

**A NEW APPROACH FOR FINDING AN OPTIMAL SOLUTION
OF UNBALANCED INTUITIONISTIC FUZZY TRANSPORTATION PROBLEMS**

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ABSTRACT

In this paper, a new method is proposed for solving unbalanced Intuitionistic fuzzy transportation problems by assuming that a decision maker is uncertain about the precise values of the transportation costs, demand and supply of the product. In this proposed method the transportation costs, demand and supply of the product are represented by triangular Intuitionistic fuzzy numbers. To illustrate the proposed method a numerical example is solved and the obtained result is compared with the results of other existing methods. The proposed method is very easy to understand and it can be applied on real life transportation problems for the decision makers.

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1. INTRODUCTION

Transportation problem is an important network structured in linear programming (LP) problem that arises in several contexts and has deservedly received a great deal of attention in the literature. The central concept in the problem is to find the minimum total transportation cost of a commodity in order to satisfy demands at destinations using available supplies at origins. Transportation problem can be used for a wide variety of situations such as scheduling, production, investment, plant location, inventory control, employment scheduling, and many others. In general, transportation problems are solved with the assumptions that the transportation costs and values of supplies and demands are specified in a precise way i.e., in crisp environment. However, in many cases the decision makers has no crisp information about the coefficients belonging to the transportation problem. In these cases, the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy sets, and the fuzzy transportation problem (FTP) appears in natural way.

The basic transportation problem was originally developed by Hitchcock [14]. The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (Variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [5] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa [7] used simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North- West Corner rule NWCR [5], Matrix Minima Method MMM [5], Vogel's Approximation Method. We can find initial basic feasible solution by using Vogel's Approximation Method VAM [5]. Many people worked on this to propose modifications to Vogel's Approximation method for obtaining initial solutions to the unbalanced transportation problem. Shimshak [25] propose a modification (SVAM) which ignores any penalty that involves a dummy row/column. Goyal [13] suggests another modification in (GVAM) where the cost of transporting goods to or from a dummy point is set equal to the highest transportation cost in the problem, rather than to zero. The method proposed by Ramakrishnan [23] consists of four steps of reduction and one step of VAM. Nagaraj Balakrishnan [21] suggests further modification in SVAM. All methods have been established for finding the optimal solution. Among these, some

methods directly attain the optimal solution namely zero suffix method [27], ASM–method [2] etc. But these two methods for finding optimal solution of a transportation problem do not reflect optimal solution proved by Mohammed [17]. In general the transportations problems are solved with the assumptions that the coefficients or cost parameters are specified precisely.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number Zedeh [28] may represent the data. Hence fuzzy decision making method is used here. Zimmermann [29] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas *et al.* [3] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficient, fuzzy supply and demand values. Chanas and Kuchta [4] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas [24] discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Edward Samuel [10, 11] showed the unbalanced fuzzy transportation problems without converting into balanced one getting an optimal solution, where the transportation cost, demand and supply are represented by triangular fuzzy number. Edward Samuel [9] proposed algorithmic approach to unbalanced fuzzy transportation problem, where the transportation cost, demand and supply are represented by triangular fuzzy number.

Liu and Kao [16] described a method for solving fuzzy transportation problem based on extension principle. Gani and Razak [12] presented a two stage cost minimizing fuzzy transportation problem (FTP) in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution and the aim is to minimize the sum of the transportation costs in two stages.

Dinagar and Palanivel [8] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [22] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers.

Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set. Nagoor Gani and Abbas [20] proposed a new average method for solving intuitionistic fuzzy transportation problem, there is no uncertainty about transportation costs but demand and supply are represented by triangular intuitionistic fuzzy number.

In this study, basic idea is to get an optimal solution for an unbalanced Intuitionistic fuzzy transportation problem (UIFTP). This paper presents a new method, simple and easy to understand technique for solving unbalanced Intuitionistic fuzzy transportation problems. This can be an alternative to the modification distribution method (MODI) [5]. No path tracing is required in this approach. The algorithm of the approach is detailed with suitable numerical examples. Further comparative studies of the new technique with other existing algorithms are established by means of sample problem

2. PRELIMINARIES

In this section, some basic definitions, arithmetic operations and an existing method for comparing Intuitionistic fuzzy numbers are presented.

