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ONE-DIMENSIONAL SOLUTE TRANSPORT IN A HOMOGENEOUS POROUS MEDIA WITH PULSE TYPE INPUT SOURCE

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ABSTRACT

In this paper, a theoretical model is developed for advection-dispersion problem including first order decay and zero order production in a one-dimensional semi-infinite porous media. Dispersion coefficient is considered proportional to seepage velocity while seepage velocity is a temporal function. Initial concentration is assumed exponentially space dependent. Time dependent pulse type source concentration which is any smooth function of time is considered at the one end of the boundary. Concentration gradient at other end is supposed to be zero. Interpolation method is applied to reduce the input function into a polynomial. In order to eliminate the time derivative, the Laplace transform technique is applied to get the solution of advection dispersion equation. Two different time dependent functions of input are considered. Obtained result is demonstrated graphically with the help of numerical examples.

Keywords: *Advection; Dispersion; Aquifer; Porous Medium; Interpolation; Laplace Transformation.*

INTRODUCTION

The advection-dispersion equation ADE is commonly used for transport of solute in porous media. The advection is controlled by the Darcy's law whereas hydrodynamic dispersion is the combination of mechanical and molecular diffusion which accounts for contaminant arising stimulates by velocity variations. Several researchers derived various theories to investigate the fluid flow and solute transport in homogeneous / heterogeneous geological formations. Solutions of the ADEs can be obtained analytically or numerically with space or temporal dependent dispersivity.

Various analytical solutions describing solute transport through one-dimensional solute transport problem in porous formation, considering steady, unsteady flow, first order decay and zero order production have been published in literature. Huang *et al.* (1996) obtained analytical solution of solute transport in heterogeneous porous media with scale dependent dispersion. Sposito *et al.*(1986) observed that dispersion coefficient increase either with distance or time. Valocchi (1989) proposed solution for kinetically sorbing solute under conditions of horizontal flow where sorbing reaction varied as an arbitrary function in vertical direction. Goode and Konikow (1990) illustrates that fluctuation in hydraulic conductivity are not sole responsible of spatial variation in groundwater velocity. Later Watson *et al.* (2002) also observed that the hydrodynamic dispersion coefficients are non-linear function of the seepage velocity. Jaiswal *et al.* (2009&2011) studied the pulse type input phenomena in one dimensional semi-infinite porous media for temporally as well space dependent dispersion coefficient. Chen *et al.* (2011a,b) solved two-dimensional advection-dispersion equation (ADE) in cylindrical coordinates subject to the third-type inlet boundary condition with finite Hankel transform technique in combination with the Laplace transform method Almost all analytical solutions are obtained assuming uniform initial concentration in an infinite or semi-infinite medium with point or line source. Guerrero *et. al.* (2013) obtained analytical solution with time dependent boundary condition in one-dimensional porous media. Van Genuchten *et al.* (2013a&b) proposed one-dimensional solute transport through porous media with or without zero-order production and first-order decay. Singh *et al.* (2015) obtained an analytical solution in a heterogeneous porous medium with scale dependent solute dispersion. Bharati *et al.* (2015) considered dispersion coefficient and velocity are proportional to non homogeneous linear expression in position variable to get the analytical solution. Mahato *et al.* (2015) obtained solution in a finite domain with pulse type input when source entering different from origin.Using Green's function an analytical solution of one dimensional porous medium for instantaneous and continuous point source taking dispersion and velocity proportional was developed in groundwater and riverine flow (Sanskrityayn *et al.*, 2017).

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The objective of this paper is to develop a theoretical model for the solute transport problem in a one-dimensional homogeneous porous medium. The homogeneous geological formation is assumed horizontal and semi-infinite length. Dispersion coefficient is assumed proportional to the velocity which is inversely proportional to linear function of time. Initially, medium is not solute free which means some concentration already present in the medium and which depends exponentially on position. Input concentration is considered any function of time which is continuous, smooth and bounded for a finite time interval. Laplace Transform Technique LTT is used to get the solution of the present study. The impacts of different boundary conditions on the contaminant concentration distribution in the porous domain are illustrated with the help of examples.

