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TOTAL RESOLVING NUMBER OF EDGE CYCLE GRAPHS G(C_3)

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ABSTRACT

Let G = (V, E) be a simple connected graph. An ordered subset W of V is said to be a resolving set of G if every vertex is uniquely determined by its vector of distances to the vertices in W. The minimum cardinality of a resolving set is called the resolving number of G and is denoted by r(G). Total resolving number as the minimum cardinality taken over all resolving sets in which $\langle W \rangle$ has no isolates and is denoted by tr(G). In this paper, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$. Also, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.

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Keywords: *resolving number, total resolving number, edge cycle graph.*

1. INTRODUCTION

Let G = (V, E) be a finite, simple, connected and undirected graph. The *degree* of a vertex v in a graph G is the number of edges incident with v and it is denoted by d(v). The maximum degree in a graph G is denoted by $\Delta(G)$ and the minimum degree is denoted by $\delta(G)$. The *distance* d(u, v) between two vertices u and v in G is the length of a shortest u- v path in G. The maximum value of distance between vertices of G is called its *diameter*. Let P_n denote any *path* on n vertices, C_n denote any *cycle* on n vertices and K_n denote any *complete graph* on n vertices. A *complete bipartite graph* is denoted by $K_{s,\,t}$. $K_{1,\,n-1}$ is called a *star*. A tree containing exactly two vertices that are not end vertices is called a *bistar* and it is denoted by $B_{s,\,t}$. The *join* G + H consists of G ∪ H and all edges joining a vertex of G and a vertex of H. Let P denote the set of all pendant edges of G and |P| = p. Vertices which are adjacent to pendant vertices are called *support vertices*.

A graph H is called a subgraph of a graph G if V(H) ⊆ V(G) and E(H) ⊆ E(G). A subgraph F if a graph G is called an *induced subgraph* $\langle F \rangle$ of G if whenever u and v are vertices of F and uv is an edge of G, then uv is an edge of F as well. For a cut vertex v of a connected graph G, suppose that the disconnected graph G \ {v} has k components $G_1, G_2, \ldots, G_k$ $(k \geq 2)$. The induced subgraphs $B_i = G[V(G_i) \cup \{v\}]$ are connected and referred to as the *brances* of G at v. The *complement* G^c of a graph G is that graph whose vertex set is V(G) and such that for each pair u, v of vertices of G, uv is an edge of G^c if and only if uv is not an edge of G. A vertex v in a graph G is called *complete vertex* if the subgraph by its neighborhood is complete. For an integer $s \geq 2$, $sK_2 + K_1$ is called the *friendship graph* and is denoted by F_s.

If $W = \{w_1, w_2, \ldots, w_k\} \subseteq V(G)$ is an ordered set, then the ordered k-tuple $(d(v, w_1), d(v, w_2), \ldots, d(v, w_k))$ is called the representation of v with respect to W and it is denoted by $r(v \mid W)$. Since the representation for each $w_i \in W$ contains exactly one 0 in the i^{th} position, all the vertices of W have distinct representations. W is called a resolving set for G if all the vertices of V \ W also have distinct representations. The minimum cardinality of a resolving set is called the resolving number of G and it is denoted by r(G).

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In 1975, Slater [9] introduced these ideas and used *locating set* for what we have called *resolving set*. He referred to the cardinality of a minimum resolving set in G as its *location number*. In 1976, Harary and Melter [4] discovered these concepts independently as well but used the term metric dimension rather than location number. In 2003, Ping Zhang and Varaporn Saenpholphat [7, 8] studied *connected resolving number* and in 2015, we introduced and studied *total resolving number*. In this paper, we use the term *resolving number* to maintain uniformity in the current literature.

If W is a resolving set and the induced subgraph $\langle W \rangle$ has no isolates, then W is called a *total resolving set* of G. The minimum cardinality taken over all total resolving sets of G is called the *total resolving number* of G and is denoted by tr(G). We introduced edge cycle graph in [5] and studied the resolving number of edge cycle graph $G(C_k)$. An *edge cycle graph* of a graph G is the graph $G(C_k)$ formed from one copy of G and $|E(G)|$ copies of P_k, where the ends of the i^{th} edge are identified with the ends of i^{th} copy of P_k.

In this paper, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$. Also, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.

2. BUILDING BLOCKS

The following results are used in the subsequent sections.

Theorem 2.1: [6] *Let $\{w_1, w_2\} \subset V(G)$ be a total resolving set in G. Then the degrees of w_1 and w_2 are at most 3.*

Lemma 2.2: [6] *For $n \geq 3$, $tr(P_n) = 2$ and $tr(C_n) = 2$.*

Observation 2.3: [6] *Let G be a graph of order $n \geq 3$. Then $2 \leq tr(G) \leq n-1$.*

Theorem 2.4: [6] *Let G be a graph of order $n \geq 3$. Then $tr(G) = n-1$ if and only if $G = K_n$ or $K_{1,n-1}$.*

Definition 2.5: *A block of G containing exactly one cut vertex of G is called an end block of G.*

Lemma 2.6: [5] *Let G be a 1-connected graph with $\delta(G) \geq 2$. Then every resolving set contains at least one non cut vertex of each end block.*

Corollary 2.7: [5] *If G contains b end blocks, then $r(G) \geq b$.*

Definition 2.8: *A cycle C_r is called an end cycle if C_r contains exactly one vertex of degree at least 3.*

Notation 2.9: *Let e_c denote the number of end cycles of the graph G.*

Theorem 2.11: [6] *Let T be a tree of order $n \geq 3$. Then $r(T(C_3)) = p$.*

In this paper, we investigate the total resolving number of the edge cycle graphs $G(C_3)$.

3. TOTAL RESOLVING NUMBER OF EDGE CYCLE GRAPHS $G(C_3)$

In this section, we determine the exact values for the total resolving number of $T(C_3)$, $C_n(C_3)$ and $F_s(C_3)$.

Observation 3.1: *For $n = 3, 4, 5$, $tr(C_n(C_3)) = 3$*

Theorem 3.2: *For $n \geq 6$, $tr(C_n(C_3)) = 4$.*

Proof: Let $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, $E(C_n) = \{v_1v_2, v_2v_3, \ldots, v_nv_1\}$ and $u_1, u_2, \ldots, u_n$ be the new vertices in $C_n(C_3)$ corresponding to the edges $v_1v_2, v_2v_3, \ldots, v_nv_1$. Then $V(C_n(C_3)) = V \cup U$, where $V = V(C_n)$, $U = \{u_1, u_2, \ldots, u_n\}$ and $E(C_n(C_3)) = E(C_n) \cup \{u_iv_i, u_iv_{i+1} / 1 \leq i \leq n-1\} \cup \{u_nv_n, u_nv_1\}$. Let W be a total resolving set of $C_n(C_3)$.

First, we claim that $tr(C_n(C_3)) \geq 4$. Suppose that $tr(C_n(C_3)) \leq 3$. By Theorem 2.1, $tr(C_n(C_3)) = 3$. Therefore $\langle W \rangle$ is P_3 or K_3. If $\langle W \rangle$ is K_3, then without loss of generality, let $W = \{v_1, u_2, v_2\}$. Then $r(v_n | W) = r(u_n | W) = (1, 2, 3)$, which is a contradiction. If $\langle W \rangle$ is P_3, then without loss of generality, let $W \subseteq \{v_1, v_2, v_3, u_1, u_2\}$. Then $r(v_n | W) = r(u_n | W)$, which is a contradiction. Thus $tr(C_n(C_3)) \geq 4$.

Let W = $\left\{v_1, v_2, v_{\left\lfloor\frac{n}{2}\right\rfloor+1}, v_{\left\lfloor\frac{n}{2}\right\rfloor+2}\right\}$. Let x, y be two distinct vertices of $V(C_n(C_3)) \setminus W$. If $d(x, v_1) \neq d(y, v_1)$ or $d(x, v_2) \neq d(y, v_2)$, then $r(x \mid W) \neq r(y \mid W)$. So we may assume that $d(x, v_1) = d(y, v_1)$ or $d(x, v_2) = d(y, v_2)$. Then $x \in U$ and $y \in V$ or $x \in V$ and $y \in U$. Without loss of generality, let $x \in U$ and $y \in V$. But $d(x, v_{\left\lfloor\frac{n}{2}\right\rfloor+1}) = d(y, v_{\left\lfloor\frac{n}{2}\right\rfloor+1}) + 1$ and $d(x, v_{\left\lfloor\frac{n}{2}\right\rfloor+2}) = d(y, v_{\left\lfloor\frac{n}{2}\right\rfloor+2}) + 1$. It follows that $r(x \mid W) \neq r(y \mid W)$. Thus W is a resolving set of $C_n(C_3)$ and $\langle W\rangle$ has no isolates, $tr(C_n(C_3)) \leq 4$. Hence $tr(C_n(C_3)) = 4$.

Lemma 3.3: *Let G be a graph of order $n \geq 3$ and $\delta(G) = 1$. Then $tr(G(C_3)) \geq p + s$.*

Proof: Let W be a total resolving set of $G(C_3)$. Let $B_1, B_2, \ldots, B_p$ be the end blocks of $G(C_3)$. Then by Lemma 2.6, $W \cap V(B_i) \neq \emptyset$, for all $1 \leq i \leq p$. Since W is a total resolving set, $|W \cap V(B_i)| \geq 2$ for all $1 \leq i \leq p$. But some end blocks have the common vertex, $|W \cap V(G(C_3))| \geq p + s$ and hence $tr(G(C_3)) \geq p + s$.

Theorem 3.4: *Let T be a tree of order at least 3. Then $tr(T(C_3)) = p + s$.*

Proof: The proof follows from Theorem 2.11 and Lemma 3.3.

Corollary 3.5: *For $n \geq 4$, $tr(P_n(C_3)) = 4$.*

Corollary 3.6: *For $n \geq 2$, $tr(K_{1, n-1}(C_3)) = n$.*

Corollary 3.7: *For $s, t \geq 1$, $tr(B_{s,t}(C_3)) = s + t + 2$.*

Theorem 3.8: *For $s \geq 2$, $tr(F_s(C_3)) = 2s$.*

Proof: Let $V(F_s)=\{u, u_{11}, u_{12}, u_{21}, u_{22}, \ldots, u_{s1}, u_{s2}\}$ and

$E(F_s) = \{uu_{ij} / 1 \leq i \leq s \text{ and } j = 1, 2\} \cup \{u_{11}u_{12}, u_{21}u_{22}, \ldots, u_{s1}u_{s2}\}$.

For $1 \leq j \leq s$, let v_i be the new vertex of the edge $u_{i1}u_{i2}$, v_{j1} be the new vertex of the edge uu_{j1} and v_{j2} be the new vertex of the edge uu_{j2} in $F_s(C_3)$. Then we have G contains exactly s blocks, say $B_1, B_2, \ldots, B_s$. Let W be a total resolving set of $F_s(C_3)$.

First, we claim that $tr(F_s(C_3)) \geq 2s$. Suppose that $tr(F_s(C_3)) \leq 2s - 1$. Then we have W contains at most three vertices from union of two blocks. Without loss of generality, let B_1 and B_2 be such blocks. Then we have $|W \cap (V(B_1) \cup V(B_2))| \leq 3$. By Lemma 2.6, $|W \cap (V(B_1) \setminus \{u\}| \neq \emptyset$ and $|W \cap (V(B_2) \setminus \{u\}| \neq \emptyset$. Let $u, x, y \in W$, where $x \in N(u) \cap V(B_1)$ and $y \in N(u) \cap V(B_2)$. Then $d(x) = 2$ or 4 in $F_n(C_3)$. If $d(x) = 2$, then without loss of generality, let $x = v_{11}$. But we have $r(u_{12} \mid W) = r(v_{12} \mid W)$. If $d(x) = 4$, then without loss of generality, let $x = u_{11}$. But we have $r(v_{11} \mid W) = r(u_{12} \mid W)$, which is a contradiction. Hence $tr(F_s(C_3)) \geq 2s$.

Next, we claim that $tr(F_s(C_3)) \leq 2s$. Now, let $W = \{u_{11}, u_{21}, \ldots, u_{s1}\} \cup \{u_{12}, u_{22}, \ldots, u_{s2}\}$. Let x, y be two distinct vertices of $V(F_s(C_3)) \setminus W$. Then we consider the following two cases.

Case-1: $x, y \in V(B_i)$ for some $1 \leq i \leq s$.

Without loss of generality, let $x, y \in V(B_1)$. If $d(x, w) \neq d(y, w)$ for some $w \in W \cap (V(B_1)$, then $r(x \mid W) \neq r(y \mid W)$. So we may assume that $d(x, w) = d(y, w)$ for all $w \in W \cap (V(B_1))$. Then $x = v_1$ and $y = u$. But $3 = d(x, w) > d(y, w) = 1$. It follows that $r(x \mid W) \neq r(y \mid W)$.

Case-2: $x \in V(B_i)$, $y \in V(B_j)$ for some $1 \leq i \neq j \leq s$.

Then clearly, $d(x, w) < d(y, w)$ for all $w \in W \cap V(B_i)$. It follows that $r(x \mid W) \neq r(y \mid W)$.

Thus W is a resolving set and $\langle W\rangle$ has no isolates, $tr(F_3(C_3)) \leq 2s$. Hence $tr(F_3(C_3)) = 2s$.

GENERAL BOUNDS AND EXTREMAL GRAPHS

In this section, we obtain bounds for the total resolving number of $G(C_3)$ and characterize the extremal graphs.

Theorem 4.1: *Let G be a graph of order $n \geq 3$. Then $3 \leq tr(G(C_3)) \leq n$.*

Proof: By Theorem 2.1, $tr(G(C_3)) \geq 3$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and v_{ij} be the new vertex of the edge v_iv_j in $G(C_3)$, where $i, j \in \{1, 2, \ldots, n\}$ and $i \neq j$. Let $W = V(G)$. Then i^{th} and j^{th} coordinates of the representation of v_{ij} are 1. Since $i \neq j$, representation of all v_{ij} are distinct. Therefore $tr(G(C_3)) \leq n$. Hence $3 \leq tr(G(C_3)) \leq n$.

Theorem 4.2: *Let G be a graph of order $n \geq 3$. Then $tr(G(C_3)) = 3$ if and only if $G \cong$* P_3 or K_3 or C_4 or $K_4 \setminus \{e\}$ or K_4 or C_5.

Proof: Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $tr(G(C_3)) = 3$. If $n = 3$, then $G \cong P_3$ or K_3. So we may assume that $n \geq 4$. For $i, j \in \{1, 2, \ldots, n\}$ and $i \neq j$, let v_{ij} be the new vertex of the edge v_iv_j in $G(C_3)$. Let $W = \{w_1, w_2, w_3\}$ be a total resolving set of $G(C_3)$.

Let $\langle W \rangle$ be K_3. If W is not a subset of V(G), then without loss of generality, let $W = \{v_1, v_2, v_{12}\}$. Let $X = V(G) \setminus \{v_1, v_2\}$. Since G is connected, a vertex of X, say v_3 is adjacent to v_1 or v_2 or both. If v_3 is adjacent to v_1 or v_2, say v_1, then $r(v_3 \mid W) = r(v_{13} \mid W) = (1, 2, 2)$, which is a contradiction. If no vertex of X is adjacent to exactly one vertex of $\{v_1, v_2\}$, then a vertex of X, say v_3 is adjacent to v_1 and v_2. Since G is connected and $n \geq 4$, v_3 is adjacent to a vertex of X, say v_4. But we have $r(v_4 \mid W) = r(v_{34} \mid W) = (2, 2, 3)$, which is a contradiction and hence $W \subset V(G)$.

Without loss of generality, let $W = \{v_1, v_2, v_3\}$ and $X = V(G) \setminus W$. Then $r(v_{12} \mid W) = (1, 1, 2)$, $r(v_{23} \mid W) = (2, 1, 1)$, $r(v_{31} \mid W) = (1, 2, 1)$ which shows that no vertex of X has exactly two neighbors in W. If a vertex $v_i \in X$ is adjacent to exactly one vertex of W, say v_j, $j \in \{1, 2, 3\}$, then $r(v_i \mid W) = r(v_{1j} \mid W)$, which is a contradiction. If there exists a vertex of X, say v_i is adjacent to no vertex of W, then $r(v_i \mid W) = r(v_{ik} \mid W)$, where $v_iv_k \in E(G)$, which is a contradiction. Hence each vertex of X is adjacent to all the vertices of W. If $|X| > 1$, then $r(v_4 \mid W) = r(v_5 \mid W) = \ldots = r(v_n \mid W)$, which is a contradiction. Consequently, $|X| = 1$. Hence $X = \{v_4\}$ and $G \cong K_4$.

Let $\langle W \rangle$ be P_3. Then we consider the following two cases.

Case-1: W is a subset of V(G).

Then without loss of generality, let $W = \{v_1, v_2, v_3\}$, where v_2 is adjacent to v_1 and v_3. Then $r(v_{12} \mid W) = (1,1, 2)$ and $r(v_{23} \mid W) = (2, 1, 1)$. Let $X = V(G) \setminus W$. If there exists a vertex $v_i \in X$ which is adjacent to v_2 but not to v_1 and v_3, then $r(v_i \mid W) = r(v_{i2} \mid W) = (2, 1, 2)$ in $G(C_3)$, which is a contradiction. If there exist two distinct vertices $v_i, v_j \in X$ such that v_i is adjacent to v_1 & v_3 and v_j is adjacent to v_1, v_2 & v_3, then $r(v_{i1} \mid W) = r(v_{j1} \mid W) = (1, 2, 2)$ and $r(v_{3i} \mid W) = r(v_{3j} \mid W) = (2, 2, 1)$ in $G(C_3)$, which is a contradiction.

Now, we claim that $|N(W)| = 1$ or 2. Suppose $|N(W)| \geq 4$. Let $N(W) = \{v_4, v_5, \ldots, v_k\}$, $k \geq 7$. Without loss of generality, let v_4 be adjacent to v_1 but not to v_2 and v_3, v_5 be adjacent to v_3 but not to v_1 and v_2, v_6 be adjacent to v_1 & v_2 or v_1, v_2 & v_3. But a vertex of $\{v_7, v_8, \ldots, v_k\}$ is adjacent to v_1 or v_3 or v_1 & v_3. If v_7 is adjacent to v_1 or v_3, say v_1, then $r(v_{14} \mid W) = r(v_{17} \mid W) = (1, 2, 3)$, which is a contradiction. If v_7 is adjacent to v_1 and v_3, then $r(v_6 \mid W) = r(v_7 \mid W)$, which is a contradiction.

Suppose $|N(W)| = 3$. Then without loss of generality, let $N(W) = \{v_4, v_5, v_6\}$ and v_4 be adjacent to v_1, v_5 be adjacent to v_3 and v_6 be adjacent to either v_1 and v_3 or v_1, v_2 & v_3. If $\langle \{ v_4, v_5, v_6\} \rangle$ is either K_3^c or $K_2 \cup K_1$, then without loss of generality, let v_4 be not adjacent to v_5 and v_6. Then $r(v_4 \mid W) = r(v_{14} \mid W) = (1, 2, 3)$ in $G(C_3)$, which is a contradiction. If $\langle \{ v_4, v_5, v_6\} \rangle$ is either P_3 or K_3, then $r(v_4 \mid W) = r(v_{16} \mid W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction. Hence $|N(W)| = 1$ or 2. Now, we consider the following two subcases.

Subcase-1: $|N(W)| = 1$.
Then without loss of generality, let $N(W) = \{v_4\}$. We claim that $|X| = 1$. Suppose $|V_1| \geq 2$. Then v_4 is a cut vertex of G. Then there are at least two branches at v_4 in $G(C_3)$, say B_1 and B_2. Let $\langle \{ v_1, v_2, v_3, v_4\} \rangle = B_1$. Therefore B_2 contains at least one end block. But no vertex of B_2 belongs to W, which is a contradiction to Lemma 2.6 and hence $X = \{v_4\}$.

If v_4 is adjacent to v_1 but not to v_2 and v_3 in G, then $r(v_4 \mid W) = r(v_{14} \mid W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction. If v_4 is adjacent to v_1 and v_3 but not to v_2, then $G \cong C_4$ and if v_4 is adjacent to v_1, v_2 and v_3, then $G \cong K_4 \setminus \{e\}$.

Subcase-2: $|N(W)| = 2$.
Then without loss of generality, let $N(W) = \{v_4, v_5\}$. Then exactly one vertex of $\{v_4, v_5\}$ is adjacent to exactly one vertex of $\{v_1, v_3\}$. Without loss of generality, let v_4 be adjacent to v_1. If $v_4v_5 \notin E(G)$, then $r(v_4 \mid W) = r(v_{4i} \mid W)$, $i \in \{1, 2, 3\}$, which is a contradiction. Thus $v_4v_5 \in E(G)$.

If v_5 is adjacent to v_3, then we claim that $|V| = 5$. Suppose $|V| > 5$. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$, $n \geq 6$. Let $\langle\{v_4, v_5, v_i\}\rangle \cong P_3$ for some $i \in \{6, 7, \ldots, n\}$. If v_i is adjacent to v_4, then $r(v_i \mid W) = r(v_{4i} \mid W) = (2, 3, 3)$ in $G(C_3)$, which is a contradiction. If $\langle\{v_4, v_5, v_i\}\rangle \cong K_3$ for some $i \in \{6, 7, \ldots, n\}$, then $r(v_i \mid W) = r(v_{45} \mid W) = (2, 3, 2)$ in $G(C_3)$, which is a contradiction. Therefore $|V| = 5$ and hence $G \cong C_5$.

If v_5 is adjacent to v_1 and v_3 in G, then $r(v_{15}|W) = r(v_4|W) = (1, 2, 2)$ in $G(C_3)$, which is a contradiction.

Case-2: W is not a subset of V(G).
Then without loss of generality, let v_1, v_2 and v_3 be three vertices of G such that $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ or K_3, $v_3 \notin W$ and $v_2 \in W$. Let $V_1 = V(G) \setminus X$. Then clearly, no vertex of V_1 is adjacent to v_2 in G, for, if $v_i \in V_1$ is adjacent to v_2 in G, then $r(v_i \mid W) = r(v_{2i} \mid W) = (2, 1, 2)$ in $G(C_3)$, which is a contradiction.

Now, we claim that $|N(X)| = 1$. Suppose $|N(X)| \geq 4$. Let $N(X) = \{v_4, v_5, \ldots, v_k\}$, $k \geq 7$. Then without loss of generality, let v_4 be adjacent to exactly one vertex of $\{v_1, v_3\}$, say v_1, v_5 be adjacent to v_3 not to v_1 and v_6 be adjacent to v_1 and v_3. But a vertex of $\{v_7, v_8, \ldots, v_k\}$ is adjacent to v_1 or v_3 or both. Without loss of generality, let v_7 be adjacent to say v_1. Then $r(v_7 \mid W) = r(v_4 \mid W)$, which is a contradiction and hence $|N(X)| \leq 3$.

If $|N(X)| = 2$, then without loss of generality, let v_4 and v_5 be two vertices in N(X). If v_4 is adjacent to v_1, v_5 is adjacent to v_3 or v_4 is adjacent to v_1 and v_5 is adjacent to v_1 and v_3, then $r(v_4 \mid W) = r(v_{14} \mid W)$ in $G(C_3)$, which is a contradiction.

Let v_4 be adjacent to v_3 and v_5 is adjacent to v_1 and v_3. If W contains exactly one vertex of V, then $r(v_4 \mid W) = r(v_{14} \mid W) = (3, 2, 2)$ in $G(C_3)$, which is a contradiction. If W contains two vertices of V, then by our assumption $v_2 \in W$ and $v_3 \notin W$, $v_1 \in W$. If $\langle\{v_1, v_2, v_3\}\rangle \cong K_3$, then $r(v_4 \mid W) = r(v_{34} \mid W) = (2, 2, 2)$. If $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and $v_4v_5 \notin E(G)$, then $r(v_4 \mid W) = r(v_{34} \mid W) = (3, 2, 2)$. If $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and $v_4v_5 \in E(G)$, then $r(v_4 \mid W) = r(v_{35} \mid W) = (2, 2, 2)$, which is a contradiction.

If $N(X) = 3$, then without loss of generality, let v_4, v_5, v_6 be three vertices in N(X) and v_4 be adjacent to v_1, v_5 be adjacent to v_3, v_6 be adjacent to v_1 and v_3 in G. Then $r(v_4 \mid W) = r(v_{14} \mid W) = (2, 2, 3)$ in $G(C_3)$, which is a contradiction.

