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ON SOME PROPERTIES OF ROUGH APPROXIMATIONS

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ABSTRACT

In 1982, Zdzislaw Pawlak [2] introduced the theory of Rough sets to deal with the problems involving imperfect knowledge. This present research article studies some interesting properties of Rough approximations of subsets of the universe set. The properties of Rough membership function or also presented.

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1. INTRODUCTION

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of Artificial Intelligence.

There are many approaches to the problem of how to understand and manipulate imperfect knowledge. The most successful approaches to tackle this problem are the Fuzzy set theory and the Rough set theory. Theories of Fuzzy sets and Rough sets are powerful mathematical tools for modeling various types of uncertainties. Fuzzy set theory was introduced by *L. A. Zadeh* in his classical paper [5] of 1965.

A polish applied mathematician and computer scientist *Zdzislaw Pawlak* introduced Rough set theory in his classical paper [2] of 1982. Rough set theory is a new mathematical approach to imperfect knowledge. This theory presents still another attempt to deal with uncertainty or vagueness.

The Rough set theory has attracted the attention of many researchers and practitioners who contributed essentially to its development and application. Rough sets have been proposed for a very wide variety of applications.

In particular, the Rough set approach seems to be important for Artificial Intelligence and cognitive sciences, especially for machine learning, knowledge discovery, data mining, pattern recognition and approximate reasoning.

Let $\mathscr{U}$ be a finite or infinite universe set. If A is any subset of the universe set $\mathscr{U}$, we present a few properties of lower and upper Rough approximations of A. The lower and upper Rough approximations or obtained by using Rough membership function also.

Throughout this article, ϕ and $\mathscr{U}$ stand for the empty set and the universe set Respectively.

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2. PRELIMINARIES

This section is devoted to some basic definitions which are needed for the further study of this paper.

Definition 2.1: A relation R on $\mathscr{U}$ is said to be an *equivalence relation* on $\mathscr{U}$ if

(a) $(x,x) \in R$ for every $x \in \mathscr{U}$ (*reflexivity*)

(b) $(x,y) \in R \Leftrightarrow (y,x) \in R$ for every $x, y \in \mathscr{U}$ (*symmetry*)

(c) $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$ for every $x, y, z \in \mathscr{U}$ (tran*sitivity*)

Definition 2.2: If R is an equivalence relation on $\mathscr{U}$ then the *equivalence class* of an element $x \in \mathscr{U}$ is denoted by the symbol $[x]_R$ and it is defined by $[x]_R = \{y \in \mathscr{U} : yRx\}$.

Definition 2.3: An *information system* is a pair $(\mathscr{U}, \mathcal{A})$ where $\mathcal{A}$ is a set of attributes. Each attribute $f \in \mathcal{A}$ is a mapping $f : \mathscr{U} \to V_f$ where V_f is the range set of the attribute $f \in \mathcal{A}$.

Each attribute $f \in \mathcal{A}$ generates an equivalence relation on $\mathscr{U}$. Corresponding to each attribute $f \in \mathcal{A}$, a relation R_f is defined on $\mathscr{U}$ such that $(x,y) \in R_f \Leftrightarrow f(x) = f(y)$. It is easy to verify that R_f is an equivalence relation on $\mathscr{U}$ and R_f is called an indiscernability relation. If $x \in \mathscr{U}$ then $[x]_{R_f} = \{y \in \mathscr{U} : f(x) = f(y)\}$.

Definition 2.4: Let $(\mathscr{U}, \mathcal{A})$ be an information system and R_f, an indiscernible relation on $\mathscr{U}$ for some $f \in \mathcal{A}$. If X is any subset of $\mathscr{U}$ then

a) The *lower Rough approximation* of X is defined to be the set
$R^{\downarrow}(X) = \{x \in \mathscr{U} : [x]_{R_f} \subseteq X\}$

b) The *upper Rough approximation* of X is defined to be the set
$R^{\uparrow}(X) = \{x \in \mathscr{U} : [x]_{R_f} \cap X \neq \phi\}$

c) The *boundary region* of X with respect to R_f is defined to be the set
$\mathfrak{B}_{R_f}(X) = R^{\uparrow}(X) - R^{\downarrow}(X)$.

Definition 2.5: A subset X of $\mathscr{U}$ is said to be a *Rough set*, if the boundary region $\mathfrak{B}_{R_f}(X) = R^{\uparrow}(X) - R^{\downarrow}(X)$ is non- empty. Sometimes, a Rough set X can also be represented as a pair $(R^{\downarrow}(X), R^{\uparrow}(X))$ using Rough approximations.

Definition 2.6: A subset X of $\mathscr{U}$ is said to be a *Crisp set* if $R^{\downarrow}(X) = X = R^{\uparrow}(X)$.
It is easy to observe that a subset X of $\mathscr{U}$ is a Crisp set if and only if it is not a Rough set.

3. PROPERTIES OF ROUGH APPROXIMATIONS

In this section, various properties of the lower and upper Rough approximations are established.

Theorem 3.1:

a) $R^{\uparrow}(\phi) = R^{\downarrow}(\phi) = \phi$

b) $R^{\uparrow}(\mathscr{U}) = R^{\downarrow}(\mathscr{U}) = \mathscr{U}$

Proof: Since $R^{\downarrow}(\phi) \subseteq \phi$, $\mathscr{U} \subseteq R^{\uparrow}(\mathscr{U})$ it follows that $R^{\downarrow}(\phi) = \phi$ and $R^{\uparrow}(\mathscr{U}) = \mathscr{U}$.
Assume that $R^{\uparrow}(\phi) \neq \phi$.

Then there exists a point x in $\mathscr{U}$ such that $x \in R^{\uparrow}(\phi) \Rightarrow [x]_{R_f} \cap \phi \neq \phi$ which is a contradiction.

Hence $R^{\uparrow}(\phi) = \phi$.

Since R_f is an equivalence relation on $\mathscr{U}$, $[x]_{R_f} \subseteq \mathscr{U}$ for every $x \in \mathscr{U}$. This implies that $R^{\downarrow}(\mathscr{U}) = \mathscr{U}$.

This results establishes the fact that the empty set ϕ and the universe set $\mathscr{U}$ are Crisp sets.

Theorem 3.2: If A and B are any two subsets of $\mathscr{U}$, then

a) $A \subseteq B \Rightarrow R^{\uparrow}(A) \subseteq R^{\uparrow}(B)$

b) $R^{\uparrow}\left(\left(R^{\uparrow}\right)(A)\right) = R^{\uparrow}(A)$

c) $R^{\uparrow}(A \cup B) = R^{\uparrow}(A) \cup R^{\uparrow}(B)$

d) $R^{\uparrow}(A \cap B) \subseteq R^{\uparrow}(A) \cap R^{\uparrow}(B)$

Proof:

a) Suppose that $A \subseteq B$. Let $x \in R^{\uparrow}(A)$.

$\Rightarrow [x]_{R_f} \cap A \neq \phi$

$\Rightarrow$ there exists a point y in $\mathscr{U}$ such that $y \in [x]_{R_f} \cap A$

$\Rightarrow y \in [x]_{R_f}$ and $y \in A$

$\Rightarrow y \in [x]_{R_f}$ and $y \in B$

$\Rightarrow y \in [x]_{R_f} \cap B$

$\Rightarrow [x]_{R_f} \cap B \neq \phi$

$\Rightarrow x \in R^{\uparrow}(B)$

Hence $R^{\uparrow}(A) \subseteq R^{\uparrow}(B)$.

b) Since $A \subseteq R^{\uparrow}(A)$, $R^{\uparrow}(A) \subseteq R^{\uparrow}\left(R^{\uparrow}(A)\right)$ (1)

Let $x \in R^{\uparrow}\left(R^{\uparrow}(A)\right)$. Then $[x]_{R_f} \cap R^{\uparrow}(A) \neq \phi$

$\Rightarrow$ there exists a point y in $\mathscr{U}$ such that $y \in [x]_{R_f} \cap R^{\uparrow}(A)$