MATHEMATICAL FORMULATION OF THE PROBLEM

The linear advection–dispersion equation in one-dimensional horizontal plane which is derived on basis of mass conservation and Fick's law of diffusion may be written as: (Bear, 1972),

$$R\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D(x,t)\frac{\partial c}{\partial x}-u(x,t)c\right)-\mu(x,t)c+\gamma(x,t) \tag{1}$$

In which, $c\left[ML^{-3}\right]$ is the solute concentration, $u\left[LT^{-1}\right]$ is the groundwater velocity, which is commonly known as the seepage velocity, $D\left[L^2T^{-1}\right]$ is the dispersion coefficient and R is a retardation factor, which is a dimension less quantity. The dispersion coefficient D is proportional to flow velocity u (Yim and Mohsen, 1992) and assumed respectively as $u(x,t)=u_0(1+m\,t)^{-1}$, $D=D_0(1+m\,t)^{-1}$. Zero order production $\gamma(x,t)\left[ML^{-3}T^{-1}\right]$ and first order production $\mu(x,t)[T^{-1}]$ are considered respectively as, $\gamma(x,t)=\gamma_0(1+m\,t)^{-1}$ and $\mu(x,t)=\mu_0(1+m\,t)^{-1}$. Where, $D_0\left[L^2T^{-1}\right]$, $u_0\left[LT^{-1}\right]$, $\mu_0\left[T^{-1}\right]$ and $\gamma_0\left[ML^{-3}T^{-1}\right]$ are constants and $m\left[T^{-1}\right]$ is a unsteady parameter. Also $x[L]$ and $t[T]$ represent longitudinal distance and time variable respectively.

Therefore, Eq. (1) may be re-written as

$$R\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_0(1+m\,t)^{-1}\frac{\partial c}{\partial x}-u_0(1+m\,t)^{-1}c\right)-\mu_0(1+m\,t)^{-1}c+\gamma_0(1+m\,t)^{-1} \tag{2}$$

The geological formation is assumed to be of semi-infinite along horizontal in direction, which is initially polluted. Let there be exponentially space-dependent source contaminant concentration be present initially in the domain space at t = 0. It means, semi-infinite medium is supposed initially not solute free. Time dependent continuous point source is introduced at the origin of the medium and then eliminated after a certain time. Concentration gradient at infinity is considered zero. In order to formulate the proposed problem mathematically, the following initial and boundary condition may be written as:

$$c(x,t)=\ c_i\exp(-\alpha\,x);\ t=0,\ x>0 \tag{3}$$

$$c(x,t)=\begin{cases}c_0F(m't);0<t\le t_0, & \text{(4a)}\\ & x=0\\ 0\,;t>t_0, & \text{(4b)}\end{cases}$$

$$\frac{\partial c(x,t)}{\partial x}=0\,;\ \text{as}\ x\to\infty \tag{5}$$

Where, c_0 and c_i are the reference concentrations and $\alpha\left[L^{-1}\right]$ is a unsteady parameter which can take positive real number to regulate initial concentration distribution. $m'\left[T^{-1}\right]$ is an unsteady parameter and $F(m't)$ is any continuous and bounded smooth function of time t in domain $[0,t_0]$**. According to Weirstrass approximation theorem** ***any continuous function on a bounded interval can be uniformly approximated in a polynomial function***. Since $F(m't)$ is continuous and bounded in domain $[0,t_0]$, so Weirstrass approximation theorem suggests that

$F(m't)$ may be written as polynomial of degree n. Hence the proposed initial and boundary conditions Eq. (3-5) may be written as:

$$c(x,t) = c_i \exp(-\alpha\, x),\ t = 0,\ x > 0 \tag{6}$$

$$c(x,t) = \begin{cases} c_0 G_n(t)\ ; 0 < t \le t_0 & (7a) \\ x = 0 & \\ 0\ ;\ t > t_0 & (7b) \end{cases}$$

$$\frac{\partial c(x,t)}{\partial x} = 0, \quad \text{as} \quad x \to \infty \tag{8}$$

Where, $G_n(t) = a_0 + a_1 t + a_2 t^2 + \ldots\ldots\ldots\ldots + a_n t^n$ (9)

also $a_0, a_1, \ldots\ldots\ldots, a_n$ are constants and $G_n(t)$ is dimension less .

Using a transformation T as; (Crank, 1975)

$$T = \int_0^t (1 + m\, t)^{-1} dt$$

or $$T = \frac{1}{m}\log(1 + m\, t) \quad \text{and} \quad t = \frac{1}{m}\left(e^{m\,T} - 1\right) \tag{10}$$

With this transformation Eq. (10), Eq. (2) and Eqs. (6-8) are reduced into new time variable T.

$$R\frac{\partial c}{\partial T} = \frac{\partial}{\partial x}\left(D_0 \frac{\partial c}{\partial x} - u_0 c\right) - \mu_0 c + \gamma_0 \tag{11}$$

$$c(x,T) = c_i \exp(-\alpha\, x),\ T = 0,\ x > 0 \tag{12}$$

$$c(x,T) = \begin{cases} H(T)\ ;\ 0 < T \le T_0 & (13a) \\ & x = 0 \\ 0\ ;\ T > T_0 & (13b) \end{cases}$$

$$\frac{\partial c(x,T)}{\partial x} = 0, \text{ as } x \to \infty \tag{14}$$

Where, $H(T) = b_0 + b_1 e^{m\,T} + b_2 e^{2m\,T} + \ldots\ldots\ldots\ldots + b_n e^{n\,m\,T}$ and $T_0 = \frac{1}{m}\log(1 + m\, t_0)$.

also $b_0, b_1, \ldots\ldots\ldots, b_n$ are constants and have dimension of concentration.

