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SIMILARITY MEASURE BETWEEN INTERVAL-VALUED INTUITIONISTIC FUZZY SETS AND THEIR APPLICATIONS TO MEDICAL DIAGNOSIS AND PATTERN RECOGNITION

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ABSTRACT

In this paper, we present an effective method for evaluating similarity measure between Interval-valued Intuitionistic Fuzzy set (IVIFS) based on the mid points of transformed triangular fuzzy numbers. Some relevant properties of the proposed similarity measures are discussed and the related similarity measures are also compared. Several illustrative examples are given to demonstrate the practicality and effectiveness of the proposed similarity measure. Further, in order to provide the supportive evidence, the proposed similarity measure is applied for medical diagnosis problem and Pattern Recognition problem and to demonstrate benefits of using the proposed similarity measure over the existing ones.

Keywords: *Fuzzy set (FS), Intuitionistic fuzzy set (IFS), Interval-valued fuzzy sets (IVFSs), Interval-valued intuitionistic fuzzy set (IVIFS), Similarity measure.*

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1. INTRODUCTION

The concept of intