (1990) studied the point source ultrasonic generation by lasers. A new laser generation model was presented by Spicer (1990) and McDonald (1990). Various other authors Kim (1997), Chen (2002), Al-Huniti and Al-Nimr (2004) studied many problems related to laser ultrasound in thermoelastic materials. Thermoelastic behavior in metal plates due to laser interactions using fractional theory of thermoelasticity was studied by Ezzat *et al.* (2012). Comparison in context of four theories of thermoelasticity was presented by Youssef *et al.* (2014). A generalized thermoelastic diffusion problem for a thick plate irradiated by thermal laser was discussed by Elhagary (2014). Kumar, Kumar and Singh (2015) recently studied the thermo mechanical interactions of an ultra-laser pulse with microstretch thermoelastic medium.

This present research is devoted to the two-dimensional interactions of input ultra-laser heat source in a piezo-electric microstretch thermoelastic medium. The integral transform technique has been applied to derive the expressions for the displacement components, couple stress, temperature and microstress distribution under various sources. Some special cases have been deduced from the present investigation.

Basic equations:

Following Iesan (2006), Iesan & Quintanilla (2007) and Al-Qahtani and Dutta (2008), the field equations and constitutive relations for a homogeneous, isotropic piezoelectric thermo-microstretch solid are:

$$(\lambda+\mu)\nabla(\nabla.\boldsymbol{u})+(\mu+K)\nabla^2\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}+\lambda_0\nabla\phi^*-\beta_0\nabla\tau=\rho\ddot{\boldsymbol{u}}, \tag{1.1}$$

$$(\gamma\nabla^2-2K)\boldsymbol{\phi}+(\alpha+\beta)\nabla(\nabla.\boldsymbol{\phi})+K\nabla\times\boldsymbol{u}=\rho j\ddot{\boldsymbol{\phi}}, \tag{1.2}$$

$$(\alpha_0\nabla^2-\lambda_3)\phi^*-\lambda_2\nabla^2\psi+\nu_1\nabla^2\tau-\lambda_0\nabla.\boldsymbol{u}+c_0\frac{\partial}{\partial t}\tau=\frac{\rho j_0}{2}\ddot{\phi}^*, \tag{1.3}$$

$$\left(n_1K^*+n_2\frac{K_1}{T_0}\right)\nabla^2\tau-\beta_0(\nabla.\dot{\boldsymbol{u}}-Q)-a\ddot{\tau}-c_0\dot{\phi}^*+\nu_1\nabla^2\phi^*-\nu_3\nabla^2\psi=0, \tag{1.4}$$

$$\lambda_2\nabla^2\phi^*+\chi\nabla^2\psi+\nu_3\nabla^2\tau=0, \tag{1.5}$$

$$E_i=-\psi_{,i}, \tag{1.6}$$

$$t_{ij}=\left(\lambda_0\phi^*+\lambda u_{r,r}\right)\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\epsilon_{ijk}\phi_k\right)-\beta_0\delta_{ij}T, \tag{1.7}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+b_0\epsilon_{mji}\phi^*_{,m}+\lambda_1\epsilon_{ijk}E_k+\nu_2\epsilon_{ijk}\tau_{,k}\ , \tag{1.8}$$

$$\lambda_i^*=\alpha_0\phi^*_{,i}+b_0\epsilon_{ijm}\phi_{j,m}+\lambda_2E_i+\nu_1\tau_{,i}, \tag{1.9}$$

$$D_k=\lambda_1\epsilon_{ijk}\phi_{j,i}-\lambda_2\phi^*_{,k}-\nu_3\tau_{,k}+\chi E_k, \tag{1.10}$$

λ,μ are Lame's constants, $\alpha,\ \beta,\ \gamma,\ \lambda_0,\ \alpha_0,\ b_0$ are Microstretch constants, K is Thermal conductivity, $\lambda_1,\ \lambda_2,\upsilon_1,\upsilon_2,\upsilon_3$ are material constants, $\boldsymbol{u}$ is Displacement vector, $\boldsymbol{\phi}$ is microrotation vector, ϕ^* is Scalar microstretch, T represents temperature and $\dot{\tau}=T$, T_0 is reference temperature, K^* is the coefficient of thermal conductivity, c^* is specific heat at constant strain, j is the microinertia, j_0 is microinertia for the microelements, m_{ij} are components of couple stress, t_{ij} are components of stress, λ_i^* is microstress tensor, D_k is dielectric displacement vector, β_0 is the relaxation time, ψ is electric potential, n_1,n_2 are piezo-electric parameters, χ represents the dielectric susceptibility.

The laser pulse irradiation in the medium can be written by the following mathematical expression:

$$Q=I_0f(t)g(x_1)h(x_3) \tag{1.11}$$

Here, $f(t)=\frac{t}{t_0^2}e^{-\left(\frac{t}{t_0}\right)},\ g(x_1)=\frac{1}{2\pi r^2}e^{-\left(\frac{x_1^2}{r^2}\right)},\ h(x_3)=\gamma^*e^{-\gamma^*x_3}$

And I_0 is the energy absorbed, t_0 is the pulse rise time and r is the beam radius.

