International Journal of Mathematical Archive-8(12), 2017, 116-120
Available online through www.ijma.info ISSN 2229 – 5046

ON HYPER GOURAVA INDICES AND COINDICES

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(Received On: 11-11-17; Revised & Accepted On: 05-12-17)

ABSTRACT

We introduce the first and second hyper Gourava indices of a graph. Also we propose the first and second hyper Gouava coindices of a graph. In this paper, we determine the first and second hyper-Gourava indices of some standard classes of graphs. Also the first and second hyper Gourava indices of certain nanotubes are determined.

Keywords: *hyper-Gourava indices, hyper-Gourava coindices, nanotubes.*

Mathematics Subject Classification: *05C05, 05C07, 05C35.*

1. INTRODUCTION

Let $G=(V, E)$ be a finite, simple connected graph. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Any undefined term in this paper may be found in Kulli [1].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a numerical parameter mathematically derived from the graph structure. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [2].

In [3] Kulli introduced the first and second Gourava indices of a molecular graph G and they are defined as

$$GO_1(G)=\sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+\left(d_G(u)d_G(v)\right)\right]$$

$$GO_2(G)=\sum_{uv\in E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right).$$

In [3], Kulli introduced the first and second Gourava coindices of a molecular graph as follows:

The first and second Gourava coindices of a graph G are respectively defined as

$$\overline{GO_1}(G)=\sum_{uv\notin E(G)}\left[\left(d_G(u)+d_G(v)\right)+\left(d_G(u)d_G(v)\right)\right]$$

$$\overline{GO_2}(G)=\sum_{uv\notin E(G)}\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right).$$

Recently many other topological indices were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

In this paper, we introduce the first and second hyper Gourava indices and coindices of graphs. Recently many other hyper indices and coindices were studied, for example, in [14, 15].

We consider $HC_5C_7[p\ q]$ and $SC_5C_7[p\ q]$ nanotubes and we compute the first and second hyper-Gourava indices of $HC_5C_7[p\ q]$ and $SC_5C_7[p\ q]$ nanotubes. For more information about the nanotubes, see [16].

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2. THE HYPER-GOURAVA INDICES OF A GRAPH

We introduce the first and second hyper Gourava indices of a graph.

Definition 1: The first and second hyper-Gourava indices of a graph G are defined as

$$HGO_1(G) = \sum_{uv \in E(G)} \left[\left(d_G(u) + d_G(v) \right) + \left(d_G(u) d_G(v) \right) \right]^2,$$

$$HGO_2(G) = \sum_{uv \in E(G)} \left[\left(d_G(u) + d_G(v) \right) \left(d_G(u) d_G(v) \right) \right]^2$$

3. RESULTS FOR SOME STANDARD CLASSES OF GRAPHS

Proposition 1: Let C_n be a cycle with $n \geq 3$ vertices. Then $HGO_1(C_n) = 64n$.

Proof: Let C_n be a cycle with $n \geq 3$ vertices. Then $HGO_1(C_n) = n[\,(2 + 2) + (2 \times 2)]^2 = 64n$.

Proposition 2: Let K_n be a complete graph with $n \geq 2$ vertices. Then $HGO_1(K_n) = \frac{1}{2} n(n+1)^2 (n-1)^3$.

Proof: Let K_n be a complete graph with n vertices. Then K_n has $\frac{n(n-1)}{2}$ edges.

$$HGO_1(K_n) = \frac{1}{2} n(n-1) \left[(n-1) + (n-1) + (n-1)(n-1) \right]^2 = \frac{1}{2} n(n+1)^2 (n-1)^3$$

Proposition 3: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then
$HGO_1(K_{m,n}) = mn(m+n+mn)^2$.

Proof: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then $K_{m,n}$ has $m+n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Clearly every vertex of V_1 is adjacent with n vertices and every vertex of V_2 is adjacent with m vertices. To compute $HGO_1(K_{m,n})$ we see that $HGO_1(K_{m,n}) = mn(m+n+mn)^2$.

Proposition 4: If G is an r-regular graph with n vertices, then $HGO_1(G) = \frac{1}{2} nr^3 (2+r)^2$.

Proof: If G is an r-regular graph with n vertices, then G has $\frac{nr}{2}$ edges. The degree of each vertex of G is r.

$$HGO_1(G) = \frac{1}{2} nr \left[(r+r) + r^2 \right]^2 = \frac{1}{2} nr^3 (2+r)^2.$$

Proposition 5: Let P_n be a path with $n \geq 3$ vertices. Then $HGO_1(P_n) = 64n - 142$.

