International Journal of Mathematical Archive-8(12), 2017, 121-129
IJMA Available online through www.ijma.info ISSN 2229 – 5046

COMMUTATIVE PRIME Γ-NEAR-FIELD SPACES WITH PERMUTING TRI-DERIVATIONS OVER NEAR-FIELD

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(Received On: 23-11-17; Revised & Accepted On: 07-12-17)

ABSTRACT

The object of this paper is to introduce a permuting tri-derivation in a Γ-near-field space. We obtain the conditions for a prime Γ-near-field space to be a commutative Γ-near-field space over a near-field.

Keywords: *Γ-near-field space; Prime Γ-near-field space; Commutative Γ-near-field space; Permuting tri-derivation.*

2000 Mathematics Subject Classification: *43A10, 46B28, 46H25, 46H99, 46L10, 46M20, 51 M 10, 51 F, 15,03 B 30.*

SECTION-1: INTRODUCTION

The derivations in near-field spaces have been introduced and they investigated some basic properties of derivations in near-field spaces. Then obtained some commutativity conditions for a Γ-near-field spaces with derivations. Some characterizations of Γ-near-field spaces and some regularity conditions were together obtained by Smt. T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram. They introduced the notion of two-sided Γ-α-derivation of a Γ-near-field space and investigated the commutativity of prime and semi-prime Γ-near-field spaces. In depth study makes us about near-field spaces over a near-field T Madhavi Latha, Dr T V Pradeep Kumar and Dr N V Nagendram worked on prime Γ-near-field spaces with derivations and they investigated the conditions for a Γ-near-field space to be commutative.

In this paper, the notion of a permuting tri-derivation in a Γ-near-field spaces is introduced.

We investigate the conditions for a prime Γ-near-field spaces over a near-field to be a commutative Γ-near-field space.

Definition 1.1: N-sub near-field space. Let (N, +, .) be a left near-field space. A sub near-field space (M, +) is called an N-sub near-field space i.e. traditional one if there is a near-field space homo-morphism $\theta : N \rightarrow Map(M)$. As usual, we write gn to mean $g(n\theta)$ for $g \in M$ and $n \in N$. In this case the group elements distribute over the near-field spaces.

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Definition 1.2: Complementary N-near-field space. M is called a complementary N-sub near-field space or N – co sub near-field space, for short, if there is a semi sub near-field space elements distribute over the sub near-field space elements and the action of N is usually written on the left of the elements of M.

Definition 1.3: (N, T) – bi sub near-field space. Let N and T be two left near-field spaces. A sub near-field space M is called an (N, T) – bi sub near-field space if

(a) M is an N-co sub near field space

(b) M is an T-sub near-field space and (c) $(ng)t = n(gt)$, $\forall\ g \in$ M, $n \in$ N, $t \in$ T.

Definition 1.4: left strong N-sub near-field space. M is called left strong N-sub near-field space if the action of N is defined on the left of M satisfying the following conditions $\forall\ n, n' \in$ N and $g, g' \in$ M

(a) $(nn')g = n(n'g)$

(b) $n(g + g') = ng + ng'$ and (c) $(n + n')g = ng + n'g$.

Note 1.5: A right strong N-sub near-field space is defined similarly. (N, +) is an (N – N) – bi sub near-field space for the left as well as right near-field space N over a near-field. If N is distributive near-field space then (N, +) is a left as well as right strong N-sub near-field space. Many more examples of these structures are given in near-field space related topic.

Definition 1.6: N-homomorphism. Let M and K be two N-sub near-field spaces (N-co sub near-field space. A sub near-field space homomorphism θ : M $\to$ K is called an N-homomorphism if for any $g \in$ M and $n \in$ N,

$(gn)\theta = (g\theta)n$, $((rg)\theta = r(g\theta))$.

Note 1.7: An (N – T) – homomorphism for (N – T)-bi sub near-field space are defined in a similar way.

Definition 1.8: A Γ-near-field space is a triple (N, +, Γ) where

(i) (N, +) is a group (not necessarily abelian),

(ii) Γ is a non-empty set of binary operations on N such that for each $\alpha \in \Gamma$, (N, +, α) is a left near-field space.

