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INTUITIONISTIC FUZZY G_δ - e -LOCALLY CLOSED SETS

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ABSTRACT

This paper is devoted to the study of new class of sets called an intuitionistic fuzzy e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed sets and intuitionistic fuzzy G_δ - e -locally closed sets are introduced and studied. Also the concepts of an intuitionistic fuzzy G_δ - e -locally closed set intuitionistic fuzzy subspace, intuitonistic fuzzy G_δ - e -local- δ -semi (resp., δ -pre and β) spaces are introduced and interesting properties are established. In this connection, interrelations are discussed. Examples are provided where necessary.

Keywords and phrases: *Intuitionistic fuzzy - e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed set, intuitionistic fuzzy G_δ - e -locally closed set, intuitionistic fuzzy G_δ - e -local- $T_{1/2}$ - spaces and intuitionistic fuzzy G_δ - e - δ - semi (resp., δ -pre and β) spaces.*

AMS (2000) subject classification: *54A40, 54A99, 03E72, 03E99.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [14] and Atanassov [1] introduced and studied intuitionistic fuzzy sets. On the other hand, Coker [4] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy α -closed set was introduced by Biljana Krsteshka and Erdal Ekici [9]. The first step of locally closedness was done by Bourbaki [3]. Ganster and Relly used locally closed sets in [8] to define LC-continuity and LC-irresoluteness. Roja, Uma and Balasubramanian [2] discussed fuzzy G_δ continuous functions. The initiations of e -open sets, e -continuity and e -compactness in topological spaces are due to Ekici [5,6,7]. In fuzzy topology, e -open sets were introduced by Seenivasan in 2014 [11]. Sobana et.al [12] were introduced the concept of fuzzy e -open sets, fuzzy e -continuity and fuzzy e -compactness in intuitionistic fuzzy topological spaces (briefly., IFTS's). In this paper we introduce the concepts of an intuitionistic fuzzy e -locally closed sets, intuitionistic fuzzy eG_δ -sets, intuitionistic fuzzy eG_δ -locally closed sets and intuitionistic fuzzy G_δ - e -locally closed sets in IFTS's. Also the concepts of an intuitionistic fuzzy G_δ - e -locally closed intuitionistic fuzzy subsapce, intuitionistic fuzzy G_δ - e -local $T_{1/2}$ $T_{\frac{1}{2}}$ space, intuitionistic fuzzy G_δ - e -local δ - semi (resp., δ -pre and β) spaces are introduced and studied. Some interesting properties and interrelations among sets and spaces are discussed with necessary examples.

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2. PRELIMINARIES

Let X be a nonempty fixed set and I the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) [1] A is an object of the following form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$, where the mapping $\mu_A : X\to I$ and $\gamma_A : X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x\in X$ to the set A, respectively, and $0\le \mu_A(x)+\gamma_A(x)\le 1$ for each $x\in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form, $A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\}$. For the sake of simplicity, we shall use the symbol $A=\langle x,\mu_A,\gamma_A\rangle$ for the intuitionistic fuzzy set $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$. Let A and B be IFSs of the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$. Then (i) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\gamma_A(x)\ge\gamma_B(x)$; (ii) $\overline{A}=\{\langle x,\gamma_A(x),\mu_A(x)\rangle : x\in X\}$; (iii) $A\cap B=\{\langle x,\mu_A(x)\wedge\mu_B(x),\gamma_A(x)\vee\gamma_B(x)\rangle : x\in X\}$; (iv) $A\cup B=\{\langle x,\mu_A(x)\vee\mu_B(x),\gamma_A(x)\wedge\gamma_B(x)\rangle : x\in X\}$; The IFS's $\underset{\sim}{0}$ and $\underset{\sim}{1}$ are defined by, $\underset{\sim}{0}=\{< x,0,1 >: x\in X\}$ and $\underset{\sim}{1}=\{< x,1,0 >: x\in X\}$. An intuitionistic fuzzy topology (IFT) [4] in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms: (i) $\underset{\sim}{0},\underset{\sim}{1}\in T$; (ii) $G_1\cap G_2\in T$, for any $G_1,G_2\in T$; (iii) $\cup G_i\in T$ for any arbitrary family $\{G_i : i\in J\}\subseteq T$. In this paper by (X,T) or simply by X we will denote the intuitionistic fuzzy topological space(IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X. Let (X,T) be an IFTS and $A=\{< x,\mu_A,\nu_A >: x\in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior [4] of A are defined by (i) $cl(A)=\bigcap\{C : C \text{ is an IFCS in } X \text{ and } C\supseteq A\}$; (ii) $int(A)=\bigcup\{D : D \text{ is an IFOS in } X \text{ and } D\subseteq A\}$; It can be also shown that $cl(A)$ is an IFCS, $int(A)$ is an IFOS in X and A is and IFCS in X if and only if $cl(A)=A$; A is an IFOS in X if and only if $int(A)=A$ Let A be IFS in an IFTS (X,T). A is called an (i) intuitionistic fuzzy regular open set (briefly $IFROS$) [13] if $A=intcl(A)$, (ii) intuitionistic fuzzy regular closed set (briefly $IFRCS$) [13] if $A=clint(A)$. Let (X,T) be a fuzzy topological space and λ be a fuzzy set in X. λ is called G_δ set [2] if $\lambda=\bigwedge_{i=1}^{\infty}\lambda_i$ where each $\lambda_i\in T$. The complement of fuzzy G_λ is fuzzy F_σ Let (X,T) be an IFTS and $A=< x,\mu_A(x),\nu_A(x) >$ be a IFS in X. Then the fuzzy δ closure [13] of A are denoted and defined by $cl_\delta(A)=\cap\{K : K$ is an IFRCS in X and $A\subseteq K\}$ and $int_\delta(A)=\cup\{G : G$ is an IFROS in X and $G\subseteq A\}$. Let A be an IFS in an IFTS (X,T). A is called an intuitionistic fuzzy δ -semiopen (resp. δ -preopen, β -open) [12] set (IF δ SO (resp. IF δ PO, IF β O), for short), if $A\le cl(int_\delta(A))$ (resp. $A\le int(cl_\delta(A))$, $A\le cl(int(cl(A)))$). A is called an intuitionistic fuzzy δ -semiclosed (resp. δ -preclosed, β -closed)[12] set (IF δ SC (resp. IF δ PC, IF β C) (for short)) if $A\ge int(cl_\delta(A))$ (resp. $A\ge cl(int_\delta(A))$, $A\ge int(cl(int(A)))$). Let A be an IFS in an IFTS (X,T). A is called an intuitionistic fuzzy e -open set [12] (IFeOS, for short) in X if $A\subseteq clint_\delta(A)\cup intcl_\delta(A)$. Let (X,T) be an IFTS and Y be any intuitionistic fuzzy subset of X. Then $T_Y=(A/Y \mid A\in T)$ is an intuitionistic fuzzy topology on Y and is called the induced or relative intuitionistic fuzzy topology [10]. The pair (Y,T_Y) is called an intuitionistic fuzzy subspace of $(X,T):(Y,T_Y)$ is called an intuitionistic fuzzy open/intuitionistic fuzzy closed subspace if the intuitionistic fuzzy charateristic function of (Y,T_Y) viz χ_Y is intutionistic fuzzy open/intuitionistic fuzzy closed.

3. Intuitionistic fuzzy G_δ-e-locally closed sets in an IFTSs

In this section, the concepts of an intuitionistic fuzzy e-locally closed set, intuitionistic fuzzy eG_δ-set, intuitionistic fuzzy eG_δ-locally closed set and intuitionistic fuzzy G_δ-e-locally closed set are introduced and studied. The concepts of an intuitionistic fuzzy G_δ-e-locally closed intuitionistic fuzzy subspace, intuitionistic fuzzy G_δ-e-local $T_{\frac{1}{2}}$ space, intuitionistic fuzzy G_δ-e-δ-semi space, intuitionistic fuzzy G_δ-e-δ-pre space, intuitionistic fuzzy G_δ-β space are introduced and studied. In this connection, interrelation among sets and spaces are discussed with suitable examples.

Definition 3.11: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an IFS on an IFTS (X,T). Then A is said to be intuitionistic fuzzy e- locally closed set (in short, IF-e-lcs) if $A=C\cap D$, where $C=\{\langle x,\mu_C(x),\gamma_C(x)\rangle : x\in X\}$ is an IFeOS and $D=\{\langle x,\mu_D(x),\gamma_D(x)\rangle : x\in X\}$ is an IFeCS in (X,T).

Definition 3.22: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an intuitionistic fuzzy set on an IFTS X. Then A is said to be an intuitionistic fuzzy eG_δ- set(in short IF-eG_δ-set) if $A=\bigcap_{i=1}^{\infty}A_i$, where $A_i=\{\langle x,\mu_{A_i}(x),\gamma_{A_i}(x)\rangle : x\in X\}$ is an IFeOS in an IFTS (X,T).

