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MULTIOBJECTIVE FRACTIONAL TRANSPORTATION PROBLEM IN FUZZY ENVIRONMENT

SHEEMA SADIA*[1], NEHA GUPTA[2] AND QAZI M. ALI[1]
[1]Department of Statistics & Operations Research, A.M.U., Aligarh-202002, India.
[2]Faculty of Commerce and Management, SGT University, Gurugram, India.

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ABSTRACT

The fractional programming is a generalization of linear programming where the objective function is a ratio of two linear functions. Similarly, in fractional transportation problem the objective is to optimize the ratio of two cost functions or damage functions or demand functions. As the ratio of two functions is considered, the fractional programming models become more suitable for real life problems. Keeping in view the complexities associated with real life transportation problem like vagueness and uncertainty involved with the parameters. The implementation of fuzzy techniques can be very useful. Therefore, in this article a Fully Fuzzy Multi-objective Fractional Transportation Problem (FFMOFTP) is considered. All the coefficients of the parameters, demands and supplies are considered as fuzzy numbers. The purpose of using fuzzy numbers is to deal with the uncertainties and vagueness associated with the parameters. Two cases are considered, one with triangular and other with trapezoidal fuzzy numbers. The fuzzy problem is converted into crisp form by two ranking methods; Yager's ranking method and Maleki ranking method respectively. The converted crisp problem is linearized by using Taylors series. Finally the compromise solution of the converted problem is obtained using fuzzy goal programming technique. The aim of this paper is not only to provide a method for solving FFMOFTP but also to evaluate the efficiency of the two ranking methods under the same circumstances. Two numerical problems for each case are also solved at the end to demonstrate the efficiency of the proposed approach.

Keywords: *Fractional transportation problem; Fuzzy goal programming; Multiobjective programming; Fuzzy numbers; Ranking method.*

1. INTRODUCTION

The distribution of goods from manufacturer to customer is a common problem and is described as Transportation Problem (TP). It was originally formulated by Hitchcock (1941). The TP can be solved by simplex method. Some shortcuts have been developed to solve TPs. However, the real life problems are quite complex and difficult to solve by conventional methods. For instance, the traditional TPs generally deal with a single objective function of minimizing cost, but in real world, one has to deal with many objectives other than minimizing cost such as minimizing time, damage charges etc. Such problems are called as Multiobjective Transportation Problems (MOTP). Several authors worked on MOTP like Lee and Moore (1973) studied multiobjective optimization transportation problems. Wahed and Waiel (2001) worked on multi-objective transportation problem under fuzziness.Pramanik and Roy (2008) formulated MOTP with fuzzy parameters.

In real life, a situation may arise where the objective is a ratio of two linear functions, e.g. ratio of transporting costs one with the travelled route and other by the preferred route. Such type of problem is an example of Fractional Transportation Problem (FTP), was originally proposed by Swarup (1966). The TP with fractional objective functions has been extensively used by several authors like Verma and Puri (1991), Khurana and Arora (2006), Joshi and Gupta (2011), Gupta and Arora (2012). In FTP, when more than one objective is taken into consideration, then such problems become Multiobjective Fractional Transportation Problems (MOFTP). Real life transportation problems generally have multiple objectives. Mukherjee and Basu (2010) solved an assignment problem with fuzzy cost by ranking method described in Yager (1981) that transforms the fuzzy assignment problem into a crisp assignment problem. Biswas and Pramanik (2011) presented multi-objective assignment problem with fuzzy costs.

Porchelvi and Sheela(2015) considered consider a linear fractional interval transportation problem with and without budgetary constraints. Recently, Liu (2016) formulated fractional transportation problem with fuzzy parameters.

Corresponding Author: Sheema Sadia*[1],
[1]Department of Statistics & Operations Research, A.M.U., Aligarh-202002, India.

In traditional TP, the decision maker is aware about the values of transportation cost, availability and demand of the product. But in real world situations, all the parameters of the transportation problems may not be known precisely, due to many uncontrollable factors. In such situations fuzzy set theory, introduced by Zadeh (1965), is quite useful and the imprecise data can be represented by fuzzy numbers.

The fuzzy concept is used in an application, when its boundaries are not fixed. Or in other words, when the information provided regarding the application is vague or imprecise, fuzzy numbers can be used for the representation of such imprecise data. Moreover, the fuzzy logic is easy to understand and it is based on natural approach. So, it becomes quite effective when applied on real life problems. In past, many researchers used this theory to deal with imprecise or vague data such as Dubois and Prade (1980), Verdagay (1984). Bit *et.al* (1992) also used application of the method given in Zimmerman (1978, 1985, 1987). Chanas *et al.* (1984) analysed TP with fuzzy supply values and fuzzy demand values. Bit *et al*. (1993) applied fuzzy programming on MOTP. Ammar and Youness (2005) used the concept of fuzzy numbers on MOTP. Recently, Sadia *et al.* (2016) used fuzzy programming approach to solve MOFTP. Keeping in view the realistic nature of MOFTP, we have taken a fully fuzzy MOFTP. The reason behind using fuzzy numbers is the uncertainties and vagueness associated with the coefficients of the parameters and with demands and supplies.

In this article, a fully fuzzy MOFTP is formulated. The objective functions are fractional and obtained as a ratio of two linear functions. And each coefficient in the numerator and denominator of all the objective functions, supply and demand are taken as triangular and trapezoidal fuzzy numbers respectively. Fuzzy numbers are converted into crisp form by using Yager's Ranking function and Maleki ranking each time. The non-linear objective functions obtained are linearized by using Taylor series. And the linearized problem is solved by fuzzy goal programming approach.

The paper is divided into various sections; the first section gives the introduction with brief literature review. The second section describes some important terms and definitions which are used in the paper. Section 3 gives the statement of the problem. The algorithm of the problem is given in section four. The numerical problems with their solutions are presented in section 5.1 and 5.2 respectively. Results are summarised in section 6. The conclusions and discussion of the results obtained are presented in section 7.

2. SOME PRELIMINARIES

In this section, some definitions and notations related to fuzzy set theory are discussed.

Definition 2.1 (Fuzzy Set): Let X be a universal set then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}}(x): X \to [0,1]$
which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$,where the value of $\mu_{\tilde{A}}(x)$ at x represents the grade of membership of x in A. Thus, nearer the value of $\mu_{\tilde{A}}(x)$ is unity, the higher is the grade of membershipof x in $\tilde{A}$.