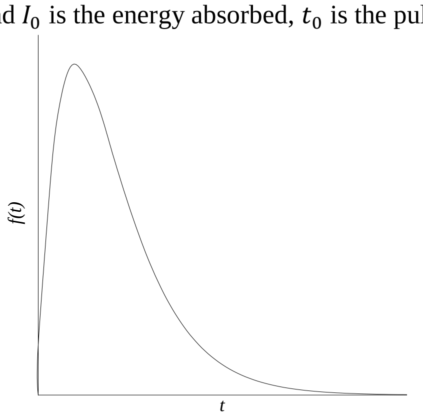

Figure-1: Temporal profile of $f(t)$

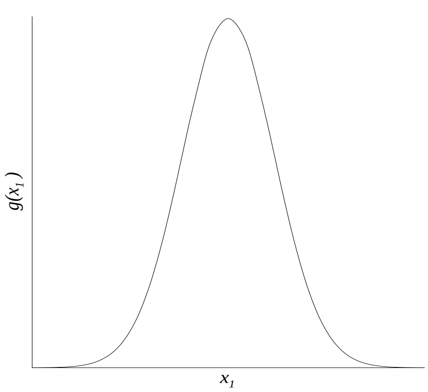

Figure-2: Profile of $g(x_1)$

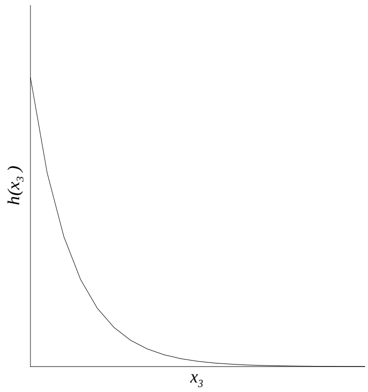

Figure-3: Profile of $h(x_3)$

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinates and a superposed dot (" ˙ ") denotes the derivative with respect to time respectively.

2. FORMULATION OF THE PROBLEM

A rectangular Cartesian coordinate system $OX_1X_2X_3$ having origin on x_3-axis with x_3-axis pointing vertically downward the medium is considered. A ramp type heating source is supposed to be acting on the origin of this rectangular Cartesian coordinate system.

Further we consider the plane strain problem with all the field variables depending on (x_1, x_3, t). For such two dimensional problems, we take:

$$\boldsymbol{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \ \boldsymbol{E} = (E_1, 0, E_3) \tag{2.1}$$

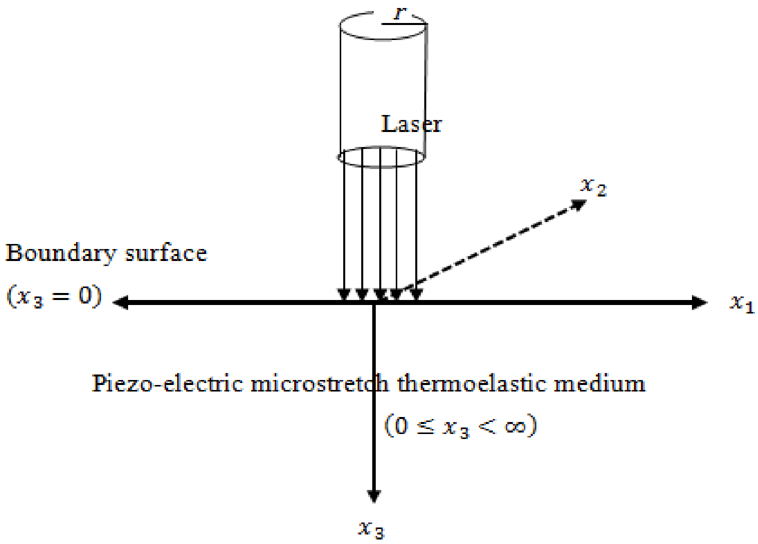

Figure- 4: Geometry of the problem

Also, it is convenient to define in equations (1.1)-(1.6) the following dimensionless quantities:

$$(x_1', x_3', u_1', u_3') = \frac{1}{L_0}(x_1, x_3, u_1, u_3), \ \phi_i' = \frac{\rho c_1^2}{\beta_1 T_0}\phi_i, \phi^{*\prime} = \frac{\rho c_1^2}{\beta_1 T_0}\phi^*, \tau' = \frac{c_1}{L_0 T_0}, t' = \frac{c_1}{L_0}t, \ t_{ij}' = \frac{1}{\rho c_1^2}t_{ij}, c_1^2 = \frac{\lambda+2\mu+K}{\rho},$$
$$m_{ij}^* = \frac{1}{\rho c_1^2 L_0} m_{ij} \ , \tag{2.2}$$

Making use of (2.1) in equations (1.1)-(1.6) and with the help of (2.2), we obtain:

$$a_1\nabla^2 u_1 + (1 - a_1)\frac{\partial}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) - a_2\frac{\partial \phi_2}{\partial x_3} + a_3\frac{\partial \phi^*}{\partial x_1} - a_4\frac{\partial \tau}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{2.3}$$

$$a_1\nabla^2 u_3 + (1 - a_1)\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + a_2\frac{\partial \phi_2}{\partial x_1} + a_3\frac{\partial \phi^*}{\partial x_3} - a_4\frac{\partial \tau}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{2.4}$$

$$a_5\nabla^2\phi_2 + a_2\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - 2a_2\phi_2 = a_6\frac{\partial^2 \phi_2}{\partial t^2}, \tag{2.5}$$

$$(a_7\nabla^2 - a_8)\phi^* - a_9\nabla^2\psi + a_{10}\nabla^2\tau - a_3\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + a_{11}\frac{\partial \tau}{\partial t} = a_{12}\ddot{\phi}^*, \tag{2.6}$$

$$\left(n_1K_1 + n_2K_2\frac{\partial}{\partial t}\right)\nabla^2\tau - a_4\frac{\partial}{\partial t}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) - a_{13}\frac{\partial^2 \tau}{\partial t^2} - a_{11}\frac{\partial \phi^*}{\partial t} + a_{10}\nabla^2\phi^* - a_{14}\nabla^2\psi = Q_0 f^*(x_1, t)e^{-\gamma^* x_3}, \tag{2.7}$$