Definition 3.33: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an IFS on an IFTS (X,T). Then A is said to be an intuitionistic fuzzy eG_δ-locally closed set (in short,IF-eG_δ-lcs) if $A=C\cap D$, where $C=\{\langle x,\mu_C(x),\gamma_C(x)\rangle : x\in X\}$ is an IF- eG_δ -set and $D=\{\langle x,\mu_D(x),\gamma_D(x)\rangle : x\in X\}$ is an IFeCS in (X,T).

Definition 3.44: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an IFS on an IFTS (X,T). Then A is said to be an intuitionistic fuzzy G_δ-e-locally closed set (in short,IF G_δ-e-lcs) if $A=B\cap C$, where $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ is an IF-G_δ -set and $C=\{\langle x,\mu_C(x),\gamma_C(x)\rangle : x\in X\}$ is an IFeCS (X,T).

The complement of an intuitionistic fuzzy G_δ-e-lcs is said to be an intuitionistic fuzzy G_δ-e-locally open set (in short, IF G_δ-e-los).

Definition 3.55: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an IFS on an IFTS (X,T). The intuitionistic fuzzy G_δ-e-locally closure of A is denoted and defined by IFG_δ-e-$lcl(A)=\bigcap\{B : B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ is an IF-G_δ-e-lcs in X and $A\subseteq B$.

Proposition 3.16: Let (X,T) be an IFTS. For any two IFSs $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ of an IFTS (X,T) then the following statements are true. (i) IFG_δ-e-$lcl(\underset{\sim}{0})=\underset{\sim}{0}$ (ii) $A\subseteq B\Rightarrow IFG_\delta$-$e$-$lcl(A)\subseteq IFG_\delta$-$e$-$lcl(B)$ (iii) IFG_δ-e-$lcl(IFG_\delta$-e-$lcl(A))=IFG_\delta$-e-$lcl(A)$ (iv) IFG_δ-e-$lcl(A\cup B)=(IFG_\delta$-e-$lcl(A))\cup(IFG_\delta$-e-$lcl(B))$

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Definition 3.67: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ be an IFS on an IFTS (X,T). The IF-G_δ-e-locally interior of A is denoted and defined by $IFG_\delta\text{-}e\text{-}lint(A)=\bigcup\{B : B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ is an IF-G_δ-e-los in X and $B\subseteq A$.

Proposition 3.28: Let (X,T) be an IFTS. Let $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ and $B=\{\langle x,\mu_B(x),\gamma_B(x)\rangle : x\in X\}$ are IFS in an IFTS (X,T). Then the following statements are true. (i) $IFG_\delta\text{-}e\text{-}lcl(A)$ is the largest IF G_δ-e-los contained in A (ii) If A is an IF G_δ-e-los then $A=IFG_\delta\text{-}e\text{-}lint(A)$ (iii) IF A is an IF-G_δ-e-los then $IFG_\delta\text{-}e\text{-}lint(IFG_\delta\text{-}e\text{-}lint(A))=IFG_\delta\text{-}e\text{-}lint(A)$ (iv) $IFG_\delta\text{-}e\text{-}lint(A)=IFG_\delta\text{-}e\text{-}lcl(\overline{A})$ (v) $IFG_\delta\text{-}e\text{-}lcl(A)=IFG_\delta\text{-}e\text{-}lint(\overline{A})$ (vi)If $A\subseteq B$ then $IFG_\delta\text{-}e\text{-}lint(A)\subseteq IFG_\delta\text{-}e\text{-}lint(B)$ (vii) $(IFG_\delta\text{-}e\text{-}lint(A))\cap(IFG_\delta\text{-}e\text{-}lint(B))\supseteq IFG_\delta\text{-}e\text{-}lint(A\cap B)$.

Remark 3.1:9 (i) $IFG_\delta\text{-}e\text{-}lcl(A)=A$ if and only if A is an IF-G_δ-e-lcs (ii) $IFG_\delta\text{-}e\text{-}lint(A)\subseteq A\subseteq IFG_\delta\text{-}e\text{-}lcl(A)$ (iii) $IFG_\delta\text{-}e\text{-}lint(\underset{\sim}{1})=\underset{\sim}{1}$ (iv) $IFG_\delta\text{-}e\text{-}lint(\underset{\sim}{0})=\underset{\sim}{0}$ (v) $IFG_\delta\text{-}e\text{-}lcl(\underset{\sim}{1})=\underset{\sim}{1}$

Proposition 3.310: Every IF-e- lcs is an IF eG_δ-lcs.

Remark 3.211: The converse of the Proposition (3.3) need not be true as show in Example (3.1).