A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ .This is often written as,
$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X)$
where, $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) that maps to the membership space. When this space contains only the two points 0 and 1, $\tilde{A}$ is non-fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a non-fuzzy set, [see Sakawa (1993)]

Definition 2.2: (Support): The support of a fuzzy set A on X , denoted by supp $(\tilde{A})$, is the set of points in X at which $\mu_{\tilde{A}}(x)$ is positive, i.e. $\text{supp}(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) > 0\}$ [see Sakawa (1993)]

Definition 2.3: (α-cut): The α-cut of a fuzzy subset $\tilde{A}$ *of* X can be defined by
$\tilde{A}(\alpha) = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha, \alpha \in [0,1]\} \ \forall \ \alpha \in [0,1]$ [see Tzeng and Huang (2013)]

Fuzzy number: A fuzzy set $\tilde{A}$ on R must have the following properties to qualify as a fuzzy number.

1. $\tilde{A}$ must be a normal fuzzy set.
2. $\tilde{A}$ must be closed interval for every $\alpha \in [0,1]$.
3. The supp ($\tilde{A}$) must be bounded in R .

Triangular Fuzzy number: It is fuzzy number represented with three points as follows in figure 1. This representation is interpreted as membership functions and holds the following conditions.

1. a_1 to a_2 is increasing function.
2. a_2 to a_3 is decreasing function.
3. $a_1 \le a_2 \le a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ x > a_3 \end{cases}$$

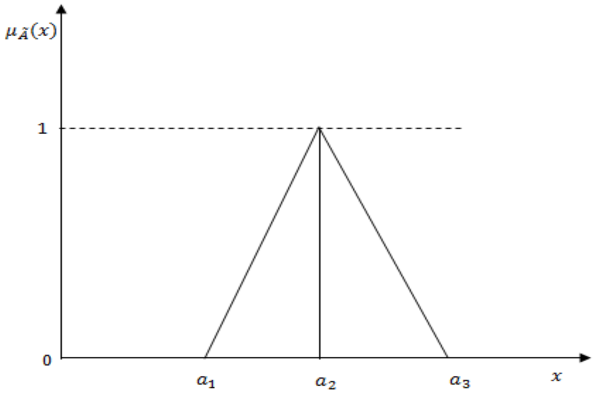

Figure 1

Trapezoidal fuzzy number: A trapezoidal fuzzy number (Figure 2) can be completely specified by the foursome $\tilde{R} = (r_1, r_2, r_3\ and\ r_4)$ with membership function as-

$$\mu_{\tilde{R}} = \begin{cases} 0, & r \le r_1 \\ \dfrac{r - r_1}{r_2 - r_1}, & r_1 \le r \le r_2 \\ 1, & r_3 \le r \le r_4 \\ 0, & r \ge r_4 \end{cases}$$

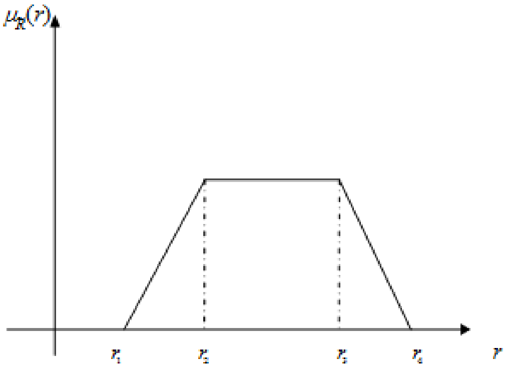

Figure 2

The Yager's Ranking Function: For a fuzzy number $\tilde{A}$, the Yager's Ranking Index will be:

$$Y(\tilde{A}) = \int_0^1 0.5(A_\alpha^L + A_\alpha^U)\,d\alpha$$

where $[A_\alpha^L, A_\alpha^U]$ is the crisp interval $\forall \alpha \in [0,1]$ obtained by $\alpha - cut$. The $\alpha - cut$ of triangular fuzzy number $\tilde{A}$ is as follows:

$$^{\alpha}A = [A_\alpha^L, A_\alpha^U] = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3],$$

The $\alpha - cut$ of trapezoidal fuzzy number $\tilde{R}$ is as follows:

$$^{\alpha}R = [R_\alpha^L, R_\alpha^U] = [(r_2 - r_1)\alpha + r_1, -(r_4 - r_3)\alpha + r_4],$$

Since, $Y(\tilde{A})$ is calculated from the values of $\alpha -$cut of $\tilde{A}$, rather than its membership function, Yager's ranking function is also applicable even if the explicit form of membership function is not known.

The Maleki Ranking Function

Maleki (2002) proposed Maleki ranking function.

Let $\tilde{T}$ be a trapezoidal fuzzy number of the form $\tilde{T} = (t^l, t^u, \gamma, \beta)$ where $(t^l - \gamma, t^u + \beta)$ is the support of $\tilde{T}$ and (t^l, t^u) is the core of it.

The Maleki ranking function of $\tilde{T}$ is:

$$\Re(\tilde{T}) = \int_0^1 (\inf \tilde{T}_\alpha + \sup \tilde{T}_\alpha)\, d\alpha$$
$$\Re(\tilde{T}) = t^l + t^u + \frac{1}{2}(\beta - \gamma)$$
$$where\ \gamma = \beta\ or\ \gamma \neq \beta$$

For our case, where $\tilde{R}$ is the trapezoidal number of the form (r_1, r_2, r_3, r_4), the Maleki ranking function is:

$$\Re(\tilde{R}) = r_2 + r_3 + \frac{1}{2}(r_4 - r_3 - r_2 + r_1)$$

Keeping in view the fact that trapezoidal fuzzy number becomes triangular fuzzy number if $r_2 = r_3$. So the Maleki Ranking function for triangular fuzzy number of the form $\tilde{A} = (a_1, a_2, a_3)$ is:

$$\Re(\tilde{A}) = 2a_2 + \frac{1}{2}(a_3 - a_2 - a_2 + a_1)$$
$$= 2a_2 + \frac{1}{2}(a_3 - 2a_2 + a_1)$$

3. STATEMENT OF THE PROBLEM

Let us consider a Fully Fuzzy Multiobjective Fractional Transportation Problem (FFMOFTP) as:

$$\left.\begin{aligned} &\text{Minimize } Z_k(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}^k x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{r}_{ij}^k x_{ij}} \\ &\text{subject to } \sum_{i=1}^{m} x_{ij} \le a_i; \\ &\qquad \sum_{j=1}^{n} x_{ij} \le b_j; \\ &\qquad x_{ij} \ge 0; \end{aligned}\right\}, \ k = 1, 2, \ldots, K. \tag{1}$$

where $Z_k(x); k = 1,2,\ldots,K$ is a vector of K objective functions. The variable x_{ij} represents the unknown quantity transported from i^{th} origin to j^{th} destination. c_{ij}^k and r_{ij}^k are the penalties associated with transporting a unit product from source O_i to destination D_j for the k^{th} criterion. In general $\tilde{c}_{ij}^k$ represents the fuzzy transporting costs or fuzzy transportation time or fuzzy damage cost either due to loss of quality or quantity of transportation. whereas $\tilde{r}_{ij}^k$ represents the fuzzy transportation cost due to preferred route or fuzzy standard transportation time or fuzzy damage cost due to preferred route.