In order to remove convective term from advection-dispersion equation Eq. (11), following transformation is used

$$c(x,T) = k(x,T)\exp\left[\frac{u_0}{2D_0}x - \frac{1}{R}\left(\frac{u_0^{\ 2}}{4D_0} + \mu_0\right)T\right] + \frac{\gamma_0}{\mu_0} \tag{15}$$

With transformation Eq. (15), Eqs. (11-14) reduced into:

$$R\frac{\partial k}{\partial T} = D_0 \frac{\partial^2 k}{\partial x^2} \tag{16}$$

$$k(x,T) = c_i \exp\{-(\alpha + \beta)\}x - \frac{\gamma_0}{\mu_0}\exp(-\beta x);\ T = 0,\ x \ge 0 \tag{17}$$

$$k(x,T) = \begin{cases} H(T)\exp(\eta^2 T) - \dfrac{\gamma_0}{\mu_0}\exp(\eta^2 T)\ ;\ 0 < T \le T_0, & (18a) \\ & x = 0 \\ -\dfrac{\gamma_0}{\mu_0}\exp(\eta^2 T); T > T_0 & (18b) \end{cases}$$

$$\frac{\partial k(x,T)}{\partial x}+\frac{u_0}{2D_0}k(x,T)=0;\ \text{as } x\to\infty,\quad T\geq 0 \tag{19}$$

Where, $\beta=\dfrac{u_0}{2D_0}$ and $\eta=\sqrt{\dfrac{1}{R}\left(\dfrac{u_0{}^2}{4D_0}+\mu_0\right)}$

Applying Laplace transformation on Eqs. (16–19) to reduce into ordinary differential equation.

$$\frac{d^2\bar{k}}{dx^2}-\frac{pR}{D_0}\bar{k}=-\frac{R}{D_0}\left[c_i\exp\{-(\alpha+\beta)x\}-\frac{\gamma_0}{\mu_0}\exp(-\beta x)\right] \tag{20}$$

$$\begin{aligned}\bar{k}(x,p)=&\left(b_0-\frac{\gamma_0}{\mu_0}\right)\frac{\left[1-\exp-\left(p-\eta^2\right)T_0\right]}{\left(p-\eta^2\right)}+b_1\frac{\left[1-\exp-\left\{p-\left(\eta^2+m\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+m\right)\right\}}\\&+b_2\times\frac{\left[1-\exp-\left\{p-\left(\eta^2+2m\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+2m\right)\right\}}\\&+\dots\dots\dots\dots+b_n\frac{\left[1-\exp-\left\{p-\left(\eta^2+n\,m\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+n\,m\right)\right\}}-\frac{\gamma_0}{\mu_0}\frac{\exp-\left(p-\eta^2\right)T_0}{\left(p-\eta^2\right)}\end{aligned}\quad,\ x=0 \tag{21}$$

$$\frac{d\bar{k}}{dX}+\frac{u_0}{2D_0}\bar{k}=0,\ \text{as}\quad x\to\infty \tag{22}$$

Where, $\bar{f}(x,p)=L\{f(x,p)\}=\int_0^\infty e^{-pt}f(x,t)d$, $p>0$ where p is a Laplace parameter.

Hence solution of Eq. (20), by using Eq. (21) and Eq. (22) may be written as:

$$\begin{aligned}\bar{k}(x,p)=&\left(b_0-\frac{\gamma_0}{\mu_0}\right)\frac{\left[1-\exp-\left(p-\eta^2\right)T_0\right]}{\left(p-\eta^2\right)}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)\\&+b_1\frac{\left[1-\exp-\left\{p-\left(\eta^2+m\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+m\right)\right\}}\times\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)\\&+b_2\frac{\left[1-\exp-\left\{p-\left(\eta^2+2m\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+2m\right)\right\}}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)\\&+\dots\dots\dots\dots+b_n\frac{\left[1-\exp-\left\{p-\left(\eta^2+nm\right)\right\}T_0\right]}{\left\{p-\left(\eta^2+nm\right)\right\}}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)-\frac{\gamma_0}{\mu_0}\frac{\exp\left\{-\left(p-\eta^2\right)T\right\}_0}{\left\{p-\eta^2\right\}}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)\\&-c_i\frac{1}{\left\{p-\frac{D_0}{R}(\alpha+\beta)^2\right\}}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)+\frac{\gamma_0}{\mu_0}\frac{1}{\left\{p-\frac{D_0}{R}\beta^2\right\}}\exp\left(-\sqrt{\frac{pR}{D_0}}x\right)\\&+c_i\frac{\exp\{-(\alpha+\beta)x\}}{\left\{p-\frac{D_0}{R}(\alpha+\beta)^2\right\}}-\frac{\gamma_0}{\mu_0}\frac{\exp\{(-\beta)x\}}{\left\{p-\frac{D_0}{R}\beta^2\right\}}\end{aligned} \tag{23}$$

Applying inverse Laplace Transform to Eq. (23) and using the transformation Eq. (15), we get the solution of the present problem as;

$$c(x,T)=\Bigg[\left(b_0-\frac{\gamma_0}{\mu_0}\right)F_\eta(x,T)+b_1F_{\sqrt{(\eta^2+m)}}(x,T)+b_2F_{\sqrt{(\eta^2+2m)}}(x,T)+\ldots\ldots\ldots\ldots+b_nF_{\sqrt{(\eta^2+nm)}}(x,T)-$$

$$c_iF_\omega(x,T)+\frac{\gamma_0}{\mu_0}F_\rho(x,T)+c_iG(x,T)-\frac{\gamma_0}{\mu_0}H(x,T)\Bigg]\times$$

$$\exp\left\{\frac{u_0}{2D_0}x-\frac{1}{R}\left(\frac{u_0^{\,2}}{4D_0}+\mu_0\right)T\right\}+\frac{\gamma_0}{\mu_0}\quad 0<T\le T_0 \tag{24a}$$

$$c(x,T)=\Bigg[\left(b_0-\frac{\gamma_0}{\mu_0}\right)\left\{F_\eta(x,T)-F_\eta(x,T-T_0)\exp(\eta^2T_0)\right\}+b_1\Big\{F_{\sqrt{(\eta^2+m)}}(x,T)-F_{\sqrt{(\eta^2+m)}}(x,T-T_0)\times$$

$$\exp\{(\eta^2+m)T_0\}\Big\}+b_2\Big\{F_{\sqrt{(\eta^2+2m)}}(x,T)-F_{\sqrt{(\eta^2+2m)}}(x,T-T_0)\exp\{(\eta^2+2m)T_0\}\Big\}+\ldots\ldots\ldots+$$

$$b_n\Big\{F_{\sqrt{(\eta^2+nm)}}(x,T)-F_{\sqrt{(\eta^2+nm)}}(x,T-T_0)\exp\{(\eta^2+nm)T_0\}\Big\}-\frac{\gamma_0}{\mu_0}F_\eta(x,T-T_0)\exp(\eta^2T_0)-$$

$$c_iF_\omega(x,T)+\frac{\gamma_0}{\mu_0}F_\rho(x,T)+c_iG(x,T)-\frac{\gamma_0}{\mu_0}H(x,T)\Bigg]\exp\left\{\frac{u_0}{2D_0}x-\frac{1}{R}\left(\frac{u_0^{\,2}}{4D_0}+\mu_0\right)T\right\}+\frac{\gamma_0}{\mu_0}$$

$$;\quad T_0<T \tag{24b}$$

Where,

$$F_\varphi(x,t)=\frac{1}{2}\left[\exp\left\{\varphi^2t-\frac{\varphi\sqrt{R}x}{\sqrt{D_0}}\right\}erfc\left\{\frac{x\sqrt{R}}{2\sqrt{D_0t}}-\varphi\sqrt{t}\right\}+\exp\left\{\varphi^2t+\frac{\varphi\sqrt{R}x}{\sqrt{D_0}}\right\}erfc\left\{\frac{x\sqrt{R}}{2\sqrt{D_0t}}+\varphi\sqrt{t}\right\}\right];$$

$$G(x,t)=\exp\left\{\omega^2t-(\alpha+\beta)x\right\};$$

$$H(x,t)=\exp\left\{\rho^2t+(-\beta)x\right\};$$

Where $\omega=\sqrt{\frac{D_0}{R}(\alpha+\beta)^2}$ and $\rho=\sqrt{\frac{D_0}{R}\beta^2}$