(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $x, y, z \in$ N *and* $\alpha, \beta \in \Gamma$.

Definition 1.9: N is a *left* Γ-near-field space because it satisfies the left distributive law. We will use the word Γ-near-field space to mean *left* Γ-near-field space. For a Γ-near-field space N, the set $N0 = \{x \in \text{N}: 0\alpha x = 0,\ \alpha \in \Gamma\}$ is called the *zero-symmetric part* of N.

Definition 1.10: A Γ-near-field space N is said to be *zero-symmetric* if N = N0.

Note 1.11: Throughout this paper N will be a zero-symmetric Γ-near-field space.

Definition 1.12: Prime Γ-near-field space. N is called *prime* if $x\Gamma\text{N}\Gamma y = \{0\}$ implies $x = 0$ or $y = 0$.

Definition 1.13: ntorsion free Γ-near-field space. N is called *ntorsion-free*, where n is a positive integer, if $nx = 0$ implies $x = 0$ for all $x \in R$.

Definition 1.14: Multiplicative center of Γ-near-field space. The symbol C(N) will represent the multiplicative center of N, that is, $C(x) = \{x \in R : x\alpha y = y\alpha x$ for all $y \in R,\ \alpha \in \Gamma\}$. For $x \in$ N, the symbol $C(x)$ will denote the centralizer of x in N.

Definition 1.15: Commutator and derivation of Γ-near-field space N. $\forall\ x, y \in$ N, $\alpha \in \Gamma$, $[x, y]\alpha$ will denote the commutator $x\alpha y - y\alpha x$, while (x, y) will indicate the additive-group commutator $x + y - x - y$. An additive map ρ : N $\to$ N is called a *derivation* if the Leibniz rule $\rho(x\alpha y) = \rho(x)\alpha y + x\alpha\rho(y)$ holds for all $x, y \in$ N, $\alpha \in \Gamma$.

Definition 1.16: Bi-derivation of Γ-near-field space N. By a *bi-derivation* we mean a bi-additive map D : N $\times N \to$ N (i.e., D is additive in both arguments) which satisfies the relations $D(x\alpha y, z) = D(x, z)\alpha y + x\alpha D(y, z)$ and $D(x, y\alpha z) = D(x, y)\alpha z + y\alpha D(x, z)$ for all $x, y, z \in$ N, $\alpha \in \Gamma$. Let D be symmetric, that is, $D(x, y) = D(y, x)$ for all $x, y \in$ N.

Definition 1.17: Trace of D in Γ-near-field space N. The map ρ : N $\to$ N defined by $\rho(x) = D(x, x)$ for all $x \in$ N is called the *trace* of D.

Definition 1.18: Permuting of Γ-near-field space N. Let us define a mapping F: N $\times N \times N \to$ N is said to be *permuting* if the equation $F(x_1, x_2, x_3) = F(x_{\pi(1)}, x_{\pi(2)}, x_{\pi(3)})$ holds for all $x_1, x_2, x_3 \in$ N and for every permutation $\{\pi(1), \pi(2), \pi(3)\}$.

SECTION-2: PERMUTING TRI-DERIVATIONS AND COMMUTATIVITY

Definition 2.1: Trace and permuting map of F on Γ-near-field space N. A map $f : N \to N$ defined by $f(x) = F(x, x, x)$ for all $x \in N$, where $F : N \times N \times N \to N$ is a permuting map, is called the *trace* of F. It is obvious that, in the case $F : N \times N \times N \to N$ is a permuting map which is also tri-additive (i.e., additive in each argument), the trace f of F satisfies the relation $f(x + y) = f(x) + 2F(x, x, y) + F(x, y, y) + F(x, x, y) + 2F(x, y, y) + f(y)$ for all $x, y \in N$.

Since we have $F(0, y, z) = F(0 + 0, y, z) = F(0, y, z) + F(0, y, z)$ for all $y, z \in N$, we obtain $F(0, y, z) = 0$ for all $y, z \in N$.

Hence we get $0 = F(0, y, z) = F(x- x, y, z) = F(x, y, z) + F(-x, y, z)$ and so we see that $F(-x, y, z) = -F(x, y, z)$ for all $x, y, z \in R$. This tells us that f is an odd function.