$$a_9\nabla^2\phi^* + \upsilon\nabla^2\psi + a_{14}\nabla^2\tau = 0, \tag{2.8}$$

Here, $a_1 = \frac{\lambda+\mu}{\rho c_1^2}, a_2 = \frac{K}{\rho c_1^2}, a_3 = \frac{\lambda_0}{\rho c_1^2}, a_4 = \frac{\beta_0 T_0}{\rho c_1^2}, a_5 = \frac{\gamma_1}{\rho c_1^2 L_0^2}, a_6 = \frac{I_1}{\rho L_0^2}, a_7 = \frac{a_0}{\rho c_1^2 L_0^2}, a_8 = \frac{\lambda_3}{\rho c_1^2}, a_9 = \frac{\lambda_2\psi_0}{\rho c_1^2 L_0^2},$

$$a_{10} = \frac{\upsilon_1 T_0}{\rho c_1^3 L_0^2}, a_{11} = \frac{c_0 T_0}{\rho c_1^2}, a_{12} = \frac{j_0}{\rho L_0^2}, a_{13} = \frac{a T_0^2}{\rho c_1^2}, a_{14} = \frac{\upsilon_{3}\psi_0 T_0}{\rho c_1^3 L_0}, K_1 = \frac{K^* T_0^2}{\rho c_1^4}, K_2 = \frac{K_1 T_0}{\rho c_1^3 L_0}, \nu = \frac{\chi\psi_0^2}{\rho c_1^2 L_0^2},$$

$$Q_0 = \frac{\rho c_1^4}{\beta_1 K^* \omega^{*2}}Q, f(x_1, t) = \left[t + \epsilon\tau_0\left(1 - \frac{t}{t_0}\right)\right]e^{-\left(\frac{x_1^2}{r^2} + \frac{t}{t_0}\right)}, \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$$

The relations connecting displacement components and microtemperature components to the potential functions in dimensionless form are:

$$u_1 = \frac{\partial \psi_1}{\partial x_1} + \frac{\partial \psi_2}{\partial x_3}, \ u_3 = \frac{\partial \psi_1}{\partial x_3} - \frac{\partial \psi_2}{\partial x_1}, \tag{19}$$

Here, the primes have been suppressed.

Using the relations defined by (2.9) in equations (2.3) - (2.8) and rewriting, we obtain:

$$\left[\nabla^2 - \frac{\partial^2}{\partial t^2}\right]\psi_1 + a_3\phi^* - a_4\frac{\partial \tau}{\partial t} = 0, \tag{20}$$

$$\left(a_1\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\psi_2 - a_2\phi_2 = 0 \tag{21}$$

$$a_2\nabla^2\psi_2 + \left(a_5\nabla^2 - 2a_2 - a_6\frac{\partial^2}{\partial t^2}\right)\phi_2 = 0 \tag{22}$$

$$\left(a_{10}\nabla^2 - a_{11}\frac{\partial}{\partial t}\right)\phi^* - a_{14}\nabla^2\psi - a_4\frac{\partial}{\partial t}\nabla^2\psi_1 + \left[\left(n_1K_1 + n_2K_2\frac{\partial}{\partial t}\right)\nabla^2 - a_{13}\frac{\partial^2}{\partial t^2}\right]\tau = 0 \tag{23}$$

$$\left(a_7\nabla^2 - a_8 - a_{12}\frac{\partial^2}{\partial t^2}\right)\phi^* + \left(a_{10}\nabla^2 + a_{11}\frac{\partial}{\partial t}\right)\tau - a_9\nabla^2\psi - a_3\nabla^2\psi_1 = 0 \tag{24}$$

$$a_9\nabla^2\phi^* + \nu\nabla^2\psi + a_{14}\nabla^2\tau = 0 \tag{25}$$

3. SOLUTION OF THE PROBLEM

Laplace transform and Fourier transform respectively are defined by the following relations:

$$\bar{f}(s, x_1, x_3) = \int_0^\infty f(t, x_1, x_3)e^{-st}dt, \tag{3.2}$$

$$\hat{f}(x_3, \xi, s) = \int_{-\infty}^\infty \bar{f}(s, x_1, x_3)e^{\iota\xi x_1}dx_1, \tag{3.3}$$

applying Laplace transform and Fourier transform defined by (3.2) & (3.3) on resulting equations, yield:

$$(D^2 - \xi_{11})\hat{\psi}_1 - a_4\hat{\tau} + a_3\widehat{\phi^*} = 0 \tag{3.4}$$

$$[-n_{11}(D^2-\xi^2) + a_{13}s^2]\hat{\tau} + a_4s(D^2-\xi^2)\hat{\psi}_1 + [a_{10}D^2 - a_{11}]\widehat{\phi^*} - a_{14}(D^2-k^2)\hat{\psi} = Q_1e^{-\gamma^* x_3} \tag{3.5}$$

$$[a_7(D^2-\xi^2) - a_8 - a_{12}s^2]\widehat{\phi^*} + [a_{10}(D^2-\xi^2) + sa_{11}]\hat{\tau} - (D^2-\xi^2)\left(a_3\hat{\psi}_1 + a_9\hat{\psi}\right) = 0 \tag{3.6}$$

$$a_9(D^2-\xi^2)\widehat{\phi^*} + \upsilon(D^2-\xi^2)\hat{\psi} + a_{14}(D^2-\xi^2)\hat{\tau} = 0 \tag{3.7}$$

$$[a_1(D^2 - \xi^2) - s^2]\hat{\psi}_2 - a_2\hat{\phi}_2 = 0 \tag{3.8}$$

$$a_2(D^2 - \xi^2)\hat{\psi}_2 + (a_5(D^2 - \xi^2) - 2a_2 + a_6s^2)\widehat{\phi_2} = 0 \tag{3.9}$$

$\xi_{11} = \xi^2 - s^2, n_{11} = n_1k_1 + sn_2k_2\,, a_{15} = a_1\xi^2 + s^2, a_{16} = a_5\xi^2 + a_6s^2 + 2a_2\,, a_{17} = a_{10}\xi^2 + a_{11}s, a_{18} = a_7\xi^2 + a_{12}s^2 + a_8, a_{16} = -a_{10}\xi^2 + a_{11}s$