RESULTS AND DISCUSSIONS

The solution obtained as in Eq. (24a,b) are discussed for sinusoidal $F(m't)=\{l+\sin(m't)\}$ and exponential $F(m't)=\exp\left(\sqrt{m't}\right)$ forms of input function for a chosen set of data taken from published literatures or empirical relationship. For example the range of seepage velocity, keeping in view the different types of soils, aquifer is lies between 2m/day to 2m/year (Todd, 1980). Concentration values c/c_0 are evaluated assuming reference concentrations as $c_0=1.0$, $c_i=0.01$ in a finite domain along longitudinal direction $0\le x(m)\le 10$ and shown graphically. Presence of source is assumed up to time $t_0(day)=9$ and then it is eliminated. Initial seepage velocity and dispersion coefficient are taken $u_0=0.01\left(m\ day^{-1}\right)$ and $D_0=1.2\left(m^2day^{-1}\right)$ respectively. Other values of common parameters are considered as $R=1.25$, $\mu_0\left(day^{-1}\right)=0.05$, $\gamma_0=0.0007$, $\alpha=0.019$. Polynomial (9) is obtained by Hermite interpolation and Lagrange interpolation for case I and case II respectively. The concentration pattern in presence of the source for both the two cases are verified by Pdepe Matlab solution.

Case-I: When input is in the sinusoidal form $F(m't) = c_0\{l + \sin(m't)\}$.

For the present case, the value of both unsteady parameters m and m' are taken 0.8, while value assigned to dimensionless parameter l is 2. Concentration value in the time domain $0 \le t(day) \le 9$ are computed at times $t(day) =$ $2,5$ and 8, while for $t(day) > 9(= t_0)$ the same is computed at times $t(day) =$ $10,12$ and 14.

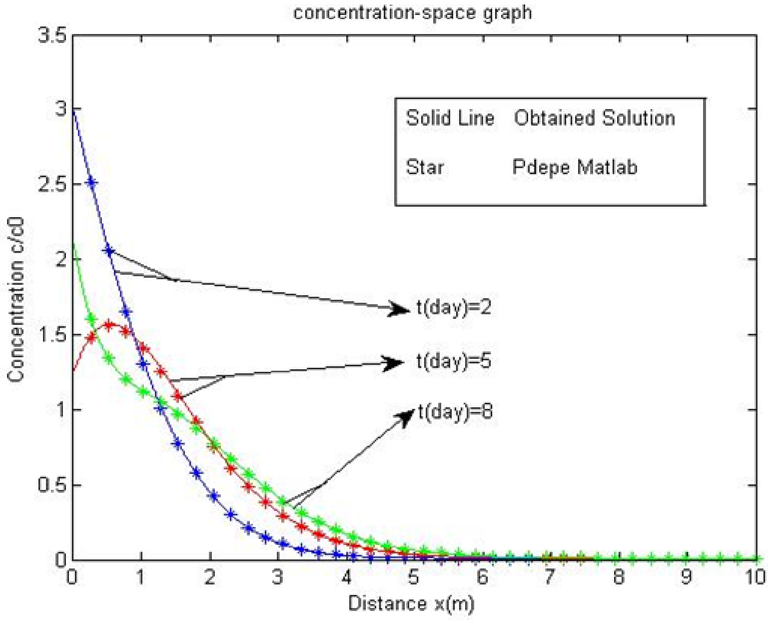

Figure-1: Distribution of dimensionless concentration for various in presence of source.

Figure1 is drawn to study the concentration pattern in presence of the source from solution Eq. (24a). Contaminant attenuates with position and time. It may be observed that the attenuation process is faster. It may also be observed that concentration near the inlet is much higher and less toward outer boundary and continuously decreases and goes on decreasing towards minimum or harmless concentration. Fluctuations at $x(m) = 0$ with time is due to periodic nature input. The obtained concentration is same in every aspect as the concentration pattern plotted with pdepe Matlab and this similarity authenticates the obtained solution.

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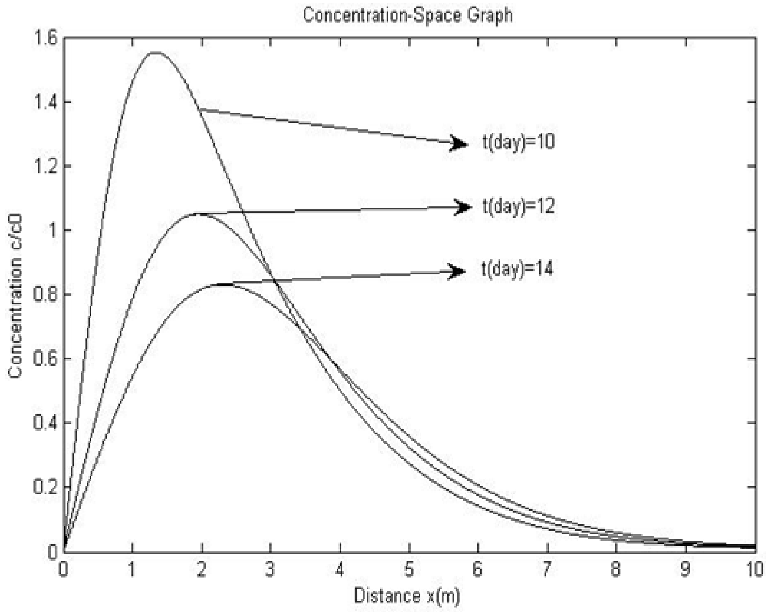

