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A STUDY ON SOME OPERATORS OVER INTERVAL VALUED INTUITIONISTIC FUZZY SETS OF SECOND TYPE

K. RAJESH*, R. SRINIVASAN**

***Full-Time Research Scholar, Department of Mathematics,**
Islamiah College (Autonomous), Vaniyambadi, Tamil Nadu, India.

****Department of Mathematics,**
Islamiah College (Autonomous), Vaniyambadi, Tamilnadu, 635751, India.

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ABSTRACT

In this paper, we introduce the new operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties.

Key Words: *Fuzzy sets (FS), Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Sets of Second Type (IFSST), Interval Valued Fuzzy Sets (IVFS), Interval Valued Intuitionistic Fuzzy Sets (IVIFS), Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST).*

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1. INTRODUCTION

An Intuitionistic Fuzzy Set (IFS) for a given underlying set X were introduced by K. T. Atanassov [2] which is the generalization of ordinary Fuzzy Sets (FS) and he introduced the theory of Interval Valued Intuitionistic Fuzzy Set (IVIFS) and established some of their properties.

The present authors further introduced the new extension of IVIFS namely Interval Valued Intuitionistic Fuzzy Sets of Second Type [4]. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we introduce the new operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type and establish some of their properties. This paper is concluded in section 4.

PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1[8]: Let X be a non - empty set. A Fuzzy Set A in X is characterized by its membership function $\mu_A: X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of the element *x* in fuzzy set A, for each $x \in X$. It is clear that A is completely determined by the set of tuples

$$A = \{< x, \mu_A(x) > | \ x \in X\}$$

Definition 2.2[2]: Let X be a non- empty set. An intuitionistic fuzzy set (IFS) A in X is defined as an object of the following form.

$$A = \{< x, \mu_A(x), \nu_A(x) > | \ x \in X\}$$

Where the functions $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0, 1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

Corresponding Author: R. Srinivasan**, **Department of Mathematics,
Islamiah College (Autonomous), Vaniyambadi, Tamilnadu, 635751, India.

Definition 2.3[2]: Let a set X be fixed. An intuitionistic fuzzy set of second type (IFSST) A in X is defined as an object of the following form.

$$A = \{< x, \mu_A(x), \nu_A(x) > \mid x \in X\}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq \mu_A{}^2(x) + \nu_A{}^2(x) \leq 1$$

Definition 2.4[2]: Let X be an universal set with cardinality n. Let [0, 1] be the set of all closed subintervals of the interval [0, 1] and elements of this set are denoted by uppercase letters. If M ∈ [0, 1] then it can be represented as $M = [M_L, M_U]$, where M_L and M_U are the lower and upper limits of M. For M ∈ [0, 1], $\overline{M} = 1 - M$ represents the interval $[1 - M_L, 1 - M_U]$ and $W_M = M_U - M_L$ is the width of M.
An interval-valued fuzzy set (IVFS) A in X is given by

$$A = \{< x, M_A(x) > \mid x \in X\}$$

where $M_A : X \to [0,1]$, $M_A(x)$ denote the degree of membership of the element x to the set A.

Definition 2.5[2] Let X be a non empty set. An interval-valued intuitionistic fuzzy set (IVIFS) A in X is given by

$$A = \{< x, M_A(x), N_A(x) > \mid x \in X\}$$

where $M_A : X \to [0, 1]$, $N_A : X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and non - membership of the element x to the set A, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M_{AU}(x) + N_{AU}(x) \leq 1 \text{ for all } x \in X.$$

Definition 2.6[4]: Let X be a non empty set. An Interval-Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) A in X is given by

$$A = \{< x, M_A(x), N_A(x) > \mid x \in X\}$$

Where $M_A: X \to [0, 1]$, $N_A: X \to [0, 1]$. The intervals $M_A(x)$ and $N_A(x)$ denote the degree of membership and the degree of non-membership of the element x to the set A, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ with the condition that

$$M^2{}_{AU}(x) + N^2{}_{AU}(x) \leq 1 \text{ for all } x \in X.$$

Definition 2.7[7]: For every two IVIFSST A and B, we have the following relations and operations