Eliminating $\bar{\psi}_1, \overline{\phi^*}$ & $\hat{\tau}$, $\bar{\psi}, \overline{\phi^*}$ & $\hat{\tau}$, $\bar{\psi}, \bar{\psi}_1$ & $\hat{\tau}$ and, $\bar{\psi}, \bar{\psi}_1, \overline{\phi^*}$ respectively from the equations (3.4)-(3.7), we obtain:

$$[\boldsymbol{D}^8 + A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E]\hat{\psi} = f_1e^{-\gamma^* x_3} \tag{3.10}$$

$$[\boldsymbol{D}^8 + A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E]\hat{\psi}_1 = f_2e^{-\gamma^* x_3} \tag{3.11}$$

$$[\boldsymbol{D}^8 + A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E]\widehat{\phi^*} = f_3e^{-\gamma^* x_3} \tag{3.12}$$

$$[\boldsymbol{D}^8 + A\boldsymbol{D}^6 + B\boldsymbol{D}^4 + C\boldsymbol{D}^2 + E]\hat{\tau} = f_4e^{-\gamma^* x_3} \tag{3.13}$$

Also eliminating ϕ_2 equations (3.8)-(3.9) yield

$$[\boldsymbol{D}^4 + F\boldsymbol{D}^2 + G]\widehat{\psi_2} = 0\,, \tag{3.14}$$

Here, $\boldsymbol{D} = \frac{d}{dx_3}$ and $A, B, C, E, F, G, f_1, f_2, f_3, f_4$ & f_5 are mentioned in Appendix A.

The solutions of the equations (3.10)-(3.14) satisfying the radiation conditions that $\left(\hat{\psi}, \hat{\psi}_1, \hat{\tau}, \widehat{\phi_2}, \widehat{\psi_2}, \widehat{\phi^*}\right) \to 0$ as $x_3 \to \infty$ are given by:

$$\left(\hat{\psi}, \hat{\psi}_1, \widehat{\phi^*}, \hat{\tau}\right) = \sum_{i=1}^4 (1, \alpha_{1i}, \alpha_{2i}, \alpha_{3i})c_ie^{-m_ix_3} + \left(\frac{f_1}{f_5}, \frac{f_2}{f_5}, \frac{f_3}{f_5}, \frac{f_4}{f_5}\right)e^{-\gamma^* x_3} \tag{3.15}$$

$$\left(\hat{\psi}_2, \widehat{\phi_2}\right) = \sum_{i=5}^6 (1, \alpha_{4i})c_ie^{-m_ix_3} \tag{3.16}$$

Here, m_i^2 $(i = 1,2,3,4)$ are the roots of the characteristic equation given (3.10) and $m_l^2(l = 5,6)$ are the roots of the characteristic equation of equation (3.14).

$$\alpha_{1i} = -\frac{\Delta_{2i}}{\Delta_{1i}},\ \alpha_{2i} = \frac{\Delta_{3i}}{\Delta_{1i}},\ \alpha_{3i} = -\frac{\Delta_{4i}}{\Delta_{1i}},\ \ i = 1,2,3,4 \text{ and } \alpha_{4i} = \frac{a_1m_i^2 + a_{15}}{a_2}, i = 5,6$$

Here, $\Delta_{1i}, \Delta_{2i}, \Delta_{3i}, \Delta_{4i}$ are defined in Appendix B.

4. BOUNDARY CONDITIONS

We consider a thermal source at the boundary surface $x_3 = 0$, mathematically, these can be written as:

$$t_{33} = 0, t_{31} = 0, m_{32} = 0, \lambda_3^* = 0, T = R(t)\delta(x_1), D_3 = 0 \tag{4.1}$$

APPLICATION:

Thermal boundary conditions (Ramp type heating):

Let us consider the boundary surface of piezo-electric microstretch thermoelastic medium is under the effect of ram type heating conditions as defined by Youssef and Al-Lehaibi []. So, the temperature distribution in the medium at $x_3 = 0$ depends on x_1 & t and is expressed in following manner:

$$T = R(t)\delta(x_1) \tag{4.2}$$

Here,

$$R(t) = \begin{cases} 0 & t \le 0 \\ T_1 \dfrac{t}{t_0} & 0 < t \le t_0 \\ T_1 & t > t_0 \end{cases}$$

And $\delta(x_1)$ is Dirac delta function.

Applying the Laplace transform on (4.2) followed by Fourier transform, we obtain the following:

$$\hat{T}(\xi, s) = T_1 \left(\frac{1-e^{-st_0}}{t_0 s^2}\right) \tag{4.3}$$

Substituting the values of $\hat{\phi}, \widehat{\phi^*}, \hat{T}, \hat{\psi}, \widehat{\phi_2}$ from the equations (3.15)-(3.16) in the boundary condition (4.1) and using (1.7)-(1.10), (2.1)-(2.2), (3.1)-(3.3) and solving the resulting equations for c_i by matrix method, we obtain:

$$\widehat{t_{33}} = \sum_{i=1}^{6} G_{1i} e^{-m_i x_3} + M_1 e^{-\gamma^* x_3} \tag{4.2}$$

$$\widehat{t_{31}} = \sum_{i=1}^{6} G_{2i} e^{-m_i x_3} + M_2 e^{-\gamma^* x_3} \tag{4.3}$$

$$\widehat{m_{32}} = \sum_{i=1}^{6} G_{3i} e^{-m_i x_3} + M_3 e^{-\gamma^* x_3} \tag{4.4}$$

$$\widehat{\lambda_3^*} = \sum_{i=1}^{6} G_{4i} e^{-m_i x_3} + M_4 e^{-\gamma^* x_3}, \tag{4.5}$$

$$\hat{T} = \sum_{i=1}^{6} G_{5i} e^{-m_i x_3} + M_5 e^{-\gamma^* x_3}, \tag{4.6}$$

$$\widehat{D_3} = \sum_{i=1}^{6} G_{6i} e^{-m_i x_3} + M_6 e^{-\gamma^* x_3}, \tag{4.7}$$

Here $G_{mi} = g_{mi} C_i$, $i = 1,2, \dots, 6$.
G_{rs} , $(r, s = 1,2, \dots, 6)$ and M_r , $(r = 1,2,3, \dots, 6)$ are described in Appendix C.