2.1. Definition [6, 15]: A fuzzy number $\tilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{x-c}{b-c} & \text{if } b \leq x \leq c \\ 0 & , \text{ otherwise} \end{cases}$$

2.2 Definition [19, 26]: A triangular intuitionistic fuzzy number A^{-I} is an intuitionistic fuzzy set in \mathfrak{R} with the following membership function $\mu_{A^{-I}}$ and non-membership function $\gamma_{A^{-I}}$ are defined by

$$\mu_{A^{-I}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 < x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \leq x < a_3 \\ 0, \text{ other wise} \end{cases} \text{ and } \gamma_{A^{-I}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^I}, a_1^I < x \leq a_2 \\ \frac{x-a_2}{a_3^I-a_2}, a_2 \leq x < a_3^I \\ 1, \text{ other wise} \end{cases}$$

Where $a_1^I \leq a_1 < a_2 < a_3 \leq a_3^I$ this TIFN is denoted by $A^{-I} = (a_1, a_2, a_3; a_1^I, a_2, a_3^I)$.

2.3. Arithmetic operations: In this section, arithmetic operations between two triangular Intuitionistic fuzzy numbers, defined on the universal set of real numbers \mathfrak{R} , are presented [18].

Let $A^{-I} = (a_1, a_2, a_3; a_1^I, a_2, a_3^I)$ and $B^{-I} = (b_1, b_2, b_3; b_1^I, b_2, b_3^I)$ be two triangular intuitionistic fuzzy numbers then the following is obtained.

- (i). $A^{-I} + B^{-I} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1^I + b_1^I, a_2 + b_2, a_3^I + b_3^I)$.
- (ii) $A^{-I} - B^{-I} = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1^I - b_3^I, a_2 - b_2, a_3^I - b_1^I)$.
- (iii) $A^{-I} \times B^{-I} = (l_1, l_2, l_3; l_1^I, l_2, l_3^I)$

Where

$$l_1 = \min [a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3], l_2 = a_2 b_2, l_3 = \max [a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3],$$

$$l_1^I = \min [a_1^I b_1^I, a_1^I b_3^I, a_3^I b_1^I, a_3^I b_3^I], l_2 = a_2 b_2, l_3^I = \max [a_1^I b_1^I, a_1^I b_3^I, a_3^I b_1^I, a_3^I b_3^I]$$

2.3 Ranking function [20,26]: Let $A^{-I} = (a_1, a_2, a_3; a_1^I, a_2, a_3^I)$ and $B^{-I} = (b_1, b_2, b_3; b_1^I, b_2, b_3^I)$ be two triangular intuitionistic fuzzy numbers

$$\text{Then } \mathfrak{R}(\tilde{A}) = \left(\frac{(a_1 + 2a_2 + a_3) + (a_1^I + 2a_2 + a_3^I)}{8} \right) \text{ and } \mathfrak{R}(\tilde{B}) = \left(\frac{(b_1 + 2b_2 + b_3) + (b_1^I + 2b_2 + b_3^I)}{8} \right).$$

2. PROPOSED METHOD

Step-1: Construct the fuzzy transportation table for the given unbalanced intuitionistic fuzzy transportation problem and then, convert it into a balanced one.

Step-2: (a) Perform row wise reduction.

Locate the largest element in each row of the given fuzzy cost table and then subtract that from each element of that row, and

(b) Perform column wise reduction.

In the reduced matrix obtained from 2(a), locate the largest element in each column and then subtract that from each element of that column.

Step-3: Verify if Total Demand (TD) exceeds Total Supply (TS) then calculate row penalty else calculate column penalty.

Step-4: Calculate penalties for each row (column) by taking the difference between the smallest and next smallest unit fuzzy transportation cost in the same row (column).

Step-5: Select the row (column) with the largest penalty and allocate as much as possible in the cell having the least cost in the selected row (column) satisfying the rim conditions.

Step-6: Adjust the supply and demand and cross out the satisfied the row or column.

Step-7: Repeat steps 4 to step 6 until the entire demand at various destinations or available supply at various sources are satisfied.

Step-8: Compute total fuzzy transportation cost for the feasible allocation from the original fuzzy cost table.

Important Remarks:

1. If there is a tie in the values of penalties then calculate their corresponding the row (column value and select the one with minimum) value and select the one with maximum.