Proof: Let $G = P_n$ be a path with $n \geq 3$ vertices. We obtain two partitions of edge set of P_n as follows:

$E_3 = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 2\}$, $|E_3| = 2$.
$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = n - 3$.

To compute $HGO_1(P_n)$, we se that

$$\begin{aligned} HGO_1(P_n) &= \sum_{uv \in E(G)} \left[\left(d_G(u) + d_G(v) \right) + \left(d_G(u) d_G(v) \right) \right]^2 \\ &= \left[(1+2) + (1 \times 2) \right]^2 2 + \left[(2+2) + (2 \times 2) \right]^2 (n-3) \\ &= 64n - 142. \end{aligned}$$

Similarly the second hyper Gourava index of some standard classes of graphs are computed.

Proposition 6:

(1) Let C_n be a cycle with $n \geq 3$ vertices. Then $HGO_2(C_n) = 256n$.
(2) Let K_n be a complete graph with $n \geq 2$ vertices. Then $HGO_2(K_n) = 2n(n-1)^7$.
(3) Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then $HGO_2(K_{m,n}) = (m+n)^2(mn)^3$.
(4) Let G is an r-regular graph with n vertices. Then $HGO_2(G) = 2nr^7$.
(5) Let P_n be a path with $n \geq 3$ vertices. Then $HGO_2(P_n) = 256n - 696$.

3. RESULTS FOR $HC_5C_7[p, q]$ NANOTUBES

We consider $HC_5C_7[p, q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternatively. The 2-dimensional lattice of nanotube $HC_5C_7[8, 4]$ is shown in Figure 1.

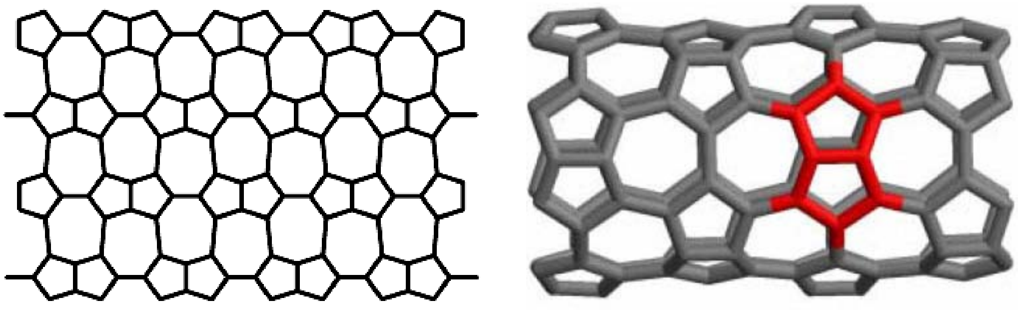

Figure-1: 2-*D* and 3-*D* lattice of nanotube $HC_5C_7[8, 4]$.

By algebraic mehod, we obtain $|V(HC_5C_7[p, q]|=4pq$ and $|E(HC_5C_7[p, q])|= 6pq - p$.

Let G be the graph of nanotube $HC_5C_7[p, q]$. It is easy to see that the vertices of G are either of degree 2 or 3.

By algebraic method, we obtain the edge partition of G based on the sum of degrees of the end vertices of each edge, as given in Table 1.

$d_G(u), d_G(u)\backslash uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$6pq - 5p$

Table-1: Edge partition of G

In the following theorem, we compute the first and second hyper Gourava indices of $HC_5C_7[p, q]$ nanotubes.

Theorem 1: The first and second hyper Gourava indices of $HC_5C_7[p, q]$ nanotube are given by
(i) $HGO_1(HC_5C_7[p, q]) = 1350pq - 641p$.
(ii) $HGO_2(HC_5C_7[p, q]) = 17496pq - 10980p$.