Figure-2: Distribution of dimensionless concentration for various time in absence of the source.

Fig. 2 illustrates the concentration profiles described by the solution in Eq. (24b), once the source of the pollution is eliminated, i.e., in the time domain $t > 9$ at $t(day) = 10,12,14$. The input concentration is nearly zero at source boundary. It may also observe that the contaminant concentration increases at the source and emerges at a point towards other boundary. The maximum value attained at lower time and lower for higher time.

Case-II: When value of input is $F(m't) = c_0 \exp.\left(\sqrt{m't}\right)$

For the present case, the value of unsteady parameters are assumed as $m'(day^{-1}) = 0.1$ and $m(day^{-1}) = 0.2$. Concentration value in the time domain in presence of source i.e. in time domain $0 \le t(day) \le 9$ are computed at times $t(day) =$ 0.05,2,5 and 8 .while for $t > 9(= t_0)$ the same is computed at times $t(day) =$ 10,12 and 14

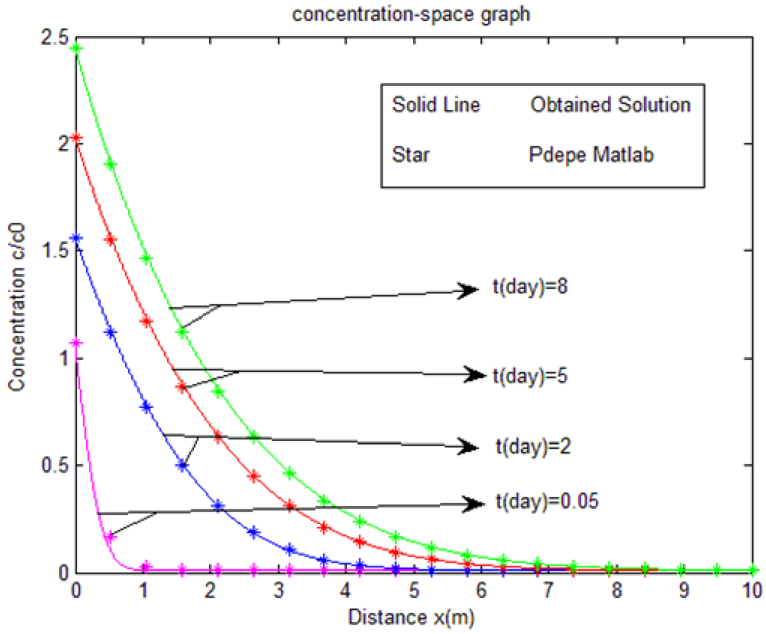

Figure-3: Distribution of dimensionless concentration for various times in presence of source.

Figure 3 displays the solute concentration from the point source along the longitudinal direction in the presence of the source of pollution, in the time domain $(0 \le t)$, at $t(day) = 0.05, 2, 5$ *and* 8. The input concentration, c/c_0 at the origin, $x(m) = 0$ are $1.05, 1.56, 2.03$ *and* 2.45 respectively. The concentration patterns are virtually identical for all values of time $t(day) = 0.05, 2, 5$ *and* 8. It attenuates with position and time. It may be observed that the contaminant concentration decreases and emerges towards the minimum or harmless concentration. The similarity between obtained solution and pdepe Matlab solution validates the solution.