4. NUMERICAL EXAMPLES

4.1. Problem 1

Table 1 gives the availability of the product supply at three sources S_1, S_2, S_3 their demand at three destinations D_1, D_2, D_3 and the approximate unit transportation cost, demand and supply of the product from each source to each destination is represented by triangular intuitionistic fuzzy numbers. Determine the intuitionistic fuzzy optimal transportation of the products such that the total transportation cost is minimum.

Table-1				
	D_1	D_2	D_3	Supply
S_1	(5,6,7;4,6,8)	(9,10,11;8,10,12)	(13,14,15;12,14,16)	(48,50,52;46,50,54)
S_2	(10,12,14;9,12,15)	(18,19,20;17,19,21)	(20,21,22;19,21,23)	(48,50,52;46,50,54)
S_3	(14,15,16;13,15,17)	(13,14,15;12,14,16)	(16,17,18;15,17,19)	(48,50,52;46,50,54)

Demand (28,30,32;26,30,34) (38,40,42;36,40,44) (54,55,56;53,55,57)

Since $\sum_{i=1}^3 a_i = (144,150,156;138,150,162) \neq \sum_{j=1}^3 b_j = (120,125,130;115,125,135)$, so the chosen problem is a unbalanced IFTP.

Iteration 1: Using step1, we get

	D_1	D_2	D_3	D_4	Supply
S_1	(5,6,7;4,6,8)	(9,10,11;8,10,12)	(13,14,15;12,14,16)	(0,0,0;0,0,0)	(48,50,52;46,50,54)
S_2	(10,12,14;9,12,15)	(18,19,20;17,19,21)	(20,21,22;19,21,23)	(0,0,0;0,0,0)	(48,50,52;46,50,54)
S_3	(14,15,16;13,15,17)	(13,14,15;12,14,16)	(16,17,18;15,17,19)	(0,0,0;0,0,0)	(48,50,52;46,50,54)

Demand (28,30,32;26,30,34) (38,40,42;36,40,44) (54,55,56;53,55,57) (24,25,26;23,25,27)

Now, $\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = (144,150,156;138,150,162)$

Iteration 2: Using step2, we get

	D_1	D_2	D_3	D_4	Supply
S_1	(-4,1,6;-8,1,10)	(-4,0,4;-8,0,8)	(-4,0,4;-8,0,8)	(5,7,9;3,7,11)	(48,50,52;46,50,54)
S_2	(-6,0,6;-10,0,10)	(-2,2,6;-6,2,10)	(-4,0,4;-8,0,8)	(-2,0,2;-4,0,4)	(48,50,52;46,50,54)
S_3	(2,7,12;-2,7,16)	(-3,1,5;-7,1,9)	(-4,0,4;-8,0,8)	(2,4,6;0,4,8)	(48,50,52;46,50,54)

Demand (28,30,32;26,30,34) (38,40,42;36,40,44) (54,55,56;53,55,57) (24,25,26;23,25,27)

Iteration 3: Using step3 (Here, Total Demand does not exceed Total Supply, i.e., $TD < TS$) and step4, we get

	D_1	D_2	D_3	D_4	Supply
S_1	(-4,1,6;-8,1,10)	(-4,0,4;-8,0,8)	(-4,0,4;-8,0,8)	(5,7,9;3,7,11)	(48,50,52;46,50,54)
S_2	(-6,0,6;-10,0,10)	(-2,2,6;-6,2,10)	(-4,0,4;-8,0,8)	(-2,0,2;-4,0,4)	(48,50,52;46,50,54)
S_3	(2,7,12;-2,7,16)	(-3,1,5;-7,1,9)	(-4,0,4;-8,0,8)	(2,4,6;0,4,8)	(48,50,52;46,50,54)

Demand (28,30,32;26,30,34) (38,40,42;36,40,44) (54,55,56;53,55,57) (24,25,26;23,25,27)

Column penalty (-10,1,12;-18,1,20) (-7,1,9;-15,1,17) (-8,0,8;-16,0,16) (0,4,8;-4,4,12)