$\Rightarrow y \in [x]_{R_f}$ and $y \in R^{\uparrow}(A)$

$\Rightarrow [y]_{R_f} = [x]_{R_f}$ and $[y]_{R_f} \cap (A) \neq \phi$

$\Rightarrow [x]_{R_f} \cap (A) \neq \phi$

$\Rightarrow x \in R^{\uparrow}(A)$

Hence $R^{\uparrow}\left(R^{\uparrow}(A)\right) \subseteq R^{\uparrow}(A)$ (2)

From (1) and (2), $R^{\uparrow}\left(\left(R^{\uparrow}\right)(A)\right) = R^{\uparrow}(A)$.

c) Since $A \subseteq A \cup B$, $R^{\uparrow}(A) \subseteq R^{\uparrow}(A \cup B)$ (3)

Similarly, $B \subseteq A \cup B \Rightarrow R^{\uparrow}(B) \subseteq R^{\uparrow}(A \cup B)$ (4)

From (3) and (4), $R^{\uparrow}(A) \cup R^{\uparrow}(B) \subseteq R^{\uparrow}(A \cup B)$ (5)

Let $x \in R^{\uparrow}(A \cup B)$. Then $(x)_{R_f} \cap (A \cup B) \neq \phi$

$\Rightarrow \left([x]_{R_f} \cap A\right) \cup \left([x]_{R_f} \cap B\right) \neq \phi$

$\Rightarrow [x]_{R_f} \cap A \neq \phi$ or $[x]_{R_f} \cap B \neq \phi$

$\Rightarrow x \in R^{\uparrow}(A)$ or $x \in R^{\uparrow}(B)$

$\Rightarrow x \in R^{\uparrow}(A) \cup R^{\uparrow}(B)$

This shows that $R^{\uparrow}(A \cup B) \subseteq R^{\uparrow}(A) \cup R^{\uparrow}(B)$ (6)

From (5) and (6), $R^{\uparrow}(A \cup B) = R^{\uparrow}(A) \cup R^{\uparrow}(B)$.

d) Since $A \cap B \subseteq A$, we have $R^{\uparrow}(A \cap B) \subseteq R^{\uparrow}(A)$ (7)

Similarly, $A \cap B \subseteq B \Rightarrow R^{\uparrow}(A \cap B) \subseteq R^{\uparrow}(B)$ (8)

From (7) and (8), $R^{\uparrow}(A \cap B) \subseteq R^{\uparrow}(A) \cap R^{\uparrow}(B)$.

Theorem 3.3: If A and B are any two subsets of $\mathscr{U}$, then

a) $A \subseteq B \Rightarrow R^{\downarrow}(A) \subseteq R^{\downarrow}(B)$

b) $R^{\downarrow}\left(\left(R^{\downarrow}\right)(A)\right) = R^{\downarrow}(A)$

c) $R^{\downarrow}(A \cap B) = R^{\downarrow}(A) \cap R^{\downarrow}(B)$

d) $R^{\downarrow}(A) \cup R^{\downarrow}(B) \subseteq R^{\downarrow}(A \cup B)$

Proof:

a) Suppose that $A \subseteq B$. Let $x \in R^{\downarrow}(A)$.

$\Rightarrow [x]_{R_f} \subseteq A$

$\Rightarrow [x]_{R_f} \subseteq A \subseteq B$

$\Rightarrow x \in R^{\downarrow}(B)$.

Hence $R^{\downarrow}(A) \subseteq R^{\downarrow}(B)$.

b) Since $R^{\downarrow}(A) \subseteq A$, $R^{\downarrow}\left(R^{\downarrow}(A)\right) \subseteq R^{\downarrow}(A)$ (1)

Let $x \in R^{\downarrow}(A)$. Then $[x]_{R_f} \subseteq A$

If $y \in [x]_{R_f}$ then $[y]_{R_f} = [x]_{R_f}$, which implies that $[y]_{R_f} \subseteq A$

$\Rightarrow y \in R^{\downarrow}(A)$.

This shows that $[x]_{R_f} \subseteq R^{\downarrow}(A)$

$\Rightarrow x \in R^{\downarrow}\left(R^{\downarrow}(A)\right)$

Hence $R^{\downarrow}(A) \subseteq R^{\downarrow}\left(R^{\downarrow}(A)\right)$ (2)

From (1) and (2) $R^{\downarrow}\left(\left(R^{\downarrow}\right)(A)\right) = R^{\downarrow}(A)$.

c) Since $A \cap B \subseteq A$, $R^{\downarrow}(A \cap B) \subseteq R^{\downarrow}(A)$ (3)

Similarly, $A \cap B \subseteq B \Rightarrow R^{\downarrow}(A \cap B) \subseteq R^{\downarrow}(B)$ (4)

From (3) and (4), $R^{\downarrow}(A \cap B) \subseteq R^{\downarrow}(A) \cap R^{\downarrow}(B)$ (5)

Let $x \in R^{\downarrow}(A) \cap R^{\downarrow}(B)$.

$\Rightarrow$ $x \in R^{\downarrow}(A)$ and $x \in R^{\downarrow}(B)$

$\Rightarrow$ $[x]_{R_f} \subseteq A$ and $[x]_{R_f} \subseteq B$

$\Rightarrow$ $[x]_{R_f} \subseteq A \cap B$

$\Rightarrow$ $x \in R^{\downarrow}(A \cap B)$

Hence $R^{\downarrow}(A) \cap R^{\downarrow}(B) \subseteq R^{\downarrow}(A \cap B)$ (6)

From (5) and (6), $R^{\downarrow}(A \cap B) = R^{\downarrow}(A) \cap R^{\downarrow}(B)$.

d) $A \subseteq A \cup B$ and $B \subseteq A \cup B$

$\Rightarrow R^{\downarrow}(A) \subseteq R^{\downarrow}(A \cup B)$ and $R^{\downarrow}(B) \subseteq R^{\downarrow}(A \cup B)$

$\Rightarrow R^{\downarrow}(A) \cup R^{\downarrow}(B) \subseteq R^{\downarrow}(A \cup B)$.

Theorem 3.4: For any $x \in \mathscr{U}$, $R^{\downarrow}\left([x]_{R_f}\right) = R^{\uparrow}\left([x]_{R_f}\right) = [x]_{R_f}$.

Proof: Clearly, $R^{\downarrow}\left([x]_{R_f}\right) \subseteq [x]_{R_f}$. (1)

Let $y \in [x]_{R_f}$. Then $[y]_{R_f} = [x]_{R_f}$.

$\Rightarrow y \in R^{\downarrow}\left([x]_{R_f}\right)$. Hence $[x]_{R_f} \subseteq R^{\downarrow}\left([x]_{R_f}\right)$. (2)

From (1) and (2), $R^{\downarrow}\left([x]_{R_f}\right) = [x]_{R_f}$.

We have $[x]_{R_f} \subseteq R^{\uparrow}\left([x]_{R_f}\right)$ (3)

Let $y \in R^{\uparrow}\left([x]_{R_f}\right)$. Then $[y]_{R_f} \cap [x]_{R_f} \neq \phi$.

$\Rightarrow$ There exists a point z in $\mathscr{U}$ such that $z \in [y]_{R_f} \cap [x]_{R_f}$

$\Rightarrow z \in [y]_{R_f}$ and $z \in [x]_{R_f}$

$\Rightarrow (z, y) \in R_f$ and $(z, x) \in R_f$

$\Rightarrow (y, x) \in R_f$

$\Rightarrow y \in [x]_{R_f}$

This proves that $R^{\uparrow}\left([x]_{R_f}\right) \subseteq [x]_{R_f}$. (4)

From (3) and (4), $R^{\uparrow}\left([x]_{R_f}\right) = [x]_{R_f}$.

This result shows that $[x]_{R_f}$ is a Crisp set for any $x \in \mathscr{U}$.

Theorem 3.5: For any $A \subseteq \mathscr{U}$,

a) $R^{\uparrow}\left(R^{\downarrow}(A)\right) = R^{\downarrow}(A)$

b) $R^{\downarrow}\left(R^{\uparrow}(A)\right) = R^{\uparrow}(A)$

Proof: suppose that $A \subseteq \mathscr{U}$.

a) Clearly, $R^{\downarrow}(A) \subseteq R^{\uparrow}\left(R^{\downarrow}(A)\right)$ (1)

Let $x \in R^{\uparrow}\left(R^{\downarrow}(A)\right)$. Then $[x]_{R_f} \cap R^{\downarrow}(A) \neq \phi$.

Since $R^{\downarrow}(A) \subseteq A$, $[x]_{R_f} \cap R^{\downarrow}(A) \subseteq [x]_{R_f} \cap A$

$\Rightarrow [x]_{R_f} \cap A \neq \phi$

$\Rightarrow x \in R^{\downarrow}(A)$

Hence $R^{\uparrow}\left(R^{\downarrow}(A)\right) \subseteq R^{\downarrow}(A)$ (2)

From (1) and (2), $R^{\uparrow}\left(R^{\downarrow}(A)\right) = R^{\downarrow}(A)$.

b) Clearly, $R^{\downarrow}\left(R^{\uparrow}(A)\right) \subseteq R^{\uparrow}(A)$ (3)

$x \in R^{\uparrow}(A) \Rightarrow [x]_{R_f} \cap A \neq \phi$

$\Rightarrow [x]_{R_f} = [y]_{R_f} \cap A \neq \phi$ for any $y \in [x]_{R_f}$

$\Rightarrow [x]_{R_f} \subseteq R^{\uparrow}(A)$

$\Rightarrow x \in R^{\downarrow}\left(R^{\uparrow}(A)\right)$

Then $R^{\uparrow}(A) \subseteq R^{\downarrow}\left(R^{\uparrow}(A)\right)$ (4)

From (3) and (4), $R^{\downarrow}\left(R^{\uparrow}(A)\right) = R^{\uparrow}(A)$.

4. ROUGH MEMBERSHIP FUNCTION

In most of the cases, the universe set is finite. The Rough membership function seems to be a very useful tool to deal with such conditions. The lower and upper Rough approximations can be obtained by using a rough membership function when the universe set is finite. Throughout this section, the universe set $\mathscr{U}$ is a non-empty finite set.

Definition 4.1: Let $(\mathscr{U}, \mathscr{A})$ be a finite information system. Fix an indiscernible relation R_f corresponding to an attribute $f \in \mathscr{A}$. If A is any subset of $\mathscr{U}$ then the Rough membership function $\lambda_A : \mathscr{U} \to [0,1]$ is defined as follows.

$$\lambda_A(x) = \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} \ \forall \ x \in \mathscr{U}.$$

The Rough membership function expresses conditional probability that $x \in A$ given R_f and can be interpreted as a degree that $x \in A$ in view of information about x expressed by R_f .

Theorem 4.2: If $A \subseteq \mathscr{U}$, then

a) $R^{\uparrow}(A) = \{x \in \mathscr{U} : \lambda_A(x) > 0\}$

b) $R^{\downarrow}(A) = \{x \in \mathscr{U} : \lambda_A(x) = 1\}$

c) $\mathfrak{B}_{R_f}(A) = \{x \in \mathscr{U} : 0 < \lambda_A(x) < 1\}$

Proof: suppose that $A \subseteq \mathscr{U}$ and R_f is an indiscernible relation on $\mathscr{U}$.

a)
$$x \in \{x \in \mathscr{U} : \lambda_A(x) > 0\} \Leftrightarrow \lambda_A(x) > 0$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} > 0$$
$$\Leftrightarrow \left|[x]_{R_f} \cap A\right| > 0$$
$$\Leftrightarrow [x]_{R_f} \cap A \neq \phi$$
$$\Leftrightarrow x \in R^{\uparrow}(A)$$

Thus $R^{\uparrow}(A) = \{x \in \mathscr{U} : \lambda_A(x) > 0\}$.

b)
$$x \in \{x \in \mathscr{U} : \lambda_A(x) = 1\} \Leftrightarrow \lambda_A(x) = 1$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} = 1$$
$$\Leftrightarrow \left|[x]_{R_f} \cap A\right| = \left|[x]_{R_f}\right|$$
$$\Leftrightarrow [x]_{R_f} \cap A = [x]_{R_f}$$
$$\Leftrightarrow [x]_{R_f} \subseteq A$$
$$\Leftrightarrow x \in R^{\downarrow}(A)$$