1. $A \subset B$ iff $M_{AU}(x) \leq M_{BU}(x)$ & $M_{AL}(x) \leq M_{BL}(x)$ & $N_{AU}(x) \geq N_{BU}(x)$ & $N_{AL}(x) \geq N_{BL}(x)$
2. $A = B$ iff $A \subset B$ & $B \subset A$
3. $\bar{A} = \{< x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)] > \mid x \in X\}$
4. $A \cup B = \{< x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))],$
   $[\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))] > \mid x \in X\}$
5. $A \cap B = \{< x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))],$
   $[\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))] > \mid x \in X\}$
6. $A + B = \{< x, [M^2{}_{AL}(x) + M^2{}_{BL}(x) - M^2{}_{AL}(x)M^2{}_{BL}(x),$
   $M^2{}_{AU}(x) + M^2{}_{BU}(x) - M^2{}_{AU}(x)M^2{}_{BU}(x)], [N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$
7. $A.B = \{< x, [M^2{}_{AL}(x)M^2{}_{BL}(x), M^2{}_{AU}(x)M^2{}_{BU}(x)],$
   $[N^2{}_{AL}(x) + N^2{}_{BL}(x) - N^2{}_{AL}(x)N^2{}_{BL}(x), N^2{}_{AU}(x) + N^2{}_{BU}(x) - N^2{}_{AU}(x)N^2{}_{BU}(x)] \mid x \in X\}$
8. $A \$ B = \{< x, \left[\sqrt{M_{AL}(x)M_{BL}(x)}, \sqrt{M_{AU}(x)M_{BU}(x)}\right], \left[\sqrt{N_{AL}(x)N_{BL}(x)}, \sqrt{N_{AU}(x)N_{BU}(x)}\right] > \mid x \in X\}$
9. $A \# B = \{< x, \left[\frac{2M_{AL}(x)M_{BL}(x)}{M^2{}_{AL}(x)+M^2{}_{BL}(x)}, \frac{2M_{AU}(x)M_{BU}(x)}{M^2{}_{AU}(x)+M^2{}_{BU}(x)}\right], \left[\frac{2N_{AL}(x)N_{BL}(x)}{N^2{}_{AL}(x)+N^2{}_{BL}(x)}, \frac{2N_{AU}(x)N_{BU}(x)}{N^2{}_{AU}(x)+N^2{}_{BU}(x)}\right] > \mid x \in X\}$
10. $A @ B = \{< x, \left[\frac{M^2{}_{AL}(x)+M^2{}_{BL}(x)}{2}, \frac{M^2{}_{AU}(x)+M^2{}_{BU}(x)}{2}\right], \left[\frac{N^2{}_{AL}(x)+N^2{}_{BL}(x)}{2}, \frac{N^2{}_{AU}(x)+N^2{}_{BU}(x)}{2}\right] > \mid x \in X\}$

3. SOME OPERATORS ON IVIFSST

In this section, we define the new operators and their extensions on IVIFSST and establish some of their properties.

Definition 3.1: For an IVIFSST A and for $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, we define the following operators:

$$\boldsymbol{P_{\alpha,\beta}(A) = \{< x, [\max(\alpha, M_{AL}(x)), \max(\alpha, M_{AU}(x))], [\min(\beta, N_{AL}(x)), \min(\beta, N_{AU}(x))] > \mid x \in X\}}$$
$$\boldsymbol{Q_{\alpha,\beta}(A) = \{< x, [\min(\alpha, M_{AL}(x)), \min(\alpha, M_{AU}(x))], [\max(\beta, N_{AL}(x)), \max(\beta, N_{AU}(x))] > \mid x \in X\}}$$

Definition 3.2 For an IVIFSST A and for $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$, we define the two operators which are the extension of $P_{\alpha,\beta}(A)$ and $Q_{\alpha,\beta}(A)$

$$\boldsymbol{P_{\alpha,\beta,\gamma,\delta}(A) = \{< x, [\max(\alpha, M_{AL}(x)), \max(\beta, M_{AU}(x))], [\min(\gamma, N_{AL}(x)), \min(\delta, N_{AU}(x))] > \mid x \in X\}}$$
$$\boldsymbol{Q_{\alpha,\beta,\gamma,\delta}(A) = \{< x, [\min(\alpha, M_{AL}(x)), \min(\beta, M_{AU}(x))], [\max(\gamma, N_{AL}(x)), \max(\delta, N_{AU}(x))] > \mid x \in X\}}$$

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Proposition 3.1: *Let X be a non empty set. For every IVIFSST A and B in X and for all* $\alpha, \beta \in [0, 1]$ *such that* $\alpha + \beta \leq 1$, *we have the following*

$$\mathrm{P}_{\alpha,\beta}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}_{\alpha,\beta}(\mathrm{A}) \cup \mathrm{P}_{\alpha,\beta}(\mathrm{B})$$