Here the fuzzy coefficients are considered as triangular and trapezoidal fuzzy numbers. The purpose of using fuzzy numbers is to overcome the uncertainty associated with parameters.

4. ALGORITHM

Step-1: The first step is to convert all the fuzzy numbers (triangular or trapezoidal) into crisp form by using Yager's and Maleki ranking function.

Now, the problem (1) reduces to the crisp problem as:

$$\left.\begin{array}{ll} \text{Minimize } Z_k(x) = \dfrac{\sum_{i=1}^{m}\sum_{j=1}^{n}\Re(\tilde{c}_{ij}^{k})x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\Re(\tilde{r}_{ij}^{k})x_{ij}} & \\ \text{subject to } \quad \sum_{i=1}^{m} x_{ij} \le a_i & \\ \qquad\qquad\quad \sum_{j=1}^{n} x_{ij} \le b_j & \\ \qquad\qquad\quad x_{ij} \ge 0 & \end{array}\right\}, \; k = 1, 2, \ldots, K. \tag{2}$$

Step-2: To obtain the fuzzy goal programming model of FFMOFTP as given in (2), we transform the objective functions $Z_k(x);\ k = 1,2,\ldots,K$, into fuzzy goals by assigning an imprecise aspiration level to each of them.

Let $Z_k^*(x);\ k = 1,2,\ldots,K$, be the optimal solutions of the K objective functions $Z_k(x)$ of FFMOFTP, when calculated in isolation subject to the system constraints. Then, the fuzzy goals appear in the form:
$Z_k(x) \,\tilde{\ge}\, Z_k^*(x).$

A payoff matrix is obtained using the individual best solutions as follows:

$$\text{Payoff Matrix} = \begin{array}{c} \\ x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ \vdots \\ x_{ij}^{(K)} \end{array} \begin{array}{c} \begin{array}{cccc} Z_1(x) & Z_2(x) & \cdots & Z_K(x) \end{array} \\ \begin{pmatrix} Z_1(x_{ij}^{(1)}) & Z_2(x_{ij}^{(1)}) & \cdots & Z_K(x_{ij}^{(1)}) \\ Z_1(x_{ij}^{(2)}) & Z_2(x_{ij}^{(2)}) & \cdots & Z_K(x_{ij}^{(2)}) \\ \vdots & \vdots & \ddots & \vdots \\ Z_1(x_{ij}^{(K)}) & Z_2(x_{ij}^{(K)}) & \cdots & Z_K(x_{ij}^{(K)}) \end{pmatrix} \end{array}$$

where $x_{ij}^{(k)}$ are the individual optimal points of the objective functions of FFMOFTP.

The maximum value of each column gives the upper bound for the objective functions whereas the minimum value of each column gives lower bound for the objective functions.

The objective values which are equal to or larger than $Z_k^*(x)$ should be absolutely satisfactory to the problem. A satisfactory optimal solution of the problem is reached, if the individual best solutions are identical. However, this situation arises rarely because the objective functions are conflicting in general.

The membership function $\mu_k(x); k = 1,2,\ldots K.$ is not linear and is corresponding to the objective function $Z_k^*(x)$ .It can be formulated as:

$$\mu_k(x) = \begin{cases} 0, & \text{if } Z_k(x) \ge Z_k^U(x); \\ \dfrac{Z_k^u(x) - Z_k(x)}{Z_k^u(x) - Z_k^L(x)}, & \text{if } Z_k^L(x) \le Z_k(x) \le Z_k^U(x); \\ 1, & \text{if } Z_k(x) \le Z_k^L(x). \end{cases}$$

Where, $Z_k^U(x)$ and $Z_k^L(x)$ are respectively the upper and lower bounds of the fuzzy objective goals for FFMOFTP.

Before step 3, the problem (2) reduces to

$$\left.\begin{array}{ll} \text{Minimize} & \mu_k(x) \\ \text{subject to} & \sum_{i=1}^{m} x_{ij} \le a_i \\ & \sum_{j=1}^{n} x_{ij} \le b_j \\ & x_{ij} \ge 0 \end{array}\right\}, \; k = 1,2,\ldots,K. \tag{3}$$

Step-3: Let $x_{ij}^{(k)*}; k=1,2,\ldots,K$, be the individual best solutions of the non-linear membership functions $\mu_k(x)$ subject to constraints. Then, the above membership functions $\mu_k(x)$ are transformed into equivalent linear form at individual best solution point by first order Taylor's series as follows:

$$\mu_k(x) \cong \mu_k\left(x_{ij}^{(k)*}\right) + \left(x_{11} - x_{11}^{(k)*}\right)\frac{\partial}{\partial x_{11}}\mu_k\left(x_{ij}^{(k)*}\right) + \ldots + \left(x_{mn} - x_{mn}^{(k)*}\right)\frac{\partial}{\partial x_{mn}}\mu_k\left(x_{ij}^{(k)*}\right) = \lambda_k.$$

Using the Taylor's Series expansion, the FFMOFTP represented by (3) reduces to the following problem

$$\left.\begin{array}{ll} \text{Minimize} & \lambda_k(x) \\ \text{subject to} & \sum\limits_{i=1}^{m} x_{ij} \le a_i \\ & \sum\limits_{j=1}^{n} x_{ij} \le b_j \\ & x_{ij} \ge 0 \end{array}\right\}, \; k=1,2,\ldots,K. \tag{4}$$

Step-4: As the maximum value of a membership function is one, so for the defined membership functions in (3), the flexible membership goals having the aspiration level unity is presented as:

$$\lambda_k(x) + \delta_k = 1; \; k=1,2,\ldots,K.$$

Here $\delta_k \ge 0; k=1,2,\ldots,K$ are the deviational variables.

Finally, the fuzzy goal programming (FGP) model will be:

$$\left.\begin{array}{ll} \text{Minimize} & \delta_k \\ \text{subject to} & \lambda_k(x) + \delta_k = 1 \\ & \sum\limits_{i=1}^{m} x_{ij} \le a_i \\ & \sum\limits_{j=1}^{n} x_{ij} \le b_j \\ & \delta_k,\; x_{ij} \ge 0 \end{array}\right\}, \; k=1,2,\ldots,K. \tag{5}$$

Step-5: The fuzzy goal programming model obtained in step 4 is solved to obtain the final compromise solution. LINGO 13 Software is used for doing all the calculations.

5.1 Problem description with data

In order to demonstrate the problem and the utility of the approach, discussed above, two numerical problems are presented. The first problem is considered in which data is of the form of triangular fuzzy numbers. And it is in the form of trapezoidal fuzzy number for second problem.

A MOFTP in fuzzy environment is considered. There are three origins and three destinations. The TP cost (ratio of transporting cost one with travelled route and the other by preferred route, time) (ratio of actual transportation time and standard transportation time, damage charges (ratio of damage charges one with travelled route and the other by preferred route), demands and supplies during the transportation are considered as fuzzy numbers (triangular or trapezoidal) and are presented in matrices below.