Thus $R^{\downarrow}(A) = \{x \in \mathscr{U} : \lambda_A(x) = 1\}$.

c) Clearly,
$$\mathfrak{B}_{R_f}(A) = R^{\uparrow}(A) - R^{\downarrow}(A)$$
$$= \{x \in \mathscr{U} : \lambda_A(x) > 0\} - \{x \in \mathscr{U} : \lambda_A(x) = 1\}$$
$$= \{x \in \mathscr{U} : 0 < \lambda_A(x) < 1\}.$$

Remark 4.3: Suppose that $A \subseteq \mathscr{U}$ and R_f is an indiscernible relation on $\mathscr{U}$. We write λ instead of writing λ_A for simplicity. If we define another indiscernible relation R_λ on $\mathscr{U}$ corresponding to the Rough membership function λ such that

$$(x, y) \in R_\lambda \Leftrightarrow \lambda(x) = \lambda(y)$$

Then we can observe the following.

$$(x, y) \in R_f \Leftrightarrow [x]_{R_f} = [y]_{R_f}$$
$$\Leftrightarrow \frac{\left|[x]_{R_f} \cap A\right|}{\left|[x]_{R_f}\right|} = \frac{\left|[y]_{R_f} \cap A\right|}{\left|[y]_{R_f}\right|}$$
$$\Leftrightarrow \lambda(x) = \lambda(y)$$
$$\Leftrightarrow (x, y) \in R_\lambda$$

Hence $R_f = R_\lambda$. Thus the process of generating indiscernible relation using a Rough membership function terminates at the first stage itself.

Theorem 4.4: If A and B are any two subsets of $\mathscr{U}$, then

a) $\lambda_{\phi} = 0$

b) $\lambda_{\mu} = 1$

c) $\lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}$

d) $\lambda_{A\cup B} = \lambda_A + \lambda_B$ provided $A \cap B = \phi$

e) $\lambda_{A^c} = 1 - \lambda_A$

Proof: Clearly, $\lambda_{\phi}(x) = \dfrac{\left|[x]_{R_f} \cap \phi\right|}{\left|[x]_{R_f}\right|} = \dfrac{|\phi|}{\left|[x]_{R_f}\right|} = 0$ and

$$\lambda_{\mathscr{U}}(x) = \frac{\left|[x]_{R_f} \cap \mathscr{U}\right|}{\left|[x]_{R_f}\right|} = \frac{\left|[x]_{R_f}\right|}{\left|[x]_{R_f}\right|} = 1.$$

Now we prove (c).

$$\lambda_{A\cup B}(x) = \frac{\left|[x]_{R_f} \cap (A\cup B)\right|}{\left|[x]_{R_f}\right|} = \frac{\left|\left([x]_{R_f} \cap A\right) \cup \left([x]_{R_f} \cap B\right)\right|}{\left|[x]_{R_f}\right|}$$

$$= \frac{\left|[x]_{R_f} \cap A\right| + \left|[x]_{R_f} \cap B\right| - \left|[x]_{R_f} \cap A \cap B\right|}{\left|[x]_{R_f}\right|}$$

$$= \lambda_A(x) + \lambda_B(x) - \lambda_{A\cap B}(x)\ \forall\ x \in \mathscr{U}$$

$$\Rightarrow \lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}.$$

Now, suppose that $A \cap B = \phi$. Then $\lambda_{A\cup B} = \lambda_A + \lambda_B - \lambda_{A\cap B}$.

$$= \lambda_A + \lambda_B - \lambda_{\phi}$$

$$= \lambda_A + \lambda_B.$$

Since $A \cup A^c = \mathscr{U}$ and $A \cap A^c = \phi$, $\lambda_{A\cup A^c} = \lambda_A + \lambda_{A^c}$.

$$\Rightarrow \lambda_{\mathscr{U}} = \lambda_A + \lambda_{A^c}$$

$$\Rightarrow 1 = \lambda_A + \lambda_{A^c}$$

$$\Rightarrow \lambda_{A^c} = 1 - \lambda_A.$$

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