Proof:
Let

$$\mathrm{A} = \{< x, [\mathrm{M_{AL}}(x), \mathrm{M_{AU}}(x)], [\mathrm{N_{AL}}(x), \mathrm{N_{AU}}(x)] > | \ x \in \mathrm{X}\}$$

and

$$\mathrm{B} = \{< x, [\mathrm{M_{BL}}(x), \mathrm{M_{BU}}(x)], [\mathrm{N_{BL}}(x), \mathrm{N_{BU}}(x)] > | \ x \in \mathrm{X}\}$$

Then,

$$\boldsymbol{P_{\alpha,\beta}(A)} = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{AL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |\boldsymbol{x} \in \mathbf{X}\}$$

And

$$A \cup B = \left\{< x, [\max\left(\mathrm{M_{AL}}(x),\ \mathrm{M_{BL}}(x)\right), \max\left(\mathrm{M_{AU}}(x),\ \mathrm{M_{BU}}(x)\right)], [\min\left(\mathrm{N_{AL}}(x),\ \mathrm{N_{BL}}(x)\right), \min\left(\mathrm{N_{AU}}(x),\ \mathrm{N_{BU}}(x)\right)] > | \ x \in \mathrm{X}\right\}$$

Now

$$P_{\alpha,\beta}(\mathrm{A} \cup B) = \left\{< x, \left[\max\left(\alpha, \max\left(\mathrm{M_{AL}}(x), \mathrm{M_{BL}}(x)\right)\right), \max\left(\alpha, \max\left(\mathrm{M_{AU}}(x), \mathrm{M_{BU}}(x)\right)\right)\right], \left[\min\left(\beta, \min\left(\mathrm{N_{AL}}(x), \mathrm{N_{BL}}(x)\right)\right), \min\left(\beta, \min\left(\mathrm{N_{AU}}(x), \mathrm{N_{BU}}(x)\right)\right)\right] > |x \in \mathrm{X}\right\} \quad (1)$$

By the definition

$$\boldsymbol{P_{\alpha,\beta}(A)} = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{AL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |x \in \mathrm{X}\},$$

$$\boldsymbol{P_{\alpha,\beta}}(\mathbf{B}) = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{BL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{BU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{BL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |x \in \mathbf{X}\}$$

$$P_{\alpha,\beta}(A) \cup P_{\alpha,\beta}(\mathrm{B}) = \{< x, [\max(\max\ (\alpha, \mathrm{M_{AL}}(x)), \max\ (\alpha, \mathrm{M_{BL}}(x))), \max(\max\ (\alpha, \mathrm{M_{AU}}(x)), \max\ (\alpha, \mathrm{M_{BU}}(x)))], [\min(\min\ (\beta, \mathrm{N}_{AL}(x)), \min\ (\beta, \mathrm{N}_{BL}(x))), \min(\min\ (\beta, N_{AU}(x)), \min\ (\beta, N_{AU}(x)))] > | \ x \in \mathrm{X}\}$$

$$P_{\alpha,\beta}(\mathrm{A}) \cup \mathrm{P}_{\alpha,\beta}(B) = \left\{< x, \left[\max\left(\alpha, \max\left(\mathrm{M_{AL}}(x), \mathrm{M_{BL}}(x)\right)\right), \max\left(\alpha, \max\left(\mathrm{M_{AU}}(x), \mathrm{M_{BU}}(x)\right)\right)\right], \left[\min\left(\beta, \min\left(\mathrm{N_{AL}}(x), \mathrm{N_{BL}}(x)\right)\right), \min\left(\beta, \min\left(\mathrm{N_{AU}}(x), \mathrm{N_{BU}}(x)\right)\right)\right] > |x \in \mathrm{X}\right\} \quad (2)$$

From (1) and (2), we get

$$\mathrm{P}_{\alpha,\beta}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}_{\alpha,\beta}(\mathrm{A}) \cup \mathrm{P}_{\alpha,\beta}(\mathrm{B})$$

Proposition 3.2: *Let X be a non empty set. For every IVIFSST A and B in X and for all* $\alpha, \beta \in [0, 1]$ *such that* $\alpha + \beta \leq 1$, *we have the following*

$$\mathrm{P}_{\alpha,\beta}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}_{\alpha,\beta}(\mathrm{A}) \cap \mathrm{P}_{\alpha,\beta}(\mathrm{B})$$