Inversion of the transform:
The transformed displacements, stresses and temperature changes are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence these are of the form $f(s, \xi, x_3)$. To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar (2005).

5. NUMERICAL RESULTS AND DISCUSSIONS

In order to illustrate the theoretical results obtained in the previous sections, some numerical results are presented. For numerical computation, the values for relevant parameters are taken for Aluminum epoxy like material, the values of physical parameters are given below:

$\lambda = 7.59 \times 10^9\ Nm^{-2}$ $\mu = 1.89 \times 10^9\ Nm^{-2}, K = 1.49 \times 10^7\ Nm^{-2},\ \rho = 2190\ Kgm^{-3}, j = 0.2 \times 10^{-19} m^2,$
$\gamma = 2.63 \times 10^3\ N, \lambda_1 = 0.5 \times 10^{10}\ Nm^{-2}, T_0 = 298\ K, I = 19.6 \times 10^{-8} m^2$
$K^* = 1.7 \times 10^6\ Jm^{-1}s^{-1}K^{-1}, a = 9.6 \times 10^2 m^2 s^{-2} K^{-1}, b = 32 \times 10^2\ Kg^{-1} m^5 s^{-2}, j_0 = 0.19 \times 10^{-6} m^2,$
$\alpha_0 = 0.9 \times 10^3\ N,\quad b_0 = 9.1 \times 10^2\ N,\quad \lambda_0 = 0.5 \times 10^9\ Nm^{-2}, \lambda_1 = .5 \times 10^9 Cm^{-1}, \lambda_2 = 1.7 \times 10^4 Cm^{-1},$
$\lambda_3 = 0.7 \times 10^9 Nm^{-2}$
$\nu_1 = 0.3 \times 10^6\ Ns^{-1}, \nu_2 = 0.457 \times 10^9\ NK^{-1}s^{-1}, \nu_3 = 2.4 \times 10^3\ Cm^{-1}s^{-1}, \chi = 318, L_0 = 1m, \psi_0 = 1NmC^{-1}$

A comparison of the dimensionless form of the field variables for the cases of piezo microstretch thermoelastic medium with a laser pulse (PZMTL), piezo microstretch thermoelastic medium without a laser pulse (PZMT) and microstretch thermoelastic with laser (MTL) subjected to ramp type heating source is presented in Figures 5-10. The values of all physical quantities for both cases are shown in the range $0 \le x_1 \le 20$.

Solid lines, dash lines and lines with large dash corresponds to piezo microstretch thermoelastic medium with a laser pulse (PZMTL), piezo microstretch thermoelastic medium without a laser pulse (PZMT) and microstretch thermoelastic medium with laser (MTL) respectively. The computations were carried out in the absence and presence of laser pulse ($I_0 = 10^5, 0$) and on the surface of plane $x_3 = 1, t = 0.1$

Fig. 5 shows the variation of normal stress t_{33} with the distance x_1. It is noticed that for PZMTL and PZMT, the normal stress t_{33} show like behavior. Initially the value of normal stress for PZMT monotonically decreases as x_1 increases also for PZMTL t_{33} decreases monotonically. The value of t_{33} exhibit this trend near the application of the source due to the heating effect by laser and then remain oscillating for all values of x_1.

Fig. 6 displays the variation of tangential stress t_{31} with the distance x_1. It is noticed that the behavior of t_{31} for PZMTL and PZMT show similar trend. Initially t_{31} decrease monotonically for MTL which is different from that of the behavior of PZMTL and PZMT.

Fig. 7 shows the variation of couple stress m_{32} with distance x_1. The behavior and variation of m_{32} for PZMTL and PZMT remain almost similar to each other for all values of x_1 indicating that the effect of input laser heat source on couple tangential stress is not significant under normal source. However the opposite trend of variation of couple tangential stress m_{32} is observed in microstretch thermoelastic medium (MTL).

Fig. 8 depicts the variation of micro stress λ_3^* with distance x_1. The initial trend and variation of λ_3^* is similar to each other for PZMTL and PZMT and tends to zero away from the source. The curve representing the microstress in MTL is also similar but the magnitude of microstress in case of MTL is very small in comparison to PZMTL and PZMT.

Fig. 9 depicts the variation of D_3 with distance x_1. The behavior and variation of D_3 is similar to each other for PZMTL and PZMT. The initial trend of variation of $D_3$is monotonically decreasing which approaches to the boundary surface away from the point of application of ramp type heating source.

Fig. 10 displays the variation of temperature T with distance x_1. The values of temperature change for PZMTL show oscillatory trend while for PZMT the temperature change show a monotonically decreasing trend for initial values of x_1.