Iteration 5: Using step5 and step6, we get

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(-4,1,6;-8,1,10)	(-4,0,4;-8,0,8)	(-4,0,4;-8,0,8)	*	(48,50,52;46,50,54)
S ₂	(-6,0,6;-10,0,10)	(-2,2,6;-6,2,10)	(-4,0,4;-8,0,8)	(-2,0,2;-4,0,4) (24,25,26;23,25,27)	(22,25,28;19,25,31)
S ₃	(2,7,12;-2,7,16)	(-3,1,5;-7,1,9)	(-4,0,4;-8,0,8)	(2,4,6;0,4,8)	(48,50,52;46,50,54)
Demand	(28,30,32;26,30,34)	(38,40,42;36,40,44)	(54,55,56;53,55,57)	*	
Column penalty	(-10,1,12;-18,1,20)	(-7,1,9;-15,1,17)	(-8,0,8;-16,0,16)	(0,4,8;-4,4,12)	

Iteration 5: Using step7, we get

	D ₁	D ₂	D ₃	D ₄	Supply
S ₁	(-4,1,6;-8,1,10) (0,5,10;-5,5,15)	(-4,0,4;-8,0,8) (38,40,42;36,40,44)	(-4,0,4;-8,0,8) (-4,5,14;-13,5,23)	(5,7,9;3,7,11)	*
S ₂	(-6,0,6;-10,0,10) (22,25,28;19,25,31)	(-2,2,6;-6,2,10)	(-4,0,4;-8,0,8)	(-2,0,2;-4,0,4) (24,25,26;23,25,27)	*
S ₃	(2,7,12;-2,7,16)	(-3,1,5;-7,1,9)	(-4,0,4;-8,0,8) (48,50,52;46,50,54)	(2,4,6;0,4,8)	*
Demand	*	*	*	*	

Iteration 6: Using step 8, we get

	D ₁	D ₂	D ₃	Supply
S ₁	(5,6,7;4,6,8) (0,5,10;-5,5,15)	(9,10,11;8,10,12) (38,40,42;36,40,44)	(13,14,15;12,14,16) (-4,5,14;-13,5,23)	*
S ₂	(10,12,14;9,12,15) (22,25,28;19,25,31)	(18,19,20;17,19,21)	(20,21,22;19,21,23)	*
S ₃	(14,15,16;13,15,17)	(13,14,15;12,14,16)	(16,17,18;15,17,19) (48,50,52;46,50,54)	*
Demand	*	*	*	

The minimum fuzzy transportation cost is

$$= (0,5,10;-5,5,15) \times (5,6,7;4,6,8) + (38,40,42;36,40,44) \times (9,10,11;8,10,12) \\ + (22,25,28;19,25,31) \times (10,12,14;9,12,15) + (-4,5,14;-13,5,23) \times (13,14,15;12,14,16) \\ + (48,50,52;46,50,54) \times (1,17,18;15,17,19)$$

$$= (1270,1650,2070; 901,1650,2507)$$

$$R(A) = 1668.50$$

4.2. Problem 2

	D ₁	D ₂	D ₃	Supply
S ₁	(5,6,7;4,6,8)	(7,8,9;6,8,10)	(2,4,6;1,4,7)	(13,14,15;12,14,16)
S ₂	(2,4,6;1,4,7)	(8,9,10;7,9,11)	(7,8,9;6,8,10)	(11,12,13;10,12,14)
S ₃	(0,1,2;-1,1,3)	(1,2,3;0,2,4)	(5,6,7;4,6,8)	(4,5,6;3,5,7)
Demand	(4,6,8;2,6,10)	(14,15,16;13,15,17)	(14,15,16;13,15,17)	

4.3. Comparison among different existing transportation methods:

S.NO	ROW	COLUMN	NWCR	MMM	VAM	Optimal	Proposed Method
1.	3	4	1851.25	1816.25	1765.25	1671.25	1668.50
2.	4	3	248.75	182.25	157.50	156.25	142.00

4.4. Results and Discussion

From the investigations and the results given above, it is clear that the proposed method is better than other existing methods and our method for solving unbalanced fuzzy transportation problems and it has its merit that it produces an optimal solution.

5. CONCLUSION

In this paper a new method is proposed for finding optimal solution of intuitionistic fuzzy transportation problem in which the transportation costs, demand and supply of the product are represented by triangular intuitionistic fuzzy numbers. So this technique can be applied for solving intuitionistic fuzzy transportation problems occurring in real life situations.

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