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INTUITIONISTIC FUZZY SETS IN MEDICAL DIAGNOSIS

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ABSTRACT

In this paper, tangent inverse distance and sine similarity measure between intuitionistic fuzzy sets are proposed and some of its properties are discussed herein. The concept of the above methods are the essential tool for dealing with uncertainties and shortcomings that affect the existing methods. Implementation of medical diagnosis is presented to find out the disease impacting the patient.

Keywords and Phrases: *Intuitionistic fuzzy set, tangent inverse distance, sine similarity measure, medical diagnosis*

2000 Mathematics subject classification: *03E72, 03F55, 62P10, 92C50.*

1. INTRODUCTION

Kumbakonam finds its place inthe 37^{th} rank in the selection list of cleanest cities in India among the 434 cities participated in this competition in the year 2017.Kumbakonam could have gained one of the first 10 spots but for the existence of few open drainage channels. These channelsserve as breeding centers of mosquitoes and thereby cause several diseases. A significant number of people have been affected by this.An urge to seek a fruitful remedy to this socio-medical malaise persuaded us to carryoutour research in the medical field. By pursuing not-so-far-attempted methods in the research, the diseases can be diagnosed with minimal loss of time and with more accuracy.

A number of real life problems in engineering, medical sciences, social sciences, economics *etc*., involve imprecise data and their solution involves the use of mathematical principles based on uncertainty and imprecision. Such uncertainties are being dealt with the help of topics like probability theory, fuzzy set theory[18], rough set theory [13] *etc*., Healthcare industry has been trying to complement the services offered by conventional clinical decision making systems with the integration of fuzzy logic techniques in them. As it is not an easy task for a clinician to derive a fool proof diagnosis it is advantageous to automate few initial steps of diagnosis which would not require intervention from an expert doctor. Intuitionistic fuzzy set possesses all attributes necessary to encode medical knowledge base and capture medical inputs.

As medical diagnosis demands large amount of information processing, large portion of which is quantifiable, also intuitive thought process involve rapid unconscious data processing and combines available information by law of average, the whole process offers low intra and inter personal consistency. So contradictions, inconsistency and fuzziness should be accepted as unavoidable as they are integrated in the behavior of biological systems as well as in their characterization. To model an expert doctor it is imperative that it should not disallow uncertainty as it would be then inapt to capture fuzzy or incomplete knowledge that might lead to the danger of fallacies due to misplaced precision.

As medical diagnosis contains lots of uncertainties and increased volume of information available to physicians from new medical technologies, the process of classifying different sets of symptoms under a single name of disease becomes difficult. In some practical situations, there is the possibility of each element having truth membership and false membership functions. So, intuitionistic fuzzy sets play a vital role in medical diagnosis.

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In 1965, Fuzzy set theory was firstly given by Zadeh [18] which is applied in many real applications to handle uncertainty. Sometimes membership function itself is uncertain and hard to be defined by a crisp value. So, the concept of interval valued fuzzy sets was proposed to capture the uncertaintyof grade of membership. In 1986, Atanassov [1]

introduced the intuitionistic fuzzy sets which consider both truth-membership and falsity-membership. Atanassov [2] introduced some concept of intuitionistic fuzzy sets. Hung, W.L and Yang,M.S[5] proposed the similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. Hung, W.L and Yang,M.S[6] examined the j-divergence of intuitionistic fuzzy sets. Similarity measures for the intuitionistic fuzzy sets were proposed by Hung, W.L and Yang,M.S[7], Intarapaiboon,P[8] & Liang,Z and Shi,P [11]. Khatibi,V and Montazer,G.A[10] examined the intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition. Pramanik,S and Mondal,K[14] introduced the weighted fuzzy similarity measure based on tangent function.Xu,Z [16] examined the intuitionistic preference relations and their application in group decision making. Ye, J [17] introduced the cosine similarity measures for intuitionistic fuzzy sets. Ejegwa P.A [3] examined the four distance measures of intuitionistic fuzzy multisets. Hanen Jemal, Zied Kechaou, and Mounir Ben Ayed [4] developed a prototype model for detecting the patients risk degree in intensive care unit. Muthukumar, P and Saisundara Krishnan, G [12] proposed similarity measure for intuituionistic fuzzy soft sets. Sarala, N and Suganya, B [15] introduced similarity measure for intuituionistic fuzzy soft sets.

Tangent inverse distance and Sine similarity measure have more accuracy and perfection than others. In some of the existing methods in intuitionistic fuzzy sets, the results are not exact and cannot be defined (if $\mu_A = \nu_A = \gamma_A = 0$). So, the new methods (tangent inverse distance and sine similarity measure) has been developed overcoming such limitations. The results obtained will be precise.

In this paper, using the notion of intuitionistic fuzzy set, it was provided an exemplary for medical diagnosis. In order to make this, various types of methods are executed.

Rest of the article is structured as follows.In Section 2, the basic definitions are briefly presented.Section 3 deals with proposed definitions and some of its properties. Sections 4, 5 & 6 contains methodology, algorithm and case study related to medical diagnosis respectively. Conclusion is givenin Section 7.