Without loss of generality, let $N(X) = \{v_4\}$. We claim that $V_1 = \{v_4\}$. Suppose $V_1 = \{v_4, v_5, \ldots, v_n\}$, $n \geq 5$. If H is $\langle V_1 \rangle$, then $H(C_3)$ contains at least one end block. But no vertex of $H(C_3)$ belongs to W, which is a contradiction to Lemma 2.6. Therefore $X = \{v_4\}$. If v_4 is adjacent to either v_1 or v_3, say v_1, then $r(v_4 \mid W) = r(v_{14} \mid W) = (1, 2, 3)$ in $G(C_3)$, which is a contradiction and hence v_4 is adjacent to v_1 and v_3. But if $\langle\{v_1, v_2, v_3\}\rangle \cong K_3$, then $r(v_4 \mid W) = r(v_{13} \mid W)$ in $G(C_3)$, which is a contradiction. Therefore $\langle\{v_1, v_2, v_3\}\rangle \cong P_3$ and hence in this case, $G \cong C_4$.

Conversely, let $G \cong P_3$ or K_3 or C_4 or $K_4 \setminus \{e\}$ or K_4 or C_5. Let $W = \{v_1, v_2, v_3\}$ and $v_1v_2, v_2v_3 \in E(G)$. Then W is a total resolving set of $G(C_3)$.

Thus $tr(G(C_3)) \leq 3$. By Theorem 4.1, $tr(G(C_3)) \geq 3$ and hence $tr(G(C_3)) = 3$.

Theorem 4.3: *Let G be a graph of order $n \geq 3$. Then $tr(G(C_3)) = n$ if and only if each non support vertex is a complete vertex of degree 2.*

Proof: Assume that $tr(G(C_3)) = n$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. Let v_{ij} be the new vertex of the edge v_iv_j in $G(C_3)$. Then we claim that each non support vertex is a complete vertex of degree 2. Suppose not. Then we consider the following two cases.

Case-1: There exists a non support vertex v_i for some i such that $d(v_i) \geq 3$ in G.
Then without loss of generality, let v_1 be such vertex and $N(v_1) = \{v_1, v_3, \ldots, v_{k+1}\}$, $k \geq 3$ in G. Let $W = \{v_2, v_3, \ldots, v_n\}$. Then for $2 \leq i \neq j \leq n$, $i - 1^{th}$ and $j - 1^{th}$ coordinates of the representation of v_{ij} are 1, 1^{st} k coordinates of the representation of v_1 are 1 and $j - 1^{th}$ coordinate of the representation of v_{1j}, $2 \leq j \leq k + 1$ is 1 in $G(C_3)$. Therefore each vertex of $V(G(C_3)) \setminus W$ have distinct representations. Since $\langle W \rangle$ has no isolates, $tr(G(C_3)) \leq n - 1$, which is a contradiction.

Case-2: There exists a non support vertex v_i for some i such that $d(v_i) = 2$ and v_i is not a complete vertex in G.
Then without loss of generality, let v_i be such vertex in G. Let $N(v_1) = \{v_2, v_3\}$ and $W = \{v_2, v_3, \ldots, v_n\}$. Then for $2 \leq i \neq j \leq n$, $i - 1^{th}$ and $j - 1^{th}$ coordinates of the representation of v_{ij} are 1, 1^{st} and 2^{nd} coordinates of v_1 are 1, 1^{st} coordinate of v_{12} is 1 and 2^{nd} coordinate of v_{13} is 1 in $G(C_3)$. Thus each vertex of $V(G(C_3)) \setminus W$ have distinct representations. Since $\langle W \rangle$ has no isolates, $tr(G(C_3)) \leq n - 1$, which is a contradiction.

Hence each non support vertex is a complete vertex of degree 2.

Conversely, suppose that each non support vertex is a complete vertex of degree 2. By Theorem 4.1, $tr(G(C_3)) \leq n$. Let W be a total resolving set for G (C_3). Let $d(v_i) = 2$, v_i is a complete non support vertex and $N(v_i) = \{v_j, v_k\}$. Then $d(v_i, v) = d(v_{jk}, v)$ for all $v \in V(G(C_3)) \setminus v_i, v_{jk}$. Therefore v_i or $v_{jk} \in W$ and by Lemma 3.3, $tr(G(C_3) \geq p + s + s' = n$, where s' denote the number of non support vertices of G. Thus $tr(G(C_3)) \geq n$ and hence $tr(G(C_3)) = n$.

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