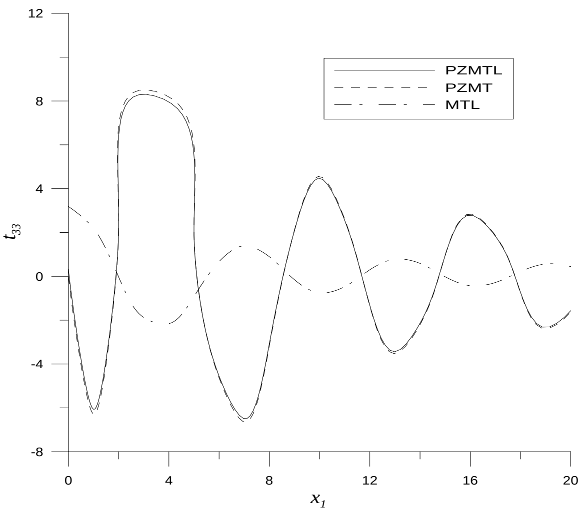

Figure-5: Variation of normal stress

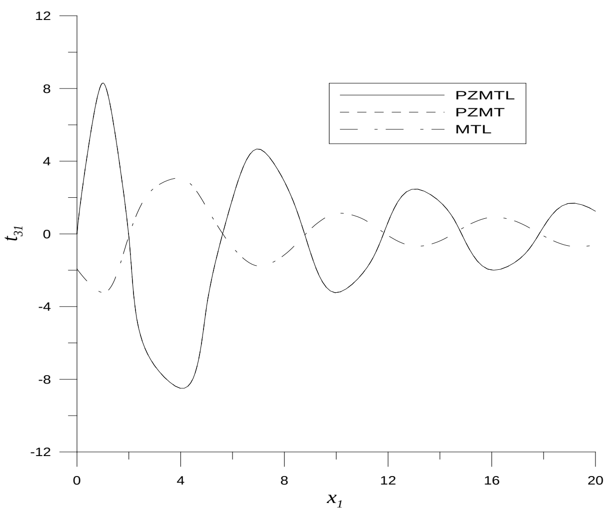

Figure-6: Variation of tangential stress

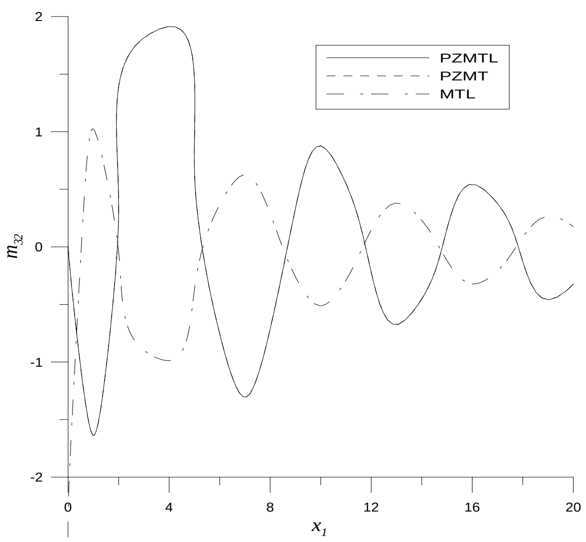

Figure-7: Variation of coupled tangential stress

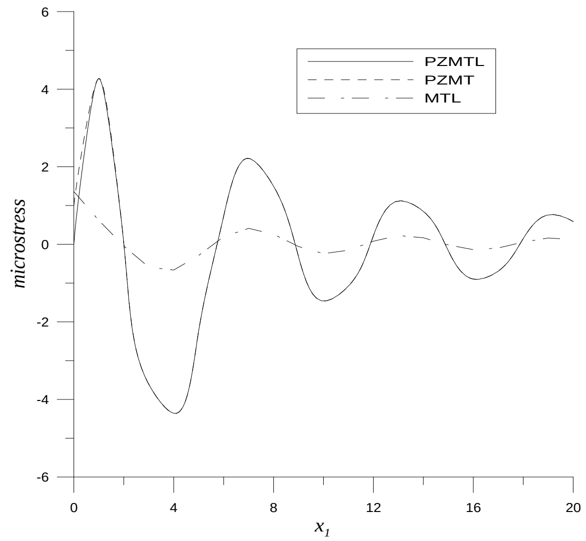

Figure-8: Variation of microstress

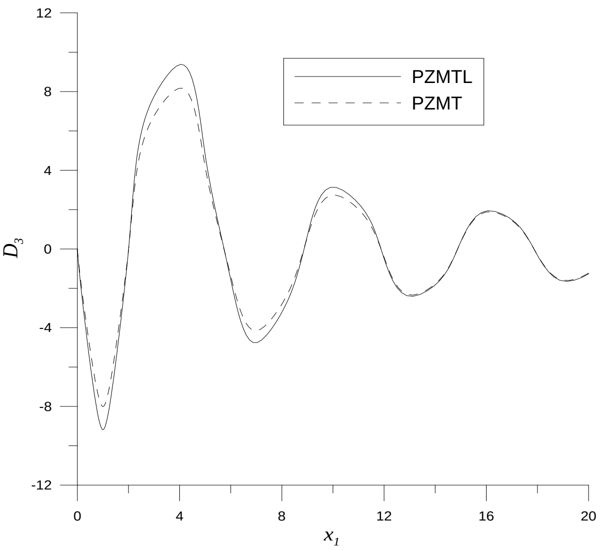

Figure-9: Variation of D_3

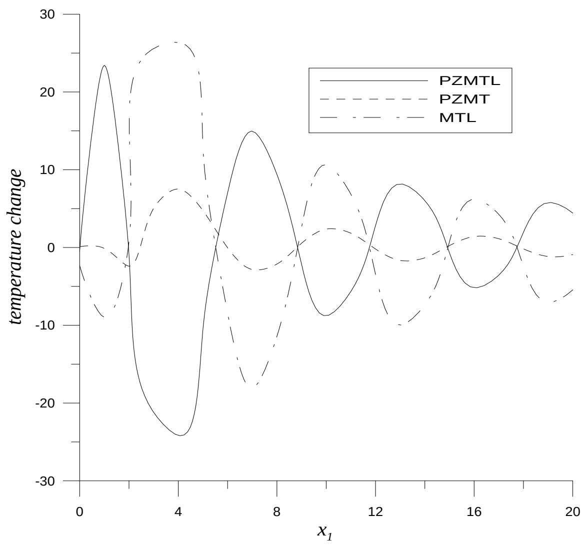

Figure-10: Variation of temperature

6. CONCLUSIONS

In this problem we have investigated the displacement components, stress components, dielectric component and temperature change in a thermo piezo-electric microstretch medium. Laplace transform and Fourier transform technique has been used to solve the set of equations mathematically. Theoretically computed variables are also exemplified through a specific model to present the results in the transformed domain.