Proof:
Let

$$\mathrm{A} = \{< x, [\mathrm{M_{AL}}(x), \mathrm{M_{AU}}(x)], [\mathrm{N_{AL}}(x), \mathrm{N_{AU}}(x)] > | \ x \in \mathrm{X}\}$$

and

$$\mathrm{B} = \{< x, [\mathrm{M_{BL}}(x), \mathrm{M_{BU}}(x)], [\mathrm{N_{BL}}(x), \mathrm{N_{BU}}(x)] > | \ x \in \mathrm{X}\}$$

Then,

$$\boldsymbol{P_{\alpha,\beta}(A)} = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{AL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |\boldsymbol{x} \in \mathbf{X}\}$$

And

$$A \cap B = \left\{< x, [\min\left(\mathrm{M_{AL}}(x),\ \mathrm{M_{BL}}(x)\right), \min\left(\mathrm{M_{AU}}(x),\ \mathrm{M_{BU}}(x)\right)], [\max\left(\mathrm{N_{AL}}(x),\ \mathrm{N_{BL}}(x)\right), \max\left(\mathrm{N_{AU}}(x),\ \mathrm{N_{BU}}(x)\right)] > | \ x \in \mathrm{X}\right\}$$

Now

$$P_{\alpha,\beta}(\mathrm{A} \cap B) = \left\{< x, \left[\max\left(\alpha, \min\left(\mathrm{M_{AL}}(x), \mathrm{M_{BL}}(x)\right)\right), \max\left(\alpha, \min\left(\mathrm{M_{AU}}(x), \mathrm{M_{BU}}(x)\right)\right)\right], \left[\min\left(\beta, \max\left(\mathrm{N_{AL}}(x), \mathrm{N_{BL}}(x)\right)\right), \min\left(\beta, \max\left(\mathrm{N_{AU}}(x), \mathrm{N_{BU}}(x)\right)\right)\right] > |x \in \mathrm{X}\right\} \quad (3)$$

By the definition

$$\boldsymbol{P_{\alpha,\beta}(A)} = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{AU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{AL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |\boldsymbol{x} \in \mathbf{X}\},$$

$$\boldsymbol{P_{\alpha,\beta}}(\mathbf{B}) = \{< \boldsymbol{x}, [\mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{BL}}(\boldsymbol{x})), \mathbf{max}\ (\boldsymbol{\alpha}, \mathbf{M_{BU}}(\boldsymbol{x}))], [\mathbf{min}\ (\boldsymbol{\beta}, \mathbf{N}_{\boldsymbol{BL}}(\boldsymbol{x})), \mathbf{min}\ (\boldsymbol{\beta}, \boldsymbol{N_{AU}}(\boldsymbol{x}))] > |\boldsymbol{x} \in \mathbf{X}\}$$

Now

$$P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(\mathrm{B}) = \{< x, [\min(\max\ (\alpha, \mathrm{M_{AL}}(x)), \max\ (\alpha, \mathrm{M_{BL}}(x))), \min(\max\ (\alpha, \mathrm{M_{AU}}(x)), \max\ (\alpha, \mathrm{M_{BU}}(x)))], [\max(\min\ (\beta, \mathrm{N}_{AL}(x)), \min\ (\beta, \mathrm{N}_{BL}(x))), \max(\min\ (\beta, N_{AU}(x)), \min\ (\beta, N_{AU}(x)))] > | \ x \in \mathrm{X}\}$$

$$P_{\alpha,\beta}(\mathrm{A}) \cap \mathrm{P}_{\alpha,\beta}(B) = \left\{< x, \left[\max\left(\alpha, \min\left(\mathrm{M_{AL}}(x), \mathrm{M_{BL}}(x)\right)\right), \max\left(\alpha, \min\left(\mathrm{M_{AU}}(x), \mathrm{M_{BU}}(x)\right)\right)\right], \left[\min\left(\beta, \max\left(\mathrm{N_{AL}}(x), \mathrm{N_{BL}}(x)\right)\right), \min\left(\beta, \max\left(\mathrm{N_{AU}}(x), \mathrm{N_{BU}}(x)\right)\right)\right] > |x \in \mathrm{X}\right\} \quad (4)$$

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From (3) and (4), we get

$$P_{\alpha,\beta}(A \cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$$

Proposition 3.3: *Let X be a non empty set. For every IVIFSST A in X and for all* $\alpha, \beta,\ \gamma, \delta \in [0, 1]$ *such that* $\alpha \leq \beta$, $\gamma \leq \delta$ *and* $\beta + \delta \leq 1$, *we have the following*

$$\overline{P_{\alpha,\beta}(\bar{A})} = Q_{\beta,\alpha}(A)$$

Proof:

Let

$$A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\}$$

Then,

$$\bar{A} = \{< x, [N_{AL}(x), N_{AU}(x)], [M_{AL}(x), M_{AU}(x)] > \mid x \in X\},$$

$$\boldsymbol{P_{\alpha,\beta}(A) = \{< x, [\max(\alpha, M_{AL}(x)), \max(\alpha, M_{AU}(x))], [\min(\beta, N_{AL}(x)), \min(\beta, N_{AU}(x))] > \mid x \in X\}}$$

$$Q_{\alpha,\beta}(A) = \{< x, [\min(\alpha, M_{AL}(x)), \min(\alpha, M_{AU}(x))], [\max(\beta, N_{AL}(x)), \max(\beta, N_{AU}(x))] > \mid x \in X\}$$

Now

$$P_{\alpha,\beta}(\bar{A}) = \{< x, [\max(\alpha, N_{AL}(x)), \max(\alpha, N_{AU}(x))], [\min(\beta, M_{AL}(x)), \min(\beta, M_{AU}(x))] > \mid x \in X\}$$

$$\overline{P_{\alpha,\beta}(\bar{A})} = \left\{< x, \left[\min\left(\beta, M_{AL}(x)\right), \min\left(\beta, M_{AU}(x)\right)\right], [\max(\alpha, N_{AL}(x)), \max(\alpha, N_{AU}(x))] > \mid x \in X\right\} \qquad (5)$$

By the definition

$$Q_{\beta,\alpha}(A) = \{< x, [\min(\beta, M_{AL}(x)), \min(\beta, M_{AU}(x))], [\max(\alpha, N_{AL}(x)), \max(\alpha, N_{AU}(x))] > \mid x \in X\} \qquad (6)$$

From (5) and (6), we get

$$\overline{P_{\alpha,\beta}(\bar{A})} = Q_{\beta,\alpha}(A)$$

Proposition 3.4: *Let X be a non empty set. For every IVIFSST A and B in X and for all* $\alpha, \beta,\ \gamma, \delta \in [0, 1]$ *such that* $\alpha \leq \beta,\ \gamma \leq \delta$ *and* $\beta + \delta \leq 1$, *we have the following*

$$Q_{\alpha,\beta,\gamma,\delta}(A \cap B) = Q_{\alpha,\beta,\gamma,\delta}(A) \cap Q_{\alpha,\beta,\gamma,\delta}(B)$$

Proof:

Let

$$A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > \mid x \in X\}$$

and

$$B = \{< x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)] > \mid x \in X\}$$

Then,

$$A \cap B = \left\{< x, \left[\min\left(M_{AL}(x),\ M_{BL}(x)\right), \min\left(M_{AU}(x),\ M_{BU}(x)\right)\right], \left[\max\left(N_{AL}(x),\ N_{BL}(x)\right), \max\left(N_{AU}(x),\ N_{BU}(x)\right)\right] > \mid x \in X\right\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(A \cap B) = \left\{< x, \left[\min\left(\alpha, \min\left(M_{AL}(x), M_{BL}(x)\right)\right), \min\left(\beta, \min\left(M_{AU}(x), M_{BU}(x)\right)\right)\right], \left[\max\left(\gamma, \max\left(N_{AL}(x), N_{BL}(x)\right)\right), \max\left(\delta, \max\left(N_{AU}(x), N_{BU}(x)\right)\right)\right] > \mid x \in X\right\} \qquad (7)$$

By the definition

$$Q_{\alpha,\beta,\gamma,\delta}(A) = \{< x, [\min(\alpha, M_{AL}(x)), \min(\beta, M_{AU}(x))], [\max(\gamma, N_{AL}(x)), \max(\delta, N_{AU}(x))] > \mid x \in X\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(B) = \{< x, [\min(\alpha, M_{BL}(x)), \min(\beta, M_{BU}(x))], [\max(\gamma, N_{BL}(x)), \max(\delta, N_{AU}(x))] > \mid x \in X\}$$

Then

$$Q_{\alpha,\beta,\gamma,\delta}(A) \cap Q_{\alpha,\beta,\gamma,\delta}(B)$$
$$= \left\{< x, \left[\min\left(\min(\alpha, M_{AL}(x)), \min(\alpha, M_{BL}(x))\right), \min\left(\min(\beta, M_{AU}(x)), \min(\beta, M_{BU}(x))\right)\right] \left[\max\left(\max(\gamma, N_{AL}(x)), \max(\gamma, N_{BL}(x))\right), \max\left(\max(\delta, N_{AU}(x)), \max(\delta, N_{BU}(x))\right)\right] > \mid x \in X\right\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(A) \cap Q_{\alpha,\beta,\gamma,\delta}(B) = \left\{< x, \left[\min\left(\alpha, \min\left(M_{AL}(x), M_{BL}(x)\right)\right), \min\left(\beta, \min\left(M_{AU}(x), M_{BU}(x)\right)\right)\right], \left[\max\left(\gamma, \max\left(N_{AL}(x), N_{BL}(x)\right)\right), \max\left(\delta, \max\left(N_{AU}(x), N_{BU}(x)\right)\right)\right] > \mid x \in X\right\} \qquad (8)$$

From (7) and (8), we get

$$Q_{\alpha,\beta,\gamma,\delta}(A \cap B) = Q_{\alpha,\beta,\gamma,\delta}(A) \cap Q_{\alpha,\beta,\gamma,\delta}(B)$$

Proposition 3.5: *Let X be a non empty set. For every IVIFSST A and B in X and for all* $\alpha, \beta, \gamma, \delta \in [0,1]$ *such that* $\alpha \leq \beta$, $\gamma \leq \delta$ *and* $\beta + \delta \leq 1$, *we have the following*

$$Q_{\alpha,\beta,\gamma,\delta}(A \cup B) = Q_{\alpha,\beta,\gamma,\delta}(A) \cup Q_{\alpha,\beta,\gamma,\delta}(B)$$

Proof:
Let

$$A = \{< x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)] > | \, x \in X\}$$

and

$$B = \{< x, [M_{BL}(x), M_{BU}(x)], [N_{BL}(x), N_{BU}(x)] > | \, x \in X\}$$

Then,

$$A \cup B = \{< x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))], [\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))] > | \, x \in X\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(A \cup B) = \{< x, [\min(\alpha, \max(M_{AL}(x), M_{BL}(x))), \min(\beta, \max(M_{AU}(x), M_{BU}(x)))], [\max(\gamma, \min(N_{AL}(x), N_{BL}(x))), \max(\delta, \min(N_{AU}(x), N_{BU}(x)))] > | x \in X\} \quad (9)$$

By the definition

$$Q_{\alpha,\beta,\gamma,\delta}(A) = \{< x, [\min(\alpha, M_{AL}(x)), \min(\beta, M_{AU}(x))], [\max(\gamma, N_{AL}(x)), \max(\delta, N_{AU}(x))] > | x \in X\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(B) = \{< x, [\min(\alpha, M_{BL}(x)), \min(\beta, M_{BU}(x))], [\max(\gamma, N_{BL}(x)), \max(\delta, N_{AU}(x))] > | x \in X\}$$

Then

$$Q_{\alpha,\beta,\gamma,\delta}(A) \cup Q_{\alpha,\beta,\gamma,\delta}(B) = \Big\{< x, \Big[\max\Big(\min(\alpha, M_{AL}(x)), \min(\alpha, M_{BL}(x))\Big), \max\Big(\min(\beta, M_{AU}(x)), \min(\beta, M_{BU}(x))\Big)\Big] \Big[\min\Big(\max(\gamma, N_{AL}(x)), \max(\gamma, N_{BL}(x))\Big), \min\Big(\max(\delta, N_{AU}(x)), \max(\delta, N_{BU}(x))\Big)\Big] > | x \in X\Big\}$$

$$Q_{\alpha,\beta,\gamma,\delta}(A) \cap Q_{\alpha,\beta,\gamma,\delta}(B) = \{< x, [\min(\alpha, \max(M_{AL}(x), M_{BL}(x))), \min(\beta, \max(M_{AU}(x), M_{BU}(x)))], [\max(\gamma, \min(N_{AL}(x), N_{BL}(x))), \max(\delta, \min(N_{AU}(x), N_{BU}(x)))] > | x \in X\} \quad (10)$$

From (9) And (10), We Get

$$Q_{\alpha,\beta,\gamma,\delta}(A \cup B) = Q_{\alpha,\beta,\gamma,\delta}(A) \cup Q_{\alpha,\beta,\gamma,\delta}(B)$$

4. CONCLUSION

We have introduced new operators on Interval Valued Intuitionistic Fuzzy Sets of Second Type (IVIFSST) and established some of their properties. It is still open to define some more operators on IVIFSST.

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