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ON SEMI ROUGH SETS

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ABSTRACT

This present research article introduces semi Rough sets by using Rough membership function. Some interesting properties of semi Rough sets and Rough membership functions are studied. A Rough membership function was introduced by Zdzislaw Pawlak together with Skowron [4] in 1991 and it is used to generate semi Rough sets in this work in a finite universe set. Some of their properties are discussed in this context.

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INTRODUCTION

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of Artificial Intelligence.

There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approaches to tackle this problem are the Fuzzy set theory and the Rough set theory. Theories of Fuzzy sets and Rough sets are powerful mathematical tools for modeling various types of uncertainties. Fuzzy set theory was introduced by *L. A. Zadeh* in his classical paper [5] of 1965.

A polish applied mathematician and computer scientist *Zdzislaw Pawlak* introduced Rough set theory in his classical paper [2] of 1982. Rough set theory is a new mathematical approach to imperfect knowledge. This theory presents still another attempt to deal with uncertainty or vagueness.

The Rough set theory has attracted the attention of many researchers and practitioners who contributed essentially to its development and application. Rough sets have been proposed for a very wide variety of applications. In particular, the Rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially for machine learning, knowledge discovery, data mining, pattern recognition and approximate reasoning.

Throughout this research article, let ϕ and $\mathscr{U}$ stand for the empty set and a finite universe set respectively. Let $|A|$ denote the number of elements in A, where A is any subset of $\mathscr{U}$.

1. PRELIMINARIES

This section is devoted to some basic definitions which are needed for the further study of this Article.

Definition 1.1: A relation R on $\mathscr{U}$ is said to be an *equivalence relation* on $\mathscr{U}$ if

(a) $(x,x)\in R$ for every $x\in\mathscr{U}$ (*reflexivity*)

(b) $(x,y)\in R \Leftrightarrow (y,x)\in R$ for every $x,y\in\mathscr{U}$ (*symmetry*)

(c) $(x,y)\in R$ and $(y,z)\in R \Rightarrow (x,z)\in R$ for every $x,y,z\in\mathscr{U}$ (tran*sitivity*)

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Definition 1.2: If R is an equivalence relation on $\mathscr{U}$ then the *equivalence class* of an element $x \in \mathscr{U}$ is denoted by the symbol $[x]_R$ and it is defined by $[x]_R = \{y \in \mathscr{U} : yRx\}$.

Definition- 1.3: An *information system* is a pair $(\mathscr{U}, \mathcal{A})$ where $\mathcal{A}$ is a set of attributes. Each attribute $f \in \mathcal{A}$ is a mapping $f : \mathscr{U} \to V_f$ where V_f is the range set of the attribute $f \in \mathcal{A}$.
Each attribute $f \in \mathcal{A}$ generates an equivalence relation on $\mathscr{U}$.

Corresponding to each attribute $f \in \mathcal{A}$, a relation R_f is defined on $\mathscr{U}$ such that $(x, y) \in R_f \Leftrightarrow f(x) = f(y)$. It is easy to verify that R_f is an equivalence relation on $\mathscr{U}$ and R_f is called an *indiscernible relation*. If $x \in \mathscr{U}$ then $[x]_{R_f} = \{y \in \mathscr{U} : f(x) = f(y)\}$.

Definition 1.4: Let $(\mathscr{U}, \mathcal{A})$ be an information system and R_f, an indiscernible relation on $\mathscr{U}$ for some $f \in \mathcal{A}$. If X is any subset of $\mathscr{U}$ then

a) The *lower Rough approximation* of X is defined to be the set

$$R_*(X) = \left\{x \in \mathscr{U} : [x]_{R_f} \subseteq X\right\}$$

b) The *upper Rough approximation* of X is defined to be the set

$$R^*(X) = \left\{x \in \mathscr{U} : [x]_{R_f} \cap X \neq \phi\right\}$$

c) The *boundary region* of X with respect to R_f is defined to be the set

$$\mathfrak{B}_{R_f}(X) = R^*(X) - R_*(X)$$

Definition 1.5: A subset X of $\mathscr{U}$ is said to be a *Rough set*, if the boundary region $\mathfrak{B}_{R_f}(X) = R^*(X) - R_*(X)$ is non- empty. Sometimes, a Rough set X can also be represented as a pair $(R_*(X), R^*(X))$ using Rough approximations.

Definition 1.6: A subset X of $\mathscr{U}$ is said to be a *Crisp set* if $R_*(X) = X = R^*(X)$.
It is easy to observe that a subset X of $\mathscr{U}$ is a Crisp set if and only if it is not a Rough set.

2. ROUGH MEMBERSHIP FUNCTION

In most of the cases, the universe set is finite. The Rough membership function seems to be a very useful tool to deal with such conditions. The lower and upper Rough approximations can be obtained by using a rough membership function when the universe set is finite. Let the universe set $\mathscr{U}$ be a non-empty finite set.

Definition 2.1: Let $(\mathscr{U}, \mathcal{A})$ be a finite information system. Fix an indiscernible relation R_f corresponding to an attribute $f \in \mathcal{A}$. If A is any subset of $\mathscr{U}$ then the Rough membership function $\lambda_A : \mathscr{U} \to [0,1]$ is defined as follows.

$$\lambda_A(x) = \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} \;\forall\; x \in \mathscr{U}.$$

The Rough membership function expresses conditional probability that $x \in A$ given R_f and can be interpreted as a degree that $x \in A$ in view of information about x expressed by R_f.

Theorem 2.2: If $A \subseteq \mathscr{U}$, then

a) $R^*(A) = \{x \in \mathscr{U} : \lambda_A(x) > 0\}$

b) $R_*(A) = \{x \in \mathscr{U} : \lambda_A(x) = 1\}$

c) $\mathfrak{B}_{R_f}(A) = \{x \in \mathscr{U} : 0 < \lambda_A(x) < 1\}$

Proof: suppose that $A \subseteq \mathscr{U}$ and R_f is an indiscernible relation on $\mathscr{U}$.

a) $$x \in \{x \in \mathscr{U} : \lambda_A(x) > 0\} \Leftrightarrow \lambda_A(x) > 0$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} > 0$$
$$\Leftrightarrow \left|[x]_{R_f} \cap A\right| > 0$$
$$\Leftrightarrow [x]_{R_f} \cap A \neq \phi$$
$$\Leftrightarrow x \in R^*(A)$$

Thus $R^*(A) = \{x \in \mathscr{U} : \lambda_A(x) > 0\}$.

b) $$x \in \{x \in \mathscr{U} : \lambda_A(x) = 1\} \Leftrightarrow \lambda_A(x) = 1$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} = 1$$
$$\Leftrightarrow \left|[x]_{R_f} \cap A\right| = \left|[x]_{R_f}\right|$$
$$\Leftrightarrow [x]_{R_f} \cap A = [x]_{R_f}$$
$$\Leftrightarrow [x]_{R_f} \subseteq A$$
$$\Leftrightarrow x \in R_*(A)$$

Thus $R_*(A) = \{x \in \mathscr{U} : \lambda_A(x) = 1\}$.

c) Clearly, $$\mathfrak{B}_{R_f}(A) = R^*(A) - R_*(A)$$
$$= \{x \in \mathscr{U} : \lambda_A(x) > 0\} - \{x \in \mathscr{U} : \lambda_A(x) = 1\}$$
$$= \{x \in \mathscr{U} : 0 < \lambda_A(x) < 1\}.$$

Remark 2.3: Suppose that $A \subseteq \mathscr{U}$ and R_f is an indiscernible relation on $\mathscr{U}$. We write λ instead of writing λ_A for simplicity. If we define another indiscernible relation R_λ on $\mathscr{U}$ corresponding to the Rough membership function λ such that

$$(x, y) \in R_\lambda \Leftrightarrow \lambda(x) = \lambda(y)$$

Then we can observe the following.

$$(x, y) \in R_f \Leftrightarrow [x]_{R_f} = [y]_{R_f}$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} = \frac{\left|[y]_{R_f} \cap A\right|}{\left|[y]_{R_f}\right|}$$
$$\Leftrightarrow \lambda(x) = \lambda(y)$$
$$\Leftrightarrow (x, y) \in R_\lambda$$

Hence $R_f = R_\lambda$. Thus the process of generating indiscernible relation using a Rough membership function terminates at the first stage itself.

Theorem 2.4: If A and B are any two subsets of $\mathscr{U}$, then

a) $\lambda_\phi = 0$

b) $\lambda_{\mathscr{U}} = 1$

c) $\lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}$

d) $\lambda_{A\cup B} = \lambda_A + \lambda_B$ provided $A\cap B = \phi$

e) $\lambda_{A^c} = 1 - \lambda_A$ where $A^c = \mathscr{U} - A$

Proof: Clearly, $\lambda_\phi(x) = \dfrac{\left|[x]_{R_f} \cap \phi\right|}{\left|[x]_{R_f}\right|} = \dfrac{|\phi|}{\left|[x]_{R_f}\right|} = 0$ and

$$\lambda_{\mathscr{U}}(x) = \frac{\left|[x]_{R_f} \cap \mathscr{U}\right|}{\left|[x]_{R_f}\right|} = \frac{\left|[x]_{R_f}\right|}{\left|[x]_{R_f}\right|} = 1.$$

Now we prove (c).

$$\lambda_{A\cup B}(x) = \frac{\left|[x]_{R_f} \cap (A\cup B)\right|}{\left|[x]_{R_f}\right|} = \frac{\left|\left([x]_{R_f} \cap A\right)\cup\left([x]_{R_f} \cap B\right)\right|}{\left|[x]_{R_f}\right|}$$

$$= \frac{\left|[x]_{R_f} \cap A\right| + \left|[x]_{R_f} \cap B\right| - \left|[x]_{R_f} \cap A \cap B\right|}{\left|[x]_{R_f}\right|}$$

$$= \lambda_A(x) + \lambda_B(x) - \lambda_{A\cap B}(x) \ \forall\ x \in \mathscr{U}$$

$\Rightarrow \lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}$.

Now, suppose that $A\cap B = \phi$. Then $\lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}$.

$$= \lambda_A + \lambda_B - \lambda_\phi$$

$$= \lambda_A + \lambda_B.$$

Since $A\cup A^c = \mathscr{U}$ and $A\cap A^c = \phi$, $\lambda_{A\cup A^c} = \lambda_A + \lambda_{A^c}$.

$\Rightarrow \lambda_{\mathscr{U}} = \lambda_A + \lambda_{A^c}$

$\Rightarrow 1 = \lambda_A + \lambda_{A^c}$

$\Rightarrow \lambda_{A^c} = 1 - \lambda_A$.

3. SEMI ROUGH SETS

In this section, we introduce the notions of lower and upper semi Rough sets. The concept of exact rough set is also defined. A few properties of these notions are established.

Definition 3.1: Let A be any subset of a finite universe set $\mathscr{U}$ and $\lambda_A : \mathscr{U} \to [0,1]$ be the Rough membership function corresponding to A. Put

$$\theta_A = \min\left\{\lambda_A(x) : x \in \mathscr{U} \text{ and } \lambda_A(x) \neq 0\right\}$$

Then A is said to be

(a) A *lower semi Rough set* if, $0 < \theta_A \leq 1/2$.

(b) An *upper semi Rough set* if, $\frac{1}{2} \le \theta_A \le 1$.

(c) An *exact Rough set* if, $\theta_A = \frac{1}{2}$.

Example 3.2: Let $\mathscr{U} = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ and let d be defined on $\mathscr{U}$ by $d(n) =$ The number of divisors of n.

The indiscernible relation R_d is given by $(m,n) \in R_d \Leftrightarrow d(m) = d(n)$.

Then we have $[1]_{R_d} = \{1\}$, $[2]_{R_d} = \{2,3,5,7,11\}$, $[4]_{R_d} = \{4,9\}$, $[6]_{R_d} = \{6,8,10\}$ and $[12]_{R_d} = \{12\}$

Let $A = \{1,3,5\}$. Then $\lambda_A(1) = 1$, $\lambda_A(2) = \lambda_A(3) = \lambda_A(5) = \lambda_A(7) = \lambda_A(11) = \frac{2}{5}$ and

$\lambda_A(4) = \lambda_A(6) = \lambda_A(8) = \lambda_A(9) = \lambda_A(10) = \lambda_A(12) = 0$. Then the non-zero values of $\lambda_A(x)$ are 1 and $\frac{2}{5}$.

$\Rightarrow \theta_A = \frac{2}{5}$ hence A is a lower semi Rough set.

Let $B = \{1,4\}$. Then $\lambda_B(1) = 1$,

$\lambda_B(2) = \lambda_B(3) = \lambda_B(5) = \lambda_B(7) = \lambda_B(11) = \lambda_B(6) = \lambda_B(8) = \lambda_B(10) = \lambda_B(12) = 0$ and

$\lambda_B(4) = \lambda_B(9) = \frac{1}{2}$.

The non-zero values of $\lambda_B(x)$ are 1 and $\frac{1}{2}$.

$\Rightarrow \theta_B = \frac{1}{2}$

$\Rightarrow B$ is an exact Rough set.

If $C = \{1\}$ then C is an upper semi Rough set.

Proposition 3.3: If $A \subseteq B$, then $\theta_A \le \theta_B$

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