2. BASIC DEFINITIONS

2.1 Definition [14]: Let X be a non-empty set. A fuzzy set A drawn from X is defined as $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A(x): X \to [0,1]$ is the membership function for the fuzzy set A.

2.2 Definition [2]: Let X be a non-empty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu_A(x): X \to [0,1], \nu_A(x): X \to [0,1]$ define the degree of membership and degree of non-membership respectively of the element $x \in X$ to the set A,which is a subset of X, and every element $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$. The value $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the intuitionistic fuzzy index or hesitation margin of x in A . $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the intuitionistic fuzzy set A and $\pi_A(x) \in [0,1]$.*i.e.*, $\pi_A(x) : X \to [0,1]$ and $0 \le \pi_A(x) \le 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

3. PROPOSED DEFINITIONS

3.1 Definition: Let $A = (\mu_A(x), \nu_A(x), \pi_A(x))$ and $B = (\mu_B(x), \nu_B(x), \pi_B(x))$ be two intuitionistic fuzzy sets. Then the tangent inverse distance is defined as

$$TID_{IFS}(A,B) = \frac{1}{2(n+1)} \sum_{i=1}^{n} \left[\tan^{-1}\left[\frac{\pi}{4} \left[1 + |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|\right]\right]\right] \quad (1)$$

Proposition 1:

(i) $TID_{IFS}(A,B) > 0$

(ii) $TID_{IFS}(A,B) = TID_{IFS}(B,A)$

(iii) If $A \subseteq B \subseteq C$ then $TID_{IFS}(A,C) \ge TID_{IFS}(A,B)$ & $TID_{IFS}(A,C) \ge TID_{IFS}(B,C)$

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Proof:

(i) The proof is straightforward.

(ii) The proof is straightforward.

(iii) It was well known that,

$$\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$$
$$\nu_A(x_i) \ge \nu_B(x_i) \ge \nu_C(x_i)$$
$$\pi_A(x_i) \ge \pi_B(x_i) \ge \pi_C(x_i).$$
$$(\because A \subseteq B \subseteq C)$$

Hence,

$$\left|\mu_A(x_i) - \mu_B(x_i)\right| \le \left|\mu_A(x_i) - \mu_C(x_i)\right|$$
$$\left|\mu_B(x_i) - \mu_C(x_i)\right| \le \left|\mu_A(x_i) - \mu_C(x_i)\right|$$
$$\left|\nu_A(x_i) - \nu_B(x_i)\right| \le \left|\nu_A(x_i) - \nu_C(x_i)\right|$$
$$\left|\nu_B(x_i) - \nu_C(x_i)\right| \le \left|\nu_A(x_i) - \nu_C(x_i)\right|$$
$$\left|\pi_A(x_i) - \pi_B(x_i)\right| \le \left|\pi_A(x_i) - \pi_C(x_i)\right|$$
$$\left|\pi_B(x_i) - \pi_C(x_i)\right| \le \left|\pi_A(x_i) - \pi_C(x_i)\right|.$$

Here, the tangent inverse distance is an increasing function

$$TID_{IFS}(A,C) \ge TID_{IFS}(A,B) \;\&\; TID_{IFS}(A,C) \ge TID_{IFS}(B,C)$$

3.2 Definition: Let $A = (\mu_A(x), \nu_A(x), \pi_A(x))$ and $B = (\mu_B(x), \nu_B(x), \pi_B(x))$ be two intuitionistic fuzzy sets. Then the sine similarity measure is defined as

$$SIN\,IFS(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left[1 - \sin\left[\frac{\pi}{4}\left[\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\nu_A(x_i) - \nu_B(x_i)\right| + \left|\pi_A(x_i) - \pi_B(x_i)\right|\right]\right]\right] \quad (2)$$

Proposition 2:

(i) $SIN_{IFS}(A,B) > 0$

(ii) $SIN_{IFS}(A,B) = SIN_{IFS}(B,A)$

(iii) If $A \subseteq B \subseteq C$ then $SIN_{IFS}(A,C) \le SIN_{IFS}(A,B) \;\&\; SIN_{IFS}(A,C) \le SIN_{IFS}(B,C)$

Proof:

(i) The proof is straightforward.

(ii) The proof is straightforward.

(iii) It was well known that,

$$\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$$
$$\nu_A(x_i) \ge \nu_B(x_i) \ge \nu_C(x_i)$$
$$\pi_A(x_i) \ge \pi_B(x_i) \ge \pi_C(x_i).$$
$$(\because A \subseteq B \subseteq C)$$

Hence,

$$\left|\mu_A(x_i) - \mu_B(x_i)\right| \le \left|\mu_A(x_i) - \mu_C(x_i)\right|$$
$$\left|\mu_B(x_i) - \mu_C(x_i)\right| \le \left|\mu_A(x_i) - \mu_C(x_i)\right|$$
$$\left|\nu_A(x_i) - \nu_B(x_i)\right| \le \left|\nu_A(x_i) - \nu_C(x_i)\right|$$
$$\left|\nu_B(x_i) - \nu_C(x_i)\right| \le \left|\nu_A(x_i) - \nu_C(x_i)\right|$$
$$\left|\pi_A(x_i) - \pi_B(x_i)\right| \le \left|\pi_A(x_i) - \pi_C(x_i)\right|$$
$$\left|\pi_B(x_i) - \pi_C(x_i)\right| \le \left|\pi_A(x_i) - \pi_C(x_i)\right|.$$