Problem 1: FFMOFTP WITH TRIANGULAR FUZZY DATA

Fuzzy Cost Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(3,5,7)}{(1,3,5)}$	$\frac{(4,7,8)}{(1,4,5)}$	$\frac{(14,15,17)}{(11,13,15)}$	$\le (10,12,14)$
a_2	$\frac{(6,8,10)}{(11,12,15)}$	$\frac{(14,17,18)}{(12,14,16)}$	$\frac{(11,12,13)}{(5,7,8)}$	$\le (13,15,17)$
a_3	$\frac{(12,14,16)}{(12,15,16)}$	$\frac{(7,10,11)}{(4,6,8)}$	$\frac{(11,13,14)}{(6,8,9)}$	$\le (18,20,22)$
Demand	=(7,9,11)	=(11,13,15)	=(19,21,23)	

Fuzzy Time Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(15,17,18)}{(7,9,12)}$	$\frac{(3,5,6)}{(1,2,4)}$	$\frac{(8,10,12)}{(0,3,4)}$	$\le (10,12,14)$
a_2	$\frac{(0,1,4)}{(1,2,5)}$	$\frac{(8,11,12)}{(3,4,5)}$	$\frac{(4,6,7)}{(3,5,6)}$	$\le (13,15,17)$
a_3	$\frac{(10,13,14)}{(6,8,10)}$	$\frac{(15,16,17)}{(10,12,13)}$	$\frac{(9,10,13)}{(9,11,14)}$	$\le (18,20,22)$
Demand	=(7,9,11)	=(11,13,15)	=(19,21,23)	

Fuzzy Damage Charge Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(10,13,14)}{(6,8,11)}$	$\frac{(13,15,16)}{(7,9,10)}$	$\frac{(6,8,9)}{(9,11,12)}$	$\le (10,12,14)$
a_2	$\frac{(13,15,16)}{(9,11,12)}$	$\frac{(13,14,15)}{(4,6,8)}$	$\frac{(15,19,21)}{(5,7,8)}$	$\le (13,15,17)$
a_3	$\frac{(4,7,8)}{(6,9,10)}$	$\frac{(13,15,16)}{(4,6,7)}$	$\frac{(15,17,18)}{(6,7,9)}$	$\le (18,20,22)$
Demand	=(7,9,11)	=(11,13,15)	=(19,21,23)	

5.1 Solution procedure

Using the above data, the FFMOFTP has been formulated which cannot be solved by any standard method. To solve the fuzzy problem, firstly we have to convert it into crisp problem by using ranking function.

Case-1 (Yager's ranking function): To convert the triangular fuzzy data into crisp form. Yager's ranking function is used as follows.

The α-cut of fuzzy number (3, 5, 7) is $[C_\alpha^L, C_\alpha^U]$ = (3+2α, 7-2α) for which,

$$\Re_y(\tilde{C}_{11}^N) = \Re_y(3,5,7) = \int_0^1 0.5(C_\alpha^L + C_\alpha^U)d\alpha = 5$$

$$\Re_y(1,3,5) = 3, \Re_y(4,7,8) = 6.5.$$

Similarly, the Yager's ranking index of each element can be obtained.

$$\Re_y(\tilde{C}_{12}^n) = 6.5, \Re_y(\tilde{C}_{12}^d) = 3.5, \Re_y(\tilde{C}_{13}^n) = 15.25, \Re_y(\tilde{C}_{13}^d) = 13, \Re_y(\tilde{C}_{21}^n) = 8, \Re_y(\tilde{C}_{21}^d) = 12.5, \Re_y(\tilde{C}_{22}^n) = 16.5 \text{ and so on.}$$

$$\Re_y(\tilde{T}_{11}^n) = 16.75, \Re_y(\tilde{T}_{11}^d) = 9.25, \Re_y(\tilde{T}_{12}^n) = 4.75, \Re_y(\tilde{T}_{12}^d) = 2.25, \Re_y(\tilde{T}_{13}^n) = 10, \Re_y(\tilde{T}_{13}^d) = 2.5, \Re_y(\tilde{T}_{21}^n) = 1.5 \text{ and so on.}$$

$$\Re_y(\tilde{D}_{11}^n) = 12.5, \Re_y(\tilde{D}_{11}^d) = 8.25, \Re_y(\tilde{D}_{12}^n) = 14.75, \Re_y(\tilde{D}_{12}^d) = 8.75, \Re_y(\tilde{D}_{13}^n) = 7.75, \Re_y(\tilde{D}_{13}^d) = 10.75 \text{ and so on.}$$

Here *n* and *d* in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:

$$\left.\begin{array}{ll}
\text{Minimize } Z_1 = \dfrac{(5x_{11}+6.5x_{12}+15.25x_{13}+8x_{21}+16.5x_{22}+12x_{23}+14x_{31}+9.5x_{32}+12.75x_{33})}{(3x_{11}+3.5x_{12}+13x_{13}+12.5x_{21}+14x_{22}+6.75x_{23}+14.5x_{31}+6x_{32}+7.75x_{33})} & \\
\text{Minimize } Z_2 = \dfrac{(16.75x_{11}+4.75x_{12}+10x_{13}+1.5x_{21}+10.5x_{22}+5.75x_{23}+12.5x_{31}+16.75x_{32}+10.5x_{33})}{(9.25x_{11}+2.25x_{12}+2.5x_{13}+2.5x_{21}+4x_{22}+4.75x_{23}+8x_{31}+11.75x_{32}+11.25x_{33})} & \\
\text{Minimize } Z_3 = \dfrac{(12.5x_{11}+14.75x_{12}+7.75x_{13}+14.75x_{21}+14x_{22}+18.5x_{23}+6.5x_{31}+14.75x_{32}+16.75x_{33})}{(8.25x_{11}+8.75x_{12}+10.75x_{13}+10.75x_{21}+6x_{22}+6.75x_{23}+8.5x_{31}+5.75x_{32}+7.5x_{33})} & \\
\text{subject to} \quad x_{11}+x_{12}+x_{13}\le 12 & \\
x_{21}+x_{22}+x_{23}\le 15 & \\
x_{31}+x_{32}+x_{33}\le 20 & \\
x_{11}+x_{21}+x_{31}=9 & \\
x_{12}+x_{22}+x_{23}=13 & \\
x_{31}+x_{32}+x_{33}=21 & \\
x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33}\ge 0 \text{ and integers.} &
\end{array}\right\}$$