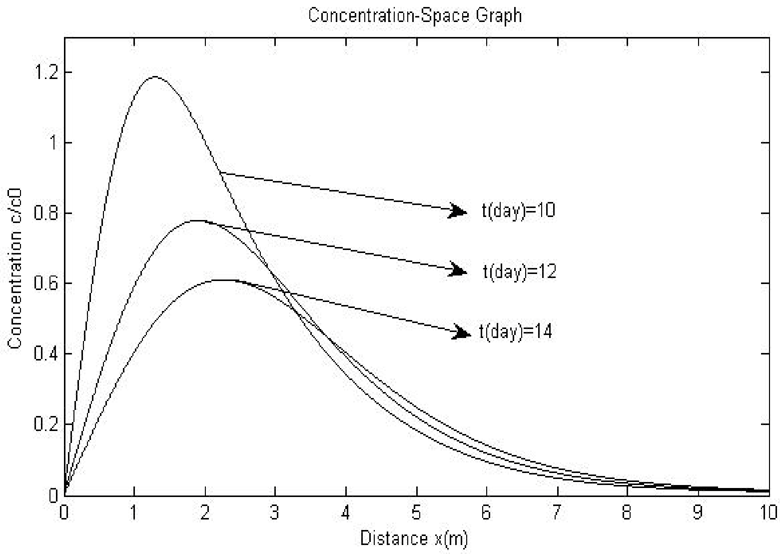

Figure-4: Distribution of dimensionless concentration for various times in absence of source.

Fig. 4 shows the solute concentration profiles at time $t(day) = 10, 12$ *and* 14 in the case of when source has been eliminated. It may be observed that the contaminant concentration profiles are virtually identical at source boundary. Initially, concentration is higher for lower time and lower for higher time near the source boundary. It may also be noticed that as the time elapses after elimination of source concentration, the peak of the concentration broadens, reduces and shifts away from origin as a result of transport phenomena of solute transport.

Since the approximation of the interpolation polynomial (9) in Lagrange method of interpolation depends on precision in selection of independent variable nodes (arguments), the solution may be refined with better approximate interpolation polynomial.

VERIFICATION OF SOLUTION

Consider the case where input $F(m't)=1/(1+m't)$ and solution (A7&A8) is obtained the way given in appendix. For a chosen set of data taken from the experimental and theoretical published literatures described as $c_0=1.0$, $c_i=0.01$ $u_0=0.01\left(m\ day^{-1}\right)$ and $D_0=1.2\left(m^2 day^{-1}\right)$, $R=1.25$, $\mu_0\left(day^{-1}\right)=0.05$, $\gamma_0=0.0007$, $\alpha=0.019$ and $m=m'=0.02$ respectively and with time of elimination of source $t_0(day)=9$,concentration- space graphs figure 5&6 are plotted from solution Eqs. (24a&24b) and Eqs. (A7&A8) in appendix in domain $0\le x(m)\le 10$ at times $t(day)=2,5 and 8$ in presence of source, while at times $t(day)=$ 10,12 and 14 in absence of source.

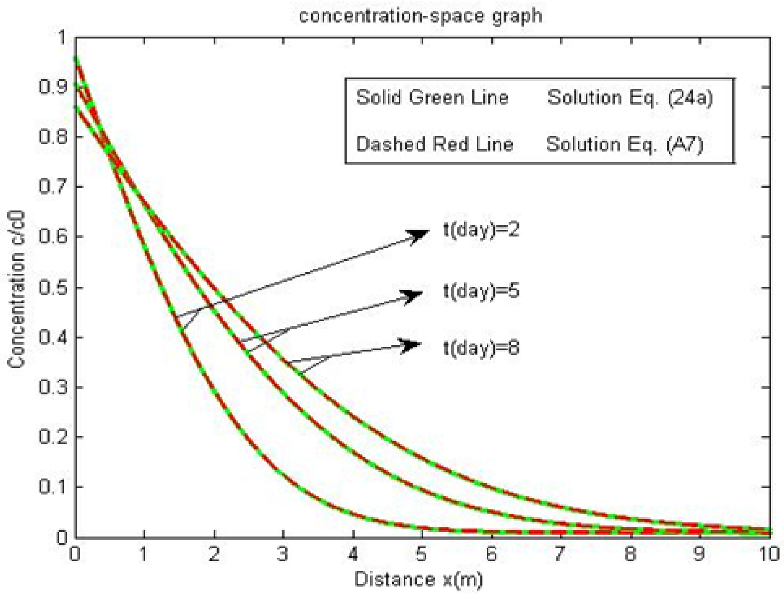

Figure-5: Distribution of dimensionless concentration for various times in presence of source.