Example 3.112: Let $X=\{a,b\}$ be a nonempty set. Let $A=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$, $B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$, $A\vee B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$ and $A\wedge B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$ be IFS of X. Then the family $T=\{0,1,A,B,A\vee B,A\wedge B\}$ is an IFT on X. Now $C=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.9},\frac{b}{0.9})\right\rangle$ be IF-eG_δ- set let $D=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{1},\frac{b}{1})\right\rangle$ be an IF- e-closed set. Hence $E=C\cap D=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{1},\frac{b}{1})\right\rangle$ is IF-eG_δ-lcs. But, E is not an IF-e-lcs. Hence, IF-eG_δ-set need not be an IF-e-lcs.

Proposition 3.413: Every IF G_δ-lcs is an IF-G_δ-e-lcs.

Remark 3.314: The converse of the Propositionn (3.4) need not be true as shown in Example (3.2).

Example 3.2:15 In Example (3.1), $A\wedge B$ is an IF- G_δ-set. Let $F=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.7},\frac{b}{0.7})\right\rangle$ be an IF-e-closed set. Hence $E=(A\wedge B)\cap F=\left\langle x,(\frac{a}{0},\frac{b}{0.1}),(\frac{a}{0.7},\frac{b}{0.7})\right\rangle$ is IF-G_δ-e-lcs. But, E is not an IF-G_δ-lcs. Hence, IF-G_δ-e-lcs need not be an IF-G_δ-lcs

Remark 3.416: *IF-* e - lcs and IF-G_δ-e-lcs are independent of each other as shown by the following Example (3.3).

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Example 3.317: In Example (3.1), $A \wedge B$ is an IF- G_δ -set. Let $G = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an IF- e -closed set. Hence $E = (A \wedge B) \cap G = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is IF- G_δ - e -lcs. But, E is not an IF- e -lcs and $H = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ be an IF- e -open set Hence $E = G \cap H = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ is IF- e -lcs. But, E is not IF- G_δ - e -lcs.

Proposition 3.5*18:* Every IF- G_δ - e -lcs is an IF- eG_δ -lcs.

Remark 3.519: The converse of the Propositionn (3.5) need not be true as shown in Example (3.4).

Example 3.420: In Example (3.1), $C = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ be IF- eG_δ - set and $G = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an IF- e -closed set. Hence $E = C \cap G = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ is IF- eG_δ -lcs. But, E is not an IF- G_δ - e -lcs. Hence, IF- eG_δ -lcs need not be an IF- G_δ - e - lcs.

Remark 3.621: IF- G_δ - lcs and IF- eG_δ - e -lcs are independent of each other as shown by the following Example (3.5).

Example 3.522: In Example (3.1), $C = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ be IF- eG_δ - set and $G = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an IF- e -closed set. Hence $E = C \cap G = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.9}, \frac{b}{0.9}) \right\rangle$ is IF- eG_δ -lcs. But, E is not an IF- G_δ -lcs and $A \wedge B$ is an IF- G_δ -set let $I = \left\langle x, (\frac{a}{0.6}, \frac{b}{0.6}), (\frac{a}{0.3}, \frac{b}{0.1}) \right\rangle$ be an intuitionistic fuzzy closed set. Hence $E = (A \wedge B) \cap I = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is IF- G_δ -lcs. But, E is not an IF- eG_δ -lcs.

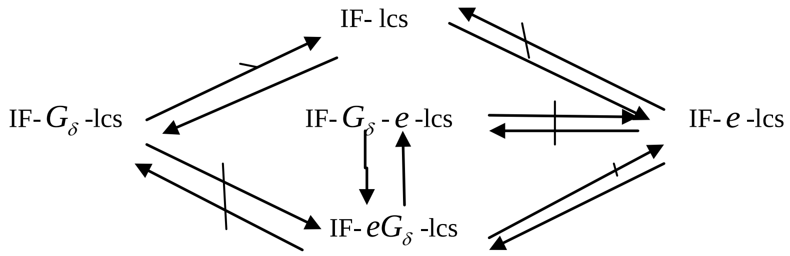
Proposition 3.623: Every IF -locally closed set is an IF- e -lcs.

Remark 3.724: The converse of the Propositionn (3.6) need not be true as shown in Example (3.6)

Example 3.6: In Example (3.1), Let $J = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ be IFeOS and $H = \left\langle x, (\frac{a}{0.1}, \frac{b}{0.2}), (\frac{a}{0.4}, \frac{b}{0.4}) \right\rangle$ be an IFeCS. Hence $E = J \cap H = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ is IF- e -lcs. But, E is not an IF- locally closed set.

Remark 3.826: Every IF- locally closed set is an IF- G_δ -lcs but the converse need not be true as shown in [10].