The pay-off matrix is obtained after solving the above problem separately for each objective function using the optimizing software LINGO is as follows:

$$\text{Payoff Matrix} = \begin{array}{c} \\ x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ x_{ij}^{(3)} \end{array}\begin{array}{c} \begin{array}{ccc} Z_1(x) & Z_2(x) & Z_3(x) \end{array} \\ \begin{pmatrix} 1.152935 & 1.525710 & 1.570192 \\ 1.185787 & 1.379630 & 1.815867 \\ 1.312642 & 1.858025 & 1.047661 \end{pmatrix}\end{array}$$

$$C_L = 1.152935, C_U = 1.312642, T_L = 1.047661, T_U = 1.815867, D_L = 1.379630, D_U = 1.858025.$$

The non-linear membership functions for objective functions $C(x), T(x)$ and $D(x)$ are obtained as follows:-

$$\mu_1 = \frac{1.312642 - Z_1}{1.312642 - 1.152935},\ \mu_2 = \frac{1.858025 - Z_2}{1.858025 - 1.379630},\ \mu_3 = \frac{1.815867 - Z_3}{1.815867 - 1.047661}$$

Then, the non-linear membership functions are transformed into linear at the individual best solution point by first order Taylor polynomial series. And it will be as follows:-

$$\begin{aligned}
\mu_1 &= 1 + (-0.0207)(x_{11}-0) + (-0.03324)(x_{12}-0) + (-0.0035316)(x_{13}-12) + 0.08546(x_{21}-6) \\
&\quad + (-0.0048407)(x_{22}-9) + (-0.05688)(x_{23}-0) + 0.03665(x_{31}-0) + (-0.034829)(x_{32}-7) \\
&\quad + (-0.051450)(x_{33}-0) = \lambda_1 \\
\mu_2 &= 1 + (-0.032441)(x_{11}-0) + (-0.010996)(x_{12}-12) + (-0.033920)(x_{13}-0) + 0.0051433(x_{21}-9) \\
&\quad + (-0.028995)(x_{22}-1) + (-0.003555)(x_{23}-0) + (-0.018928)(x_{31}-0) + (-0.0204048)(x_{32}-0) \\
&\quad + (0.005911)(x_{33}-20) = \lambda_2 \\
\mu_3 &= 1 + (-0.0066583)(x_{11}-0) + (-0.015949)(x_{12}-0) + 0.0421698(x_{13}-12) + 0.000482(x_{21}-0) \\
&\quad + (-0.034077)(x_{22}-13) + (-0.054714)(x_{23}-0) + 0.031127(x_{31}-9) + (-0.040598)(x_{32}-0) \\
&\quad + (-0.038131)(x_{33}-9) = \lambda_3
\end{aligned}$$

Then, the FGP model for solving the above problem will be as follows:

$$\left.\begin{array}{ll}\text{Minimize} & \delta_1+\delta_2+\delta_3\\ \text{subject to} & \lambda_1+\delta_1=1\\ & \lambda_2+\delta_2=1\\ & \lambda_3+\delta_3=1\\ & x_{11}+x_{12}+x_{13}\le 12;\ x_{21}+x_{22}+x_{23}\le 15; x_{31}+x_{32}+x_{33}\le 20\\ & x_{11}+x_{21}+x_{31}=9; x_{12}+x_{22}+x_{23}=13;\ x_{31}+x_{32}+x_{33}=21\\ & \delta_1,\ \delta_2,\ \delta_3\ge 0\\ & x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33}\ge 0\ \text{ and integers.}\end{array}\right\}$$

By solving the above problem using LINGO-13, we get the following results:
$(x_{11}^*, x_{12}^*, x_{13}^*, x_{21}^*, x_{22}^*, x_{23}^*, x_{31}^*, x_{32}^*, x_{33}^*) = (0,0,12,9,0,2,0,13,7)$ with minimum cost C = 1.187085, Damage Charges, D = 1.560027, and Time = 1.486371, respectively.

Case-2 (Maleki ranking function): To convert the triangular fuzzy data into crisp form. Maleki ranking function is used as follows.

The crisp form of fuzzy number (3, 5, 7) on applying Maleki Ranking will be

$$\Re_M(\tilde{C}_{11}^N)=\Re_M(3,5,7)=2*5+\frac{1}{2}(7-2*5+3)=10$$
$$\Re_M(1,3,5)=6, \Re(4,7,8)=13.$$

Similarly, the Yager's ranking index of each element can be obtained.

$$\Re_M(\tilde{C}_{12}^n)=13, \Re_M(\tilde{C}_{12}^d)=7, \Re_M(\tilde{C}_{13}^n)=30.5, \Re_M(\tilde{C}_{13}^d)=26, \Re_M(\tilde{C}_{21}^n)=16, \Re_M(\tilde{C}_{21}^d)=25, \Re_M(\tilde{C}_{22}^n)=33 \text{ and so on.}$$
$$\Re_M(\tilde{T}_{11}^n)=33.5, \Re_M(\tilde{T}_{11}^d)=18.5, \Re_M(\tilde{T}_{12}^n)=9.5, \Re_M(\tilde{T}_{12}^d)=4.5, \Re_M(\tilde{T}_{13}^n)=20, \Re_M(\tilde{T}_{13}^d)=5, \Re_M(\tilde{T}_{21}^n)=3 \text{ and so on.}$$
$$\Re_M(\tilde{D}_{11}^n)=25, \Re_M(\tilde{D}_{11}^d)=16.5, \Re_M(\tilde{D}_{12}^n)=29.5, \Re_M(\tilde{D}_{12}^d)=17.5, \Re_M(\tilde{D}_{13}^n)=15.5, \Re_M(\tilde{D}_{13}^d)=21.5 \text{ and so on.}$$

Here *n* and *d* in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:

$$\left.\begin{array}{ll}\text{Minimize } Z_1= & \dfrac{(10x_{11}+13x_{12}+30.5x_{13}+16x_{21}+33x_{22}+24x_{23}+28x_{31}+19x_{32}+25.5x_{33})}{(6x_{11}+7x_{12}+26x_{13}+25x_{21}+28x_{22}+13.5x_{23}+29x_{31}+12x_{32}+15.5x_{33})}\\ \text{Minimize } Z_2= & \dfrac{(33.5x_{11}+9.5x_{12}+20x_{13}+3x_{21}+21x_{22}+11.5x_{23}+25x_{31}+33.5x_{32}+21x_{33})}{(18.5x_{11}+4.5x_{12}+5x_{13}+5x_{21}+8x_{22}+9.5x_{23}+16x_{31}+23.5x_{32}+22.5x_{33})}\\ \text{Minimize } Z_3= & \dfrac{(25x_{11}+29.5x_{12}+15.5x_{13}+29.5x_{21}+28x_{22}+37x_{23}+13x_{31}+29.5x_{32}+33.5x_{33})}{(16.5x_{11}+17.5x_{12}+21.5x_{13}+21.5x_{21}+12x_{22}+13.5x_{23}+17x_{31}+11.5x_{32}+15x_{33})}\\ \text{subject to} & x_{11}+x_{12}+x_{13}\le 12\\ & x_{21}+x_{22}+x_{23}\le 15\\ & x_{31}+x_{32}+x_{33}\le 20\\ & x_{11}+x_{21}+x_{31}=9\\ & x_{12}+x_{22}+x_{23}=13\\ & x_{31}+x_{32}+x_{33}=21\\ & x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33}\ge 0\ \text{ and integers.}\end{array}\right\}$$

We will follow the steps given in section (4) and proceeding in the same manner as above. We will obtain the final result as follows.
$(x_{11}^*, x_{12}^*, x_{13}^*, x_{21}^*, x_{22}^*, x_{23}^*, x_{31}^*, x_{32}^*, x_{33}^*) = (0,0,12,9,0,2,0,13,7)$ with minimum cost C = 1.187085, Damage Charges, D = 1.560027, and Time = 1.486371, respectively.