Definition 2.2: Tri-derivation of Γ-near-field space N. A tri-additive map $D : N \times N \times N \to N$ will be called a *tri-derivation* if the relations $D(x_1\alpha x_2, y, z) = D(x_1, y, z)\alpha x_2 + x_1\alpha D(x_2, y, z)$, $D(x, y_1\alpha y_2, z) = D(x, y_1, z)\alpha y_2 + y_1\alpha D(x, y_2, z)$ and $D(x, y, z_1\alpha z_2) = D(x, y, z_1)\alpha z_2 + z_1\alpha D(x, y, z_2)$ are fulfilled for all $x, y, z, x_i, y_i, z_i \in N$, $i = 1, 2$, $\alpha \in \Gamma$.

We need the following lemmas to obtain our main results.

Lemma 2.3: *Let N be a prime Γ-near-field space. If* $C(N) - \{0\}$ *contains an element z for which* $z + z \in C(N)$, *then* $(N, +)$ *is abelian.*

Lemma 2.4: *Let N be a* 3!-*torsion free* G-*near-ring. Suppose that there exists a permuting tri-additive map* $F : N \times N \times N \to N$ *such that* $f(x) = 0$ *for all* $x \in N$, *where f is the trace of F. Then we have* $F = 0$.

Lemma 2.5: *Let N be a* 3!-*torsion free prime Γ-near-field space and let* $x \in N$. *Suppose that there exists a nonzero permuting tri-derivation* $D: N \times N \times N \to N$ *such that* $x\alpha d(y) = 0$ *for all* $y \in N$, $\alpha \in G$, *where d is the trace of D. Then we have* $x = 0$.

Proof: Since we have $\rho(y + z) = \rho(y) + 2D(y, y, z) + D(y, z, z) + D(y, y, z) + 2D(y, z, z) + \rho(z)$ for all $y, z \in N$, $\alpha \in G$, the hypothesis gives $2x\alpha D(y, y, z) + x\alpha D(y, z, z) + x\alpha D(y, y, z) + 2x\alpha D(y, z, z) = 0$ for all $y, z \in R$, $\alpha \in \Gamma$. (a)
Setting $y = -y$ in (a), it follows that
$2x\alpha D(y, y, z) - x\alpha D(y, z, z) + x\alpha D(y, y, z) - 2x\alpha D(y, z, z) = 0$ for all $y, z \in R$, $\alpha \in \Gamma$ (b)

On the other hand,
for any $y, z \in N$, $\rho(z + y) = \rho(z) + 2D(z, z, y) + D(z, y, y) + D(z, z, y) + 2D(z, y, y) + \rho(y)$.

So, by the hypothesis, we have
$2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0$ for all $x, y, z \in N$, $\alpha \in \Gamma$, (c)

Since D is permuting. Comparing (a) with (b), we get
$2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) = x\alpha D(y, y, z) - 3x\alpha D(y, z, z)$ which means that $2x\alpha D(y, z, z) + x\alpha D(y, y, z) + x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = x\alpha D(y, y, z) - 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z)$ for all $x, y, z \in N, \alpha \in \Gamma$.

Now, from (3), we obtain
$x\alpha D(y, y, z) - 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0$ for all $x, y, z \in N$, $\alpha \in \Gamma$ (d)

Taking $y = -y$ in (4) leads to
$x\alpha D(y, y, z) + 3x\alpha D(y, z, z) + 2x\alpha D(y, y, z) = 0$ for all $x, y, z \in N$, $\alpha \in \Gamma$ (e)

Combining (d) and (e), we obtain
$x\alpha D(y, z, z) = 0$ for all $x, y \in N$, $\alpha \in \Gamma$, since R is 6-torsion free. (f)

Replacing $z = z + w$ to linearize (f) and using the conditions show that
$x\alpha D(w, y, z) = 0$ for all $w, x, y, z \in N$, $\alpha \in \Gamma$ (g)

Substituting $w\beta v$ for w in (h), we get $x\alpha w\beta D(v, y, z) = 0$ for all $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$. Since N is prime and $D \neq 0$, we arrive at $x = 0$. This completes the proof of the theorem.