Remark 3.927: Clearly the followinng diagram holds.



Definition 3.728: Let (X,T) be an IFTS and Y be any intuitionistic fuzzy subset of X. Then $T_Y = (A/Y \mid A \in T)$ is an IFT on Y and is called the induced or relative intuitionistic fuzzy topology. The pair (Y,T_Y) is called an intuitionistic fuzzy subspace of $(X,T):(Y,T_Y)$ is called an IF- G_δ - e -locally closed intuitionistic fuzzy subspace if the intuitionistic fuzzy charateristic function of (Y,T_Y) viz χ_Y is IF- G_δ - e -lcs.

Proposition 3.729: Let (X,T) be an IFTS. Suppose $Z \subseteq Y \subseteq X$ and (Y,T_Y) is an IF- G_δ - e -locally closed intuitionistic fuzzy subspace of an IFTS (X,T). If (Z,T_Z) is an IF- G_δ - e -locally closed intuitionistic fuzzy subspace in an IFTS $(X,T) \Leftrightarrow (Z,T_Z)$ is an intuitionistic fuzzy G_δ - e -locally closed intuitionistic fuzzy subspace in an IFTS (Y,T_Y).

Definition 3.830: An IFTS (X,T) is said to be an intuitionnistic fuzzy G_δ - e -local- $T_{\frac{1}{2}}$ space if for every intuitionistic fuzzy G_δ - e -locally closed set is an intuitionistic fuzzy closed set in an IFTS (X,T).

Definition 3.931: An IFTS (X,T) is said to be an intuitionnistic fuzzy G_δ - e -local- δ -semi (resp., δ -pre and β) space if for every IF- G_δ - e -lcs is an intuitionistic fuzzy δ -semi (resp., δ -pre and β) closed set in an IFTS (X,T).

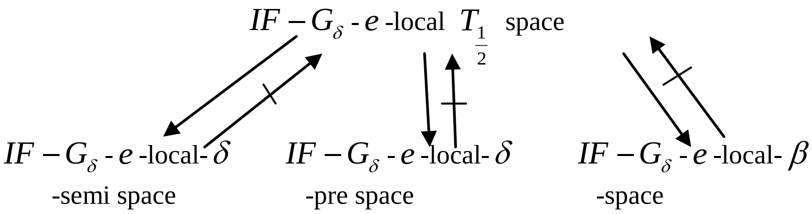
Proposition 3.832: Every intuitionistic fuzzy G_δ - e -local- $T_{\frac{1}{2}}$ space is an intuitionistic fuzzy G_δ - e -local- δ -semi (resp., G_δ - e -local- δ -pre and G_δ - e -local- β) space.

Remark 3.1033: The converse of the Proposition (3.8) need not be true as shown in Examples (3.7) and (3.8).

Example 3.7:34 Let $X = \{a,b\}$ be a nonempty set. Let $A = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$, $B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$, $A \vee B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$ and $A \wedge B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ be IFS of X. Then the family $T = \{0, 1, A, B, A \vee B, A \wedge B\}$ is an IFT on X. Now, $A \wedge B$ is an IF- G_δ -set. Let $C = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an intuitionistic fuzzy e - closed set. Hence $E = (A \wedge B) \cap C = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is δ -semi-closed. Hence (X,T) is an intuitionistic fuzzy G_δ - e -local- δ -semi space. But, E is not an intuitionistic fuzzy closed set. Thus, (X,T) is not an intuitionistic fuzzy G_δ - e -local $T_{1/2}$ space. Hence, intuitionistic fuzzy G_δ - e -local- δ -semi space need not be an intuitionistic fuzzy G_δ - e -local $T_{1/2}$.

Example 3.8:35 In Example (3.7), Let $D = \left\langle x, (\frac{a}{0}, \frac{b}{0}), (\frac{a}{0.5}, \frac{b}{0.5}) \right\rangle$ be an IFeCS. Hence $E = (A \wedge B) \cap D = \left\langle x, (\frac{a}{0}, \frac{b}{0}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$ is δ -pre-closed(resp., β -closed). Hence (X, T) is an intuitionistic fuzzy G_δ - e -local- δ -pre (G_δ - e -local- β -space). But, E is not an intuitionistic fuzzy closed set. Thus, (X, T) is not an intuitionistic fuzzy G_δ - e -local $T_{\frac{1}{2}}$ space. Hence, intuitionistic fuzzy G_δ - e -local- δ -pre (G_δ - e -local- β)space need not be an intuitionistic fuzzy G_δ - e -local $T_{\frac{1}{2}}$.

Remark 3.1136: Clearly the following diagram holds.



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