Problem 2: FFMOFTP WITH TRAPEZOIDAL FUZZY DATA

Fuzzy Cost Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(4,6,8,10)}{(1,3,4,6)}$	$\frac{(3,6,9,11)}{1,3,6,9}$	$\frac{(12,13,15,17)}{(10,11,13,16)}$	$\le (12,15,17,20)$
a_2	$\frac{(4,6,8,10)}{(10,13,15,18)}$	$\frac{(12,15,17,18)}{(10,12,16,17)}$	$\frac{(11,13,15,17)}{(6,9,12,14)}$	$\le (16,18,22,24)$
a_3	$\frac{(12,13,16,18)}{(10,12,15,17)}$	$\frac{(8,10,12,14)}{(4,6,9,13)}$	$\frac{(10,13,15,18)}{(8,10,11,12)}$	$\le (20,23,25,28)$
Demand	=(8,10,14,16)	=(10,11,13,14)	=(11,13,15,17)	

Fuzzy Time Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(12,15,18,21)}{(6,10,14,16)}$	$\frac{(2,4.6,10)}{(1,3,7,9)}$	$\frac{(6,10,14,16)}{(2,5,8,11)}$	$\le (12,15,17,20)$
a_2	$\frac{(4,5,7,9)}{(2,5,8,11)}$	$\frac{(8,10,14,16)}{(3,4,5,9)}$	$\frac{(6,8,10,13)}{(4,5,9,11)}$	$\le (16,18,22,24)$
a_3	$\frac{(10,13,16,19)}{(8,11,14,17)}$	$\frac{(13,15,19,22)}{(6,10,13,15)}$	$\frac{(7,9,13,15)}{(5,7,9,10)}$	$\le (20,23,25,28)$
Demand	=(8,10,14,16)	=(10,11,13,14)	=(11,13,15,17)	

Fuzzy Damage Matrix

	b_1	b_2	b_3	Supply
a_1	$\frac{(10,13,15,19)}{(8,9,11,15)}$	$\frac{(13,16,17,20)}{(6,7,9,11)}$	$\frac{(7,8,10,12)}{(8,9,10,11)}$	$\le (12,15,17,20)$
a_2	$\frac{(14,16,18,20)}{(6,9,13,15)}$	$\frac{(12,14,18,20)}{(6,8,10,12)}$	$\frac{(14,18,20,24)}{(3,5,7,9)}$	$\le (16,18,22,24)$
a_3	$\frac{(3,4,8,10)}{(4,6,8,10)}$	$\frac{(13,15,17,19)}{(3,4,7,9)}$	$\frac{(13,15,17,21)}{(5,7,,9,12)}$	$\le (20,23,25,28)$
Demand	=(8,10,14,16)	=(10,11,13,14)	=(11,13,15,17)	

Using the above trapezoidal data, the FFMOFTP has been formulated. It cannot be solved by any standard methods. To solve this problem, convert it into crisp problem by using ranking functions.

Case-1 (Yager's ranking function): To convert the trapezoidal fuzzy data into crisp form. Yager's ranking function is used as follows.

The α-cut of fuzzy number (4, 6, 8, 10) is $[C_\alpha^L, C_\alpha^U]$ = (4+2α, 10-2α) for which,

$$\Re_{yt}(\tilde{C}_{11}^N) = \Re_{yt}(4,6,8,10) = \int_0^1 0.5(C_\alpha^L + C_\alpha^U)d\alpha = 7$$

$$\Re_{yt}(3,6,9,11) = 7.25, \Re_{yt}(12,13,15,17) = 14.25.$$

Similarly, the Yager's ranking index of each element can be obtained.

$$\Re_{yt}(\tilde{C}_{12}^n) = 7.25, \Re_{yt}(\tilde{C}_{12}^d) = 4.25, \Re_{yt}(\tilde{C}_{13}^n) = 14.25, \Re_{yt}(\tilde{C}_{13}^d) = 12.5, \Re_{yt}(\tilde{C}_{21}^n) = 7\, \Re_{yt}(\tilde{C}_{21}^d) = 14, \Re_{yt}(\tilde{C}_{22}^n) = 15.5 \text{ and so on.}$$

$$\Re_{yt}(\tilde{T}_{11}^n) = 16.5\, \Re_{yt}(\tilde{T}_{11}^d) = 11.5, \Re_{yt}(\tilde{T}_{12}^n) = 5.5, \Re_{yt}(\tilde{T}_{12}^d) = 5, \Re_{yt}(\tilde{T}_{13}^n) = 11.5, \Re_{yt}(\tilde{T}_{13}^d) = 6.5, \Re_{yt}(\tilde{T}_{21}^n) = 6.25 \text{ and so on.}$$

$$\Re_{yt}(\tilde{D}_{11}^n) = 14.25, \Re_{yt}(\tilde{D}_{11}^d) = 10.75, \Re_{yt}(\tilde{D}_{12}^n) = 16.5, \Re_{yt}(\tilde{D}_{12}^d) = 8.25, \Re_{yt}(\tilde{D}_{13}^n) = 9.25, \Re_{yt}(\tilde{D}_{13}^d) = 9.5 \text{ and so on.}$$

Here *n* and *d* in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:-

$$\left.\begin{aligned}
&\text{Minimize } Z_1 = \frac{(7x_{11}+7.25x_{12}+14.25x_{13}+7x_{21}+15.5x_{22}+14x_{23}+14.75x_{31}+11x_{32}+14x_{33})}{(3.5x_{11}+4.25x_{12}+12.5x_{13}+14x_{21}+13.75x_{22}+10x_{23}+13.5x_{31}+8x_{32}+10.25x_{33})}\\
&\text{Minimize } Z_2 = \frac{(16.5x_{11}+5.5x_{12}+12x_{13}+6.25x_{21}+12x_{22}+9.25x_{23}+14.5x_{31}+17.25x_{32}+10.5x_{33})}{(11.5x_{11}+5x_{12}+6.5x_{13}+6.25x_{21}+5.25x_{22}+7.25x_{23}+12.5x_{31}+11x_{32}+7.75x_{33})}\\
&\text{Minimize } Z_3 = \frac{(14.25x_{11}+11.5x_{12}+9.25x_{13}+17x_{21}+16x_{22}+19x_{23}+6.25x_{31}+16x_{32}+16.5x_{33})}{(10.75x_{11}+8.25x_{12}+9.5x_{13}+10.75x_{21}+9x_{22}+6x_{23}+7x_{31}+5.75x_{32}+8.25x_{33})}\\
&\text{subject to} \quad x_{11}+x_{12}+x_{13}=16\\
&\qquad x_{21}+x_{22}+x_{23}\le 20\\
&\qquad x_{31}+x_{32}+x_{33}\le 24\\
&\qquad x_{11}+x_{21}+x_{31}=12\\
&\qquad x_{12}+x_{22}+x_{23}=12\\
&\qquad x_{31}+x_{32}+x_{33}=14\\
&\qquad x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33}\ge 0 \text{ and integers.}
\end{aligned}\right\}$$