Lemma 2.6: *Let N be a Γ-near-field space and let* $D : N \times N \times N \to N$ *be a permuting tri-derivation. Then we have* $[D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v = D(x, z, w)\alpha y\beta v + x\alpha D(y, z,w)\beta v$ *for all* $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$.

Proof: Since we have $D(x\alpha y, z, w) = D(x, z, w)\alpha y + x\alpha D(y, z, w)$ for all $w, x, y, z \in N$, $\alpha \in \Gamma$, the associative law gives $D((x\alpha y)\beta v, z, w) = D(x\alpha y, z, w)\beta v + x\alpha y\beta D(v, z, w) = [D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v + x\alpha y\beta D(v, z, w)$ for all $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$ (h)

and $D(x\alpha(y\beta v), z, w) = D(x, z, w)\alpha y\beta v + x\alpha D(y\beta v, z, w)$

$= D(x, z, w)\alpha y\beta v + x\alpha[D(y, z, w)\beta v + y\beta D(v, z, w)]$

$= D(x, z, w)\alpha y\beta v + x\alpha D(y, z, w)\beta v + x\alpha y\beta D(v, z, w)$ for all $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$ (i)

Comparing (h) and (i), we see that $[D(x, z, w)\alpha y + x\alpha D(y, z, w)]\beta v = D(x, z, w)\alpha y\beta v + x\alpha D(y, z, w)\beta v$ for all $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$.

This completes the proof of the lemma.

Now we are ready to prove our main results in this section.

Theorem 2.7: *Let N be a* 3!-*torsion free prime Γ-near-field space. Suppose that there exists a Non zero permuting tri-derivation* $D: N \times N \times N \to N$ *such that* $D(x, y, z) \in C(N)$ *for all* $x, y, z \in N$. *Then N is a commutative Γ-near-field space.*

Proof: Assume that $D(x, y, z) \in C(N)$ for all $x, y, z \in N$. Since D is nonzero, there exist $x_0, y_0, z_0 \in N$ such that $D(x_0, y_0, z_0) \in C(N) - \{0\}$ and $D(x_0, y_0, z_0) + D(x_0, y_0, z_0) = D(x_0, y_0, z_0 + z_0) \in C(N)$.

So (N, +) is abelian by Lemma 2.1 and Since the hypothesis implies that

$w\beta D(x, y, z) = D(x, y, z)\beta w$ for all $w, x, y, z \in N$, $\beta \in \Gamma$, (j)

we replace x by $x\alpha v$ in (j) to get $w\beta[D(x, y, z)\alpha v + x\alpha D(v, y, z)] = [D(x, y, z)\alpha v + x\alpha D(v, y, z)]\beta w$ and thus, from Lemma 2.4 and the hypothesis, it follows that $D(x, y, z)\beta w\alpha v + D(v, y, z)\alpha w\beta x = D(x, y, z)\alpha v\beta w + D(v, y, z)\beta x\alpha w$ which means that

$D(x, y, z)\beta[w, v]\alpha = D(v, y, z)\beta[x, w]\alpha$ for all $v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$ (k)

Setting $d(u)$ in place of v in (k) and using $d(x)$Î $C(N)$ for all $x \in N$, by the hypothesis,

We obtain

$D(d(u), y, z)\beta[x, w]\alpha = 0$ for all $u, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$ (l)

The substitution vax for x in (l) yields that $D(d(u), y, z)\beta va[x, w]\alpha = 0$ $\forall$ $u, v, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$. Since R is prime, we obtain either $D(d(u), y, z) = 0$ or $[x, w]\alpha = 0$ $\forall$ $u, w, x, y, z \in N$, $\alpha, \beta \in \Gamma$.

Assume that $D(d(u), y, z) = 0$ for all $u, y, z \in N$, $\alpha, \beta \in \Gamma$ (m)

Let us take $u + x$ instead of u in (m).