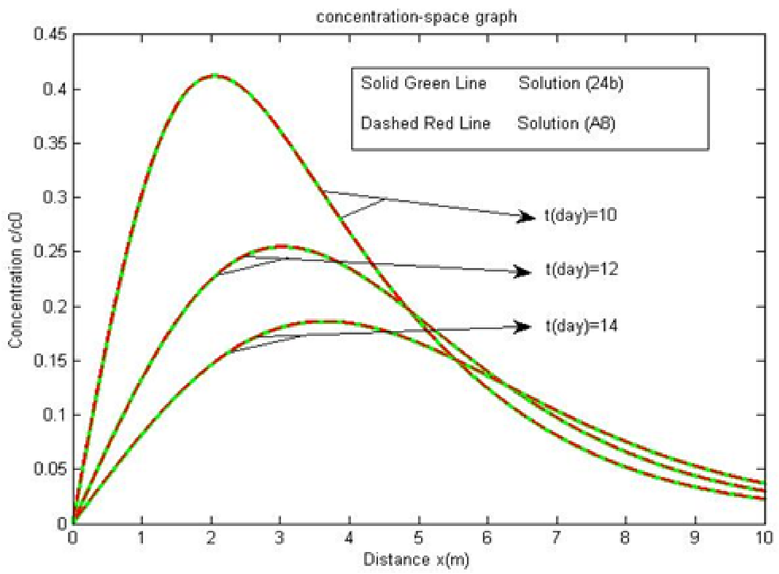

Figure-6: Distribution of dimensionless concentration for various times in absence of source.

Concentration pattern obtained from solution Eq.(A7& A8) in appendix has been in good agreement with same obtained from the Eq.(24a &24b) as shown with help of figure (5&6). It verifies the authenticity of the obtained solution Eq. (24a & 24b).

CONCLUSION

In the present study an analytical solution of advection- dispersion equation in one-dimensional semi-infinite porous media is obtained to simulate groundwater transport of a solute. Two different time dependent function are taken as input concentration. The input point source has been a general continuous and bounded smooth function of time in an interval of time and then eliminated.Laplace transformation technique is employed to get the analytical solution of the present problem. The input point source has been taken a general continuous and bounded smooth function of time dependent. The obtained result predicts the concentration profiles accurately for non-reactive contaminants and an appropriate interpolation method provides optimum result. The transport model is benchmarked against analytical solutions available in the literature for one-dimensional longitudinal porous formation. The proposed model has not been authenticated against any experimental data. The solution may help to determine minimum/maximum or harmless concentration at any position and time.

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Appendix:

The input boundary conditions with $F(m't) = 1/(1+m't)$ may be described as

$$c(x,t) = c_i \exp(-\alpha x); \; t>0, \; x>0 \tag{A1}$$

$$c(x,t) = \begin{cases} c_0/(1+m't); & 0<t\le t_0 \quad \text{(A2a)} \\ & x=0 \\ 0; & t>t_0 \quad , \quad \text{(A2b)} \end{cases}$$

$$\frac{\partial c(x,t)}{\partial x} = 0; \text{ as } x\to\infty \tag{A3}$$

With transformation Eq. (10) may written as:

$$c(x,T) = c_i \exp(-\alpha x), \; T=0, \; x>0 \tag{A4}$$

$$c(x,T) = \begin{cases} c_0 \exp(-mT); \; 0<T\le T_0 & \text{(A5a)} \\ x=0 & \\ 0; \; T>T_0, \quad x=0 & \text{(A5b)} \end{cases}$$

$$\frac{\partial c(x,T)}{\partial x} = 0, \text{ as } \quad x\to\infty \tag{A6}$$

Now, applying Laplace Transformation Technique, solution may be written as:

$$c(x,T) = \left[c_0 F_{\eta_1}(x,T) - \frac{\gamma_0}{\mu_0} F_\eta(x,T) - c_i F_\omega(x,T) + \frac{\gamma_0}{\mu_0} F_\rho(x,T) + c_i G(x,T) - \frac{\gamma_0}{\mu_0} H(x,T) \right] \times \exp\left\{ \frac{u_0}{2D_0}x - \frac{1}{R}\left(\frac{u_0^2}{4D_0} + \mu_0 \right) T \right\} + \frac{\gamma_0}{\mu_0}$$

$$0<T\le T_0 \tag{A7}$$

$$c(x,T) = \left[c_0 \left\{ F_{\eta_1}(x,T) - F_{\eta_1}(x,T-T_0)\exp(\eta_1^2 T_0) \right\} - \frac{\gamma_0}{\mu_0} F_\eta(x,T) - c_i F_\omega(x,T) + \frac{\gamma_0}{\mu_0} F_\rho(x,T) + c_i G(x,T) - \frac{\gamma_0}{\mu_0} H(x,T) \right] \times \exp\left\{ \frac{u_0}{2D_0}x - \frac{1}{R}\left(\frac{u_0^2}{4D_0} + \mu_0 \right) T \right\} + \frac{\gamma_0}{\mu_0}$$

$$T_0 < T \tag{A8}$$

Where,

$$\eta_1 = \sqrt{\eta^2 - m}$$