The pay-off matrix is obtained after solving the above problem separately for each objective function using the optimizing software LINGO is as follows:

$$\text{Payoff Matrix} = \begin{array}{c} \\ x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ x_{ij}^{(3)} \end{array}\begin{array}{c} \begin{array}{ccc} Z_1(x) & Z_2(x) & Z_3(x) \end{array} \\ \begin{pmatrix} 0.927899 & 0.980026 & 1.128099 \\ 1.602083 & 1.211087 & 1.558484 \\ 1.454420 & 1.888048 & 1.228748 \end{pmatrix}\end{array}$$

$C_L = 0.927899, C_U = 1.128099, T_L = 1.228748, T_U = 1.888048, D_L = 1.211087, D_U = 1.602083.$

The non-linear membership functions for objective functions $C(x), T(x)$ and $D(x)$ are obtained as follows:

$$\mu_1 = \frac{1.228748 - Z_1}{1.228748 - 0.927899},\ \mu_2 = \frac{1.602083 - Z_2}{1.602083 - 1.211087},\ \mu_3 = \frac{1.888048 - Z_3}{1.888048 - 1.228748}$$

We will follow the steps given in section 4 and will obtain the final result as follows.
$(x_{11}^*, x_{12}^*, x_{13}^*, x_{21}^*, x_{22}^*, x_{23}^*, x_{31}^*, x_{32}^*, x_{33}^*) = (0,2,14,12,8,0,0,2,0)$ with minimum cost C = 0.927899, Damage Charges, D = 1.454420, and Time = 1.602083, respectively.

Case-2 (Maleki ranking function): To convert the triangular fuzzy data into crisp form. Maleki ranking function is used as follows.

The crisp form of fuzzy number (3, 5, 7) on applying Maleki Ranking will be

$$\Re_M(\tilde{C}_{11}^N) = \Re_M(3,5,7) = 2*5 + \frac{1}{2}(7 - 2*5 + 3) = 10$$

$$\Re_M(1,3,5) = 6, \Re(4,7,8) = 13.$$

Similarly, the Yager's ranking index of each element can be obtained.

$$\Re_M(\tilde{C}_{12}^n) = 13, \Re_M(\tilde{C}_{12}^d) = 7, \Re_M(\tilde{C}_{13}^n) = 30.5, \Re_M(\tilde{C}_{13}^d) = 26, \Re_M(\tilde{C}_{21}^n) = 16, \Re_M(\tilde{C}_{21}^d) = 25, \Re_M(\tilde{C}_{22}^n) = 33 \text{ and so on.}$$

$$\Re_M(\tilde{T}_{11}^n) = 33.5, \Re_M(\tilde{T}_{11}^d) = 18.5, \Re_M(\tilde{T}_{12}^n) = 9.5, \Re_M(\tilde{T}_{12}^d) = 4.5, \Re_M(\tilde{T}_{13}^n) = 20, \Re_M(\tilde{T}_{13}^d) = 5, \Re_M(\tilde{T}_{21}^n) = 3 \text{ and so on.}$$

$$\Re_M(\tilde{D}_{11}^n) = 25, \Re_M(\tilde{D}_{11}^d) = 16.5, \Re_M(\tilde{D}_{12}^n) = 29.5, \Re_M(\tilde{D}_{12}^d) = 17.5, \Re_M(\tilde{D}_{13}^n) = 15.5, \Re_M(\tilde{D}_{13}^d) = 21.5 \text{ and so on.}$$

Here *n* and *d* in the superscripts denote the numerator and denominator values.

Using the above data, the MOFTP with mixed constraints can be given as follows:

$$\left.\begin{array}{l}
\text{Minimize } Z_1 = \dfrac{(14x_{11}+14.5x_{12}+28.5x_{13}+14x_{21}+31x_{22}+28x_{23}+29.5x_{31}+22x_{32}+28x_{33})}{(7x_{11}+9.5x_{12}+25x_{13}+28x_{21}+27.5x_{22}+20x_{23}+27x_{31}+16x_{32}+20.5x_{33})} \\
\text{Minimize } Z_2 = \dfrac{(33x_{11}+11x_{12}+24x_{13}+12.5x_{21}+24x_{22}+18.5x_{23}+29x_{31}+34.5x_{32}+21x_{33})}{(23x_{11}+10x_{12}+13x_{13}+12.5x_{21}+10.5x_{22}+14.5x_{23}+25x_{31}+22x_{32}+15.5x_{33})} \\
\text{Minimize } Z_3 = \dfrac{(28.5x_{11}+33x_{12}+18.5x_{13}+34x_{21}+32x_{22}+38x_{23}+12.5x_{31}+32x_{32}+33x_{33})}{(21.5x_{11}+16.5x_{12}+19x_{13}+21.5x_{21}+18x_{22}+12x_{23}+14x_{31}+11.5x_{32}+16.5x_{33})} \\
\text{subject to} \quad x_{11}+x_{12}+x_{13}=16 \\
\qquad x_{21}+x_{22}+x_{23}\le 20 \\
\qquad x_{31}+x_{32}+x_{33}\le 24 \\
\qquad x_{11}+x_{21}+x_{31}=12 \\
\qquad x_{12}+x_{22}+x_{23}=12 \\
\qquad x_{31}+x_{32}+x_{33}=14 \\
\qquad x_{11},x_{12},x_{13},x_{21},x_{22},x_{23},x_{31},x_{32},x_{33}\ge 0 \text{ and integers.}
\end{array}\right\}$$

We will follow the steps given in section 4 and will obtain the final result as follows.
$(x^*_{11}, x^*_{12}, x^*_{13}, x^*_{21}, x^*_{22}, x^*_{23}, x^*_{31}, x^*_{32}, x^*_{33}) = (0, 2,14,12,8,0,0,2,0)$ with minimum cost C = 0.927899, Damage Charges, D = 1.454420, and Time = 1.602083, respectively.