$\Rightarrow 0 = D(d(u + x), y, z) = D(d(u) + d(x) + 3D(u, u, x) + 3D(u, x, x), y, z)$

$= 3D(D(u, u, x), y, z) + 3D(D(u, x, x), y, z)$, that is,

$D(D(u, u, x), y, z) + D(D(u, x, x), y, z) = 0$ for all $v, w, x, y \in N$. (n)

Setting $u = -u$ in (14) and then comparing the result with (n), we see that

$D(D(u, u, x), y, z) = 0$ for all $u, x, y, z \in N$ (o)

Substituting ulx for x in (o) and employing (n) give the relation $d(u)lD(x, y, z) + D(u, y, z)lD(u, u, x) = 0$ and so it follows from the hypothesis that

$d(u)lD(x, y, z) + D(u, u, x)lD(u, y, z) = 0$ for all $u, x, y, z \in N$, $\alpha, \beta \in \Gamma$ (p)

We put $u = y = x$ in (p) to obtain, $\rho(x)lD(x, x, w) = 0$ for all $w, x \in N$, $l \in \Gamma$ (q)

Taking wlx in substitute for w in (q) yields $d(x)lwld(x) = 0$, for all $l \in \Gamma$, and so the primeness of N implies that $\rho(x) = 0$ for all $x \in N$.

Hence, by Lemma 3.2, we have $D = 0$ which is a contradiction. So N is a commutative Γ-near-field space over a near-field. This completes the proof of the theorem.

Theorem 2.8: Let N be a 3!-torsion free prime G-near-ring. Suppose that there exists a nonzero permuting tri-derivation $D : N \times N \times N \to$ N such that $\rho(x)$, $\rho(x) + \rho(x) \in C(D(u, v, w))$ for all $u, v, w, x \in$ N, where ρ is the trace of D. Then N is a commutative Γ-near-field space over a near-field.

Proof: Assume that $\rho(x)$, $\rho(x) + \rho(x) \in C(D(u, v, w))$ for all $u, v, w, x \in$ N (r)
From (r), we get

$$\begin{aligned} D(u+t, v, w)\alpha(\rho(x) + \rho(x)) &= (\rho(x) + \rho(x))\alpha D(u+t, v, w) \\ &= (\rho(x) + \rho(x))\alpha[D(u, v, w) + D(t, v, w)] \\ &= (\rho(x) + \rho(x))\alpha D(u, v, w) + (\rho(x) + \rho(x))\alpha D(t, v, w) \\ &= \rho(x)\alpha D(u, v, w) + \rho(x)\alpha D(u, v, w) + \rho(x)\alpha D(t, v, w) + \rho(x)\alpha D(t, v, w) \\ &= \rho(x)\alpha[D(u, v, w) + D(u, v, w) + D(t, v, w) + D(t, v, w)] \\ &= [D(u, v, w) + D(u, v, w) + D(t, v, w) + D(t, v, w)]\alpha\rho(x) \text{ for all } t, u, v, w, x \in N, \alpha \in \Gamma \quad \text{(s)} \end{aligned}$$

and

$$\begin{aligned} D(u+t, v, w)\alpha(\rho(x) + \rho(x)) &= D(u+t, v, w)\alpha\rho(x) + D(u+t, v, w)\alpha\rho(x) \\ &= [D(u, v, w) + D(t, v, w)]\alpha\rho(x) + [D(u, v, w) + D(t, v, w)]\alpha\rho(x) \\ &= [D(u, v, w) + D(t, v, w) + D(u, v, w) + D(t, v, w)]\alpha\rho(x) \text{ for all } t, u, v, w, x \in N, \alpha \in \Gamma \quad \text{(t)} \end{aligned}$$

Comparing (s) and (t), we obtain $D((u, t), v, w)\alpha\rho(x) = 0$ for all $t, u, v, w, x \in$ N, $\alpha \in \Gamma$.
Hence it follows from Lemma 2.3 that $D((u, t), v, w) = 0$ for all $t, u, v, w \in$ N (u)

We substitute $u\beta z$ for u and $u\beta t$ for t in (u) to get
$0 = D(u\beta(z, t), v, w) = D(u, v, w)\beta(z, t) + u\beta D((z, t), v, w) = D(u, v, w)\beta(z, t)$, $\beta \in \Gamma$.
That is,
$D(u, v, w)\beta(z, t) = 0$ for all $t, u, v, w, z \in$ N, $\beta \in \Gamma$ (v)

Letting $z = s$dz in (v) and comparing the results (v) we obtain,
$D(u, v, w)\beta s\rho(z, t) = 0$ for all $s, t, u, v, w, z \in$ N, $\beta, \rho \in \Gamma$ (w)

Since $D \neq 0$, we conclude, from (w) and the primeness of N, that $(z, t) = 0$ is fulfilled for all $t, z \in$ N. Therefore $(N, +)$ is abelian.