Here, the sine similarity measure is a decreasing function

$$SIN_{IFS}(A,C) \le SIN_{IFS}(A,B) \;\&\; SIN_{IFS}(A,C) \le SIN_{IFS}(B,C)$$

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4. METHODOLOGY

In this section, we present an application of intuitionistic fuzzy set in medical diagnosis. In a given pathology, suppose S is a set ofsymptoms, D is a set of diseases and P is a set of patients and let Q be aintuitionistic fuzzy relation from the set ofpatients to the symptoms. *i.e.,* $Q(P \rightarrow S)$ and R be a intuitionistic fuzzy relation from the set of symptoms to the diseases *i.e.,* $R(S \rightarrow D)$ and then the methodology involves three main jobs:

1. Determination of symptoms
2. Formulation of medical knowledge based on intuitionistic fuzzy sets
3. Determination of diagnosis on the basis of new types of methods in intuitionistic fuzzy sets

5. ALGORITHM

Step-1: The Symptoms of the patients are given to obtain the patient-symptom relation Q and are noted in Table 1

Step-2: The medical knowledge relating the symptoms with the set of disease under consideration are given to obtain the symptom-disease relation R and are noted in Table 2.

Step-3: The computation T of the relation of patients and diseases is found using (1) & (2) and are noted in Table 3 & 4 respectively.

Step-4: Finally, the minimum value from Table 3 and maximum value from Table 4 of each row were selected to find the possibility of the patient affected with the respective disease and then it was concluded that the patient $P_k \left(1, 2, 3 \,\&\, 4\right)$ was suffering from the disease $D_r \left(r = 1, 2, 3, 4 \,\&\, 5\right)$

6. CASE STUDY [9]

Let there be four patients $= \left\{P_2, P_2, P_{2\cdot}\right\}$ and the set of symptoms S ={ S_1 Temperature, S_2 Headache, S_2 Stomach pain, S_4 Cough, S_5 Chest pain}.The intuitionistic fuzzy relation $Q(P \rightarrow$ is given as in Table 1.Let the set of diseases D = { D_1 = Viral fever, D_2 = Malaria, D_3 = Typhoid, D_4 = Stomach problem, D_5 = Chest problem}.The intuitionistic fuzzy relation $R(S \rightarrow$ is given as in Table 2.

Table-1: Patient - Symptom Relation (Using step1)

Q	Temperature	Headache	Stomach pain	Cough	Chest pain
P_1	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.2,0.8,0.0)	(0.6,0.1,0.3)	(0.1,0.6,0.3)
P_2	(0.0,0.8,0.2)	(0.4,0.4,0.2)	(0.6,0.1,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
P_3	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
P_4	(0.6,0.1,0.3)	(0.5,0.4,0.1)	(0.3,0.4,0.3)	(0.7,0.2,0.1)	(0.3,0.4,0.3)

Table-2: Symptom - Disease Relation (Using step2)

R	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Temperature	(0.4,0.0,0.6)	(0.7,0.0,0.3)	(0.3,0.3,0.4)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
Headache	(0.3,0.5,0.2)	(0.2,0.6,0.2)	(0.6,0.1,0.3)	(0.2,0.4,0.4)	(0.0,0.8,0.2)
Stomach pain	(0.1,0.7,0.2)	(0.0,0.9,0.1)	(0.2,0.7,0.1)	(0.8,0.0,0.2)	(0.2,0.8,0.0)
Cough	(0.4,0.3,0.3)	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.2,0.7,0.1)	(0.2,0.8,0.0)
Chest pain	(0.1,0.7,0.2)	(0.1,0.8,0.1)	(0.1,0.9,0.0)	(0.2,0.7,0.1)	(0.8,0.1,0.1)

Table-3: Tangent inverse distance (Using step3 & step4)

T	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.3649	**0.3547**	0.3605	0.4192	0.4163
P_2	0.3855	0.4009	0.3715	**0.3276**	0.3923
P_3	0.3910	0.4012	**0.3744**	0.4053	0.4186
P_4	**0.3688**	0.3704	0.3918	0.4041	0.4242

Table-4: Sine similarity measure (Using step3)

T	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
P_1	0.2931	**0.3210**	0.2976	0.1453	0.1436
P_2	0.2339	0.1898	0.2717	**0.3913**	0.2165
P_3	0.2235	0.1950	**0.2667**	0.1761	0.1463
P_4	**0.2875**	0.2786	0.2224	0.1869	0.1299

From Table 3 & 4, it is obvious that, if the doctor agrees, then P_1 suffers from Malaria, P_2 suffers from Stomach problem, P_3 suffers from Typhoid and P_4 suffers from Viral fever.

7. CONCLUSIONS

Our propounded techniques are most decisive to hold the problems related to medical diagnosis competently. The proposed approaches can find more implementation in other areas such as decision making, cluster analysis *etc*.

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