This analysis of results obtained have some following conclusions:

(1) It can be concluded from the figures 5-13 that all the physical variables have nonzero values only in the bounded region. This indicates that all the results obtained here are in agreement with the generalized theory of thermoelasticity.
(2) It is clear from the results that the input laser heat source (value of I_0) has a significant role in the variation of all field quantities.
(3) If the piezo-electric parameters are absent and laser heat source is neglected then the results are obtained for generalized thermoelastic problem, then these results are in agreement with Elhagary [30].
(4) The variation of various stress components differs significantly due to the presence of normal force and due to the presence of thermal source.
(5) The temperature change is also affected due to input laser heat source as well as load/source applied.

The new model is employed in a piezo-electric microstretch thermoelastic medium as a new concept in the field of thermoelasticity. The subject becomes more interesting due to presence of an ultra-short input laser heat source. The method of solution in this research can be applied to a large no. of problems in engineering and science. It is hoped that this model will serve as more realistic model and will motivate the other authors to solve piezo-electric thermoelasticity.

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Appendix A:

$$A=\frac{LA2}{LA1},B=\frac{LA3}{LA1},C=\frac{LA4}{LA1},E=\frac{LA5}{LA1}$$

$$LA1=a_{17}a_{14}^2-a_{10}^2(a_{14}+\nu)-a_9a_{10}(n_{11}+a_{14})-a_7\nu n_{11}$$

$$LA2=-\xi^2(a_{17}a_{14}^2-a_{10}^2(a_{14}+\nu)-a_9a_{10}(n_{11}+a_{14})-a_7\nu n_{11})$$

$$LA3=a_{14}^2(a_3^2-a_{18})+a_{10}(a_9a_{13}+a_{14}a_{17})-a_9a_{14}a_{19}+\nu a_7a_{13}+\nu a_{10}(a_{17}-a_{19})+\nu a_{18}n_{11}-a_7a_{14}^2\xi_1+ a_{10}^2\xi_1(a_{14}+\nu)-\nu a_3^2n_{11}-a_7a_{14}^2\xi^2+a_{10}^2a_{14}\xi^2+a_7\xi^2n_{11}\nu+a_4^2(a_9a_{10}+a_7\nu)s^2+a_9a_{10}(a_{14}+n_{11})\xi_1+ a_7\nu n_{11}\xi_1+a_9a_{10}\xi^2(a_{14}+2n_{11})+a_3a_4s(2a_{10}\nu+(a_9+a_{10})a_{14})$$

$$LA4=\nu(a_{17}a_{19}-a_{13}(a_{18}+a_3^2)-a_{18}\xi^2n_{11}\nu)+a_{14}^2\left(2a_3^2\xi^2+a_{18}(\xi_1+\xi^2)\right)+\xi_1a_{14}\xi^2(a_7a_{14}-a_{10}^2)- a_4^2a_{18}\nu s^2+2a_3^2\xi^2\nu n_{11}-a_9a_{10}a_{13}\xi_1-a_{10}a_{17}\xi_1(a_{14}+\nu)+a_{19}\xi_1(a_9a_{14}+a_{10}\nu)-\xi_1\nu(a_7a_{13}+a_{18}n_{11})- a_{10}\xi^2(a_9a_{13}+a_{17}a_{14})+a_9(a_{19}a_{14}\xi^2-a_{10}\xi^4n_{11})+a_3a_4\nu s(a_{19}-a_{17})-s^2a_4^2\xi^2(2a_9a_{10}+a_7\nu)- a_9a_{10}\xi_1\xi^2(a_{14}+2n_{11})-a_7\xi_1\xi^2n_{11}\nu-2a_3a_4\xi^2s(a_9a_{14}+a_{10}^2+a_{10}\nu)$$

$$LA5=a_{14}^2(a_3^2\xi^4-a_{18}\xi_1\xi^2)-a_3^2\nu(a_{13}\xi^2+\xi^4n_{11})+\xi_1\nu(a_{13}a_{18}-a_{17}a_{19})+a_4^2s^2(a_9a_{10}\xi^4+a_{18}\xi^2\nu)+ \xi_1\xi^2(a_9a_{10}a_3+a_{14}a_{10}a_{17}-a_{14}a_9a_{19})+a_{10}a_9\xi_1\xi^4n_{11}+a_{18}\xi_1\xi^2n_{11}\nu+a_3a_4s\begin{pmatrix}(a_9+a_{10})a_{14}\xi^4+a_{14}\xi^2\nu\\ -a_{19}\xi^2\nu\end{pmatrix},$$

$$F=\frac{(a_2^2-a_1a_{16}-a_5a_{15})}{a_1a_5},G=(a_{15}a_{16}-a_2^2\xi^2)/a_1a_5$$

$$f_1=\left(\gamma^{*2}-\xi^2\right)^2\begin{vmatrix}a_4s & a_{10}\gamma^{*2}-a_{17} & -n_{11}\left(\gamma^{*2}-\xi^2\right)+a_{13}\\ -a_3 & a_7\gamma^{*2}-a_{18} & a_{10}\gamma^{*2}+a_{19}\\ 0 & a_9 & a_{14}\end{vmatrix},$$

$$f_2=\left(\gamma^{*2}-\xi^2\right)\begin{vmatrix}-a_{14} & a_{10}\gamma^{*2}-a_{17} & -n_{11}\left(\gamma^{*2}-\xi^2\right)+a_{13}\\ -a_{10} & a_7\gamma^{*2}-a_{18} & a_{10}\gamma^{*2}+a_{19}\\ \nu & a_9\left(\gamma^{*2}-\xi^2\right) & a_{14}\left(\gamma^{*2}-\xi^2\right)\end{vmatrix},$$