5. RESULT

On solving the above problems using LINGO-13, we have obtained the following results of the given problem. The result will remain same on applying Yager as well as Maleki Ranking method. So, we can use any of the two ranking function. As Maleki function is comparatively easy to apply, so, it's favourable to use it.

TABLE-1:

Problem1	x^*_{11}	x^*_{12}	x^*_{13}	x^*_{21}	x^*_{22}	x^*_{23}	x^*_{31}	x^*_{32}	x^*_{33}	Min. Cost	Min. Damage	Min. Time
Yager's method	0	0	12	9	0	2	0	13	7	1.187085	1.560027	1.486371
Maleki method	0	0	12	9	0	2	0	13	7	1.187085	1.560027	1.48637

The results obtained for problem 1 areminimum cost C = 1.187085, damage charges, D = 1.560027, and time = 1.486371, respectively. The results are same for both the ranking functions.

TABLE-2:

Problem 2	x^*_{11}	x^*_{12}	x^*_{13}	x^*_{21}	x^*_{22}	x^*_{23}	x^*_{31}	x^*_{32}	x^*_{33}	Min. Cost	Min. Damage	Min. Time
Yager's method	0	2	14	12	8	0	0	2	0	0.927899	1.454420	1.602083
Maleki method	0	2	14	12	8	0	0	2	0	0.927899	1.454420	1.602083

The results obtained for problem 2 are minimum cost C =0.927899, damage charges, D = 1.45442, and time = 1.602083, respectively. The results are same for both the ranking functions.

6. CONCLUSION AND DISCUSSIONS

In the present work, a fuzzy programming approach is used to find a compromise solution forthe fully fuzzy multi-objective fractional transportation problem. The proposed approach has the following key features:

1. It provides the decision maker with a simple and easy approach to solve fully fuzzy MOFTP.
2. Two cases are discussed one with triangular fuzzy numbers and other with trapezoidal fuzzy numbers.
3. Two ranking methods i.e. Yagers and Maleki ranking methods are used to convert the fuzzy problem into crisp problem to check the efficiency of the two ranking methods from the obtained results.
4. The remarkable conclusion obtained from solution table 1 and 2 is that the results provided by Yagers ranking method and Maleki ranking methods are same. So, any of the two methods can be used for solving the problem. However, it is advisable to use Maleki method to avoid mathematical errors as it is comparatively easier.
5. The provided approach can be very useful while dealing with fully fuzzy MOFTP and similar real life problems involving uncertainty of the coefficients of the parameters or demands and supplies.
6. The approach can also be used for solving other fractional programming problems.

7. REFERENCES

1. Adlakha, V., Kowalski, K., & Vemuganti, R. R., Heuristic algorithms for the fixed-charge transportation problem. Opsearch, New Delhi, 43(2), (2006), 132.
2. Ammar, E. E., & Youness, E. A., Study on multiobjective transportation problem with fuzzy numbers. Applied Mathematics and Computation, 166(2), (2005), 241-253.
3. Chanas, S., Kołodziejczyk, W., & Machaj, A., A fuzzy approach to the transportation problem. Fuzzy Sets and Systems, 13(3), (1984), 211-221.
4. Dubois, D., & Prade, H., Systems of linear fuzzy constraints. Fuzzy sets and systems, 3(1), (1980), 37-48.
5. El-Wahed, W. F. A., A multi-objective transportation problem under fuzziness. Fuzzy Sets and Systems, 117(1), (2001), 27-33.
6. Hitchcock, F. L., The distribution of a product from several sources to numerous localities. J. Math. Phys., 20(2), (1941), 224-230.
7. Joshi, V. D., & Gupta, N., Linear fractional transportation problem with varying demand and supply. Le Matematiche, 66(2), (2011), 3-12.
8. Kaur, A., & Kumar, A., A new method for solving fuzzy transportation problems using ranking function. Applied Mathematical Modelling, 35(12), (2011), 5652-5661.
9. Kaur, A., & Kumar, A., A new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers. Applied Soft Computing, 12(3), (2012), 1201-1213.
10. Khurana, A., & Arora, S. R., The sum of a linear and a linear fractional transportation problem with restricted and enhanced flow. Journal of Interdisciplinary Mathematics, 9(2), (2006), 373-383.
11. Lee, S. M., & Moore, L. J., Optimizing transportation problems with multiple objectives. AIIE transactions, 5(4),(1973), 333-338.
12. Liu, S. T. Fractional transportation problem with fuzzy parameters. Soft Computing, 20(9), (2016), 3629-3636.
13. Maleki, H.R. "Ranking functions and their applications to fuzzy linearprogramming", Far East Journal of Mathematical Science, 4,(2002), 283-301
14. Mukherjee, S., & Basu, K., Application of Fuzzy Ranking Method for Solving Assignment Problems with Fuzzy Costs. International Journal of Computational & Applied Mathematics 5(3), (2010), 359-368.
15. Porchelvi, R. S., & Sheela, A. A Linear Fractional Interval Transportation Problem with and Without Budgetary Constraints.
16. Pramanik, S., & Roy, T. K., Multiobjective transportation model with fuzzy parameters: priority based fuzzy goal programming approach. Journal of Transportation Systems Engineering and Information Technology, 8(3), (2008), 40-48.
17. Sadia, S., Gupta, N., & Ali, Q. M., Multiobjective Capacitated Fractional Transportation Problem with Mixed Constraints. Mathematical Sciences Letters. 5(3), (2016), 235-242.
18. Sakawa, M., Fuzzy sets and interactive multiobjective optimization. Springer Science & Business Media. (2013).
19. Swarup, K., Transportation technique in linear fractional functional programming. Journal of Royal Naval Scientific Service, 21(5), (1966), 256-260.
20. Tzeng, G. H., & Huang, J. J., Fuzzy multiple objective decision making. CRC Press, (2013).
21. Verdegay, J. L., A dual approach to solve the fuzzy linear programming problem. Fuzzy sets and systems, 14(2),(1984), 131-141.
22. Verma, V., & Puri, M. C., On a paradox in linear fractional transportation problem. Recent Developments in Australian Society of Operational Research, Gordan and Breach Science Publishers, (1991). 413-424.
23. Yager, R. R., A procedure for ordering fuzzy subsets of the unit interval. Information sciences, 24(2), (1981).143-161.
24. Zadeh, L. A., Fuzzy sets. Information and control, 8(3), (1965), 338-353.
25. Zimmermann H. J., Fuzzy sets, decision making and expert systems. Dordrecht: Kluwer Academic Publishers, (1987).
26. Zimmermann H. J., Fuzzy set theory - and its applications. Boston: Kluwer Academic Publishers, (1996).
27. Zimmermann, H. J., Fuzzy programming and linear programming with several objective functions. Fuzzy sets and systems, 1(1), (1978), 45-55.
28. Zimmermann, H. J., Applications of fuzzy set theory to mathematical programming. Information sciences, 36(1), (1985), 29-58.

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