By the hypothesis, we know that $[\rho(x), D(u, v, w)]\alpha = 0$ for all $u, v, w, x \in$ N, $\alpha \in \Gamma$ (x)

Hence if we let $x = x + y$ in (24) and since $\rho(x + y) = \rho(x) + 2D(x, x, y) + D(x, y, y) + D(x, x, y) + 2D(x, y, y) + \rho(y)$, then we deduce from (x) that $3[D(x, x, y), D(u, v, w)]\alpha + 3[D(x, y, y), D(u, v, w)]\alpha = 0$ for all $u, v, w, x, y \in$ N, $\alpha \in \Gamma$.

Since N is 3-torsion-free, we obtain,
$[D(x, x, y), D(u, v, w)]\alpha + [D(x, y, y), D(u, v, w)]\alpha = 0$ for all $u, v, w, x, y \in$ N, $\alpha \in \Gamma$ (y)

Setting $y = -y$ in (y) and comparing the result with (y), we obtain
$[D(x, y, y), D(u, v, w)]\alpha = 0$ for all $u, v, w, x, y \in$ N, $\alpha \in \Gamma$ (z)

Replacing y by $y + z$ in (z) and using (z), we have $[D(x, y, z), D(u, v, w)]\alpha = 0$, $\alpha \in \Gamma$, since D is permuting, i.e.,
$D(x, y, z)\alpha D(u, v, w) = D(u, v, w)\alpha D(x, y, z)$ for all $u, v, w, x, y, z \in$ N, $\alpha \in \Gamma$ (aa)

Taking $u\beta t$ instead of u in (aa), we obtain,
$D(u, v, w)\beta t\alpha D(x, y, z) - D(x, y, z)\alpha D(u, v, w)\beta t + u\beta D(t, v, w)\alpha D(x, y, z) - D(x, y, z)\beta u\alpha D(t, v, w) = 0$
for all $t, u, v, w, x, y, z \in$ N, $\alpha, \beta \in \Gamma$ (bb)

Substituting $\rho(u)$ for u in (bb) and then utilizing the hypothesis and (aa), we get
$D(\mathrm{d}(u), v, w)\beta[t, D(x, y, z)]\alpha = 0$ for all $t, u, v, w, x, y, z \in$ N, $\alpha, \beta \in \Gamma$ (cc)

Let us write in (cc) wds instead of w. Then we have $D(d(u), v, w)\mathrm{d}s\beta[t, D(x, y, z)]\alpha = 0$ for all $s, t, u, v, w, x, y, z \in$ N, $\alpha, \beta, \rho \in \Gamma$. Since N is prime near-field space over a near-field, we arrive at either $D(d(u), v, w) = 0$ or $[t, D(x, y, z)]\alpha = 0$ for all $t, u, v, w, x, y, z \in$ N, $\alpha, \beta \in \Gamma$.

As in the proof of Theorem 2.5, the case when $D(d(u), v, w) = 0$ holds for all $u, v, w \in$ N leads to the contradiction. Consequently, we arrive at $[t, D(x, y, z)]\alpha = 0$ for all $t, x, y, z \in$ N, $\alpha \in \Gamma$, i.e, $D(x, y, z) \in C(N)$ for all $x, y, z \in$ N. Therefore, Theorem 2.5 yields that a near-field space N is a commutative Γ-near-field space over a near-field which completes the proof.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to the referee for encouraging remarks, suggestions and Dr N V Nagendram being a Professor is indebted to the referee for his various valuable comments leading to the improvement of the advanced research article. This work was supported by the chairman Sri B Srinivasa Rao, Kakinada Institute of Technology & Science (K.I.T.S.), R&D education Department S&H (Mathematics), Divili 533 433. Andhra Pradesh INDIA.

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Source of support: Nil, Conflict of interest: None Declared.