$$f_3 = \left(\gamma^{*2} - \xi^2\right)^2 \begin{vmatrix} -a_{14} & a_4 s & a_{10}\gamma^{*2} - a_{17} \\ -a_{10} & -a_3 & a_7\gamma^{*2} - a_{18} \\ \nu & 0 & a_9\left(\gamma^{*2} - \xi^2\right) \end{vmatrix},$$

$$f_4 = \left(\gamma^{*2} - \xi^2\right)^2 \begin{vmatrix} -a_{14} & a_4 s & -n_{11}\left(\gamma^{*2} - \xi^2\right) + a_{13} \\ -a_{10} & -a_3 & a_{10}\gamma^{*2} + a_{19} \\ \nu & 0 & a_{14}\left(\gamma^{*2} - \xi^2\right) \end{vmatrix},$$

$$f_5 = \gamma^{*8} + A{\gamma^*}^6 + B\gamma^{*4} + C\gamma^{*2} + E$$

Appendix B:

$$\Delta_{1i} = (m_i^2 - \xi^2)^2 \begin{vmatrix} a_4 s & a_{10}m_i^2 - a_{17} & -n_{11}(m_i^2 - \xi^2) + a_{13} \\ -a_3 & a_7 m_i^2 - a_{18} & a_{10}m_i^2 + a_{19} \\ 0 & a_9 & a_{14} \end{vmatrix},$$

$$\Delta_{2i} = (m_i^2 - \xi^2) \begin{vmatrix} -a_{14} & a_{10}m_i^2 - a_{17} & -n_{11}(m_i^2 - \xi^2) + a_{13} \\ -a_{10} & a_7 m_i^2 - a_{18} & a_{10}m_i^2 + a_{19} \\ \nu & a_9(m_i^2 - \xi^2) & a_{14}(m_i^2 - \xi^2) \end{vmatrix},$$

$$\Delta_{3i} = (m_i^2 - \xi^2)^2 \begin{vmatrix} -a_{14} & a_4 s & a_{10}m_i^2 - a_{17} \\ -a_{10} & -a_3 & a_7 m_i^2 - a_{18} \\ \nu & 0 & a_9(m_i^2 - \xi^2) \end{vmatrix},$$

$$\Delta_{4i} = (m_i^2 - \xi^2)^2 \begin{vmatrix} -a_{14} & a_4 s & -n_{11}(m_i^2 - \xi^2) + a_{13} \\ -a_{10} & -a_3 & a_{10}m_i^2 + a_{19} \\ \nu & 0 & a_{14}(m_i^2 - \xi^2) \end{vmatrix},$$

Appendix C:

$$b_1 = \frac{\lambda}{\rho c_1^2}, b_2 = \frac{2\mu + K}{\rho c_1^2}, b_3 = \frac{\lambda_0 L_0^2}{j_0^2 \rho c_1^2}, b_4 = \frac{\beta_0 T_0}{\rho c_1^2}, b_5 = \frac{\mu + K}{\rho c_1^2}, b_6 = \frac{\mu}{\rho c_1^2}, b_7 = \frac{K L_0^2}{j^2 \rho c_1^2}, b_8 = \frac{\gamma}{j^2 \rho c_1^2}, b_9 = \frac{b_0}{j_0^2 \rho c_1^2},$$
$$b_{10} = \frac{\lambda_1 \psi_0}{L_0^2 \rho c_1^2}, b_{11} = \frac{\nu_2 T_0}{L_0 \rho c_1^3}, b_{12} = \frac{\alpha_0}{j_0^2 \rho c_1^2}, b_{13} = \frac{b_0}{j^2 \rho c_1^2}, b_{14} = \frac{\lambda_2 \psi_0}{L_0^2 \rho c_1^2}, b_{15} = \frac{\nu_1 T_0}{L_0 \rho c_1^3}, b_{16} = \frac{\lambda_1 L_0}{j^2}, b_{17} = \frac{\lambda_2 L_0}{{j_0}^2},$$
$$b_{18} = \frac{\chi \psi_0}{L_0},$$
$g_{1i} = [b_1(m_i^2 - \xi^2) + b_2 m_i^2]\alpha_{1i} + b_3\alpha_{2i} - b_4 s\alpha_{3i}, g_{2i} = -\iota\xi b_2\alpha_{1i}m_i, g_{3i} = \iota\xi(b_9\alpha_{2i} + b_{10} + sb_1\alpha_{3i}),$
$g_{4i} = -m_i b_{12}\alpha_{2i} + m_i b_4 - m_i b_{15}\alpha_{3i}s, g_{5i} = s\alpha_{3i}, g_{6i} = b_{17}m_i\alpha_{2i} + m_i b_{18}$, for $i = 1,..4$
and $g_{1i} = -\iota\xi b_2 m_i$, $g_{2i} = (b_5 m_l^2 - b_6\xi^2) - b_5\alpha_{4i}$, $g_{3i} = -b_8\alpha_{4i}m_i$,
$g_{4i} = i\xi b_{13}\alpha_{4i}$,$g_{5i} = 0$, $g_{6i} = -i\xi b_{16}\alpha_{4i}$, for $i = 5,6$

$$M_1 = \left(\frac{\left[b_1\left(\gamma^{*2} - \xi^2\right) + b_2\gamma^{*2}\right]f_2 + b_3 f_3 - b_4 s f_4}{f_5}\right), M_2 = \frac{-\iota\xi b_2\gamma^* f_2}{f_5}, M_3 = \frac{\iota\xi(b_9 f_3 + b_{10} f_1 + s b_1 f_4)}{f_5}$$
$$M_4 = \frac{-\gamma^* b_{12} f_3 + m_i b_4 f_1 - \gamma^* b_{15} f_4 s}{f_5}, M_5 = \frac{s f_4}{f_5}, M_6 = \frac{(b_{17}\gamma^* f_3 + \gamma^* b_{18} f_1)}{f_5},$$

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