Proof: Let G be the graph of $HC_5C_7[p, q]$ nanotube. The graph G has $4pq$ vertices and $6pq - p$ edges.

i) From equation (1), we have

$$HGO_1\left(HC_5C_7\left[p,q\right]\right)=\sum_{uv\in E(G)}\left[\left(d_G\left(u\right)+d_G\left(v\right)\right)+\left(d_G\left(u\right)d_G\left(v\right)\right)\right]^2$$

Using Table 1, we obtain

$$\begin{aligned}HGO_1\left(HC_5C_7\left[p,q\right]\right)&=4p\left[\left(2+3\right)+\left(2\times 3\right)\right]^2+\left(6pq-5p\right)\left[\left(3+3\right)+\left(3\times 3\right)\right]^2\\&=1350pq-641p.\end{aligned}$$

ii) From equation (2), we have

$$HGO_2\left(HC_5C_7\left[p,q\right]\right)=\sum_{uv\in E(G)}\left[\left(d_G\left(u\right)+d_G\left(v\right)\right)\left(d_G\left(u\right)d_G\left(v\right)\right)\right]^2$$

Using Table 1, we obtain

$$\begin{aligned}HGO_2\left(HC_5C_7\left[p,q\right]\right)&=4p\left[\left(2+3\right)\left(2\times 3\right)\right]^2+\left(6pq-5p\right)\left[\left(3+3\right)\left(3\times 3\right)\right]^2\\&=17496pq-10980p.\end{aligned}$$

4. RESULTS FOR $SC_5C_7[p, q]$ NANOTUBES

We consider $SC_5C_7[p, q]$ nanotubes. The 2-dimensional lattice of nanotube SC_5C_7 [8, 4] is shown in Figure 2. In 2-dimensional lattice of $SC_5C_7[p, q]$, p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternatively.

Figure-2: 2-*D* and 3-*D* lattice of nanotube $SC_5C_7[8, 4]$.

Let G be the graph of nanotube $SC_5C_7[p, q]$. By algebraic mehod, we obtain $|V(SC_5C_7[p, q]| = 4pq$ and $|E(HC_5C_7[p, q])| = 6pq - p$.

It is easy to see that the vertices of G are either of degree 2 or 3. By algebraic method, we obtain the edge partition of G based on the sum of degrees of the end vertices of each edge, as given in Table 2.

$d_G(u), d_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 3)	(3, 3)
Number of edges	q	$6q$	$6pq - p - 7q$

Table-2: Edge partition of G

In the following theorem, we compute the first and second hyper-Gourava indices of $SC_5C_7[p, q]$ nanotubes.

Theorem 2: The first and second hyper Gourava indices of $SC_5C_7[p, q]$ nanotube are respectively given by
(i) $HGO_1(SC_5C_7[p, q]) = 1350pq - 225p - 785q$.
(ii) $HGO_2(SC_5C_7[p, q]) = 17496pq - 2916p - 14756q$.

Proof: Let G be the graph of $SC_5C_7[p, q]$ nanotube. The graph G has $4pq$ vertices and $6pq - p$ edges.

i) From equation (1), we have

$$HGO_1\left(HC_5C_7\left[p,q\right]\right) = \sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)+\left(d_G(u)d_G(v)\right)\right]^2$$

Using Table 2, we obtain

$$\begin{aligned} HGO_1\left(SC_5C_7\left[p,q\right]\right) &= q\left[(2+2)+(2\times 2)\right]^2 + 6q\left[(2+3)+(2\times 3)\right]^2 \\ &\quad +\left(6pq-p-7q\right)\left[(3+3)+(3\times 3)\right]^2 \\ &= 1350pq - 225p - 785q. \end{aligned}$$

ii) From equation (2), we have

$$HGO_2\left(SC_5C_7\left[p,q\right]\right) = \sum_{uv\in E(G)}\left[\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)\right]^2$$

Using Table 2, we obtain

$$\begin{aligned} HGO_2\left(SC_5C_7\left[p,q\right]\right) &= q\left[(2+2)(2\times 2)\right]^2 + 6q\left[(2+3)(2\times 3)\right]^2 \\ &\quad +\left(6pq-p-7q\right)\left[(3+3)(3\times 3)\right]^2 \\ &= 17496pq - 2916p - 14756q. \end{aligned}$$

5. THE FIRST AND SECOND HYPER GOURAVA COINDICES

We propose the first and second hyper-Gourava coindices of a graph.

Definition 2: The first and second hyper-Gourava coindices of a graph G are defined as

$$\overline{HGO_1}(G) = \sum_{uv\notin E(G)}\left[\left(d_G(u)+d_G(v)\right)+\left(d_G(u)d_G(v)\right)\right]^2$$

$$\overline{HGO_2}(G) = \sum_{uv\notin E(G)}\left[\left(d_G(u)+d_G(v)\right)\left(d_G(u)d_G(v)\right)\right]^2$$

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Source of support: Nil, Conflict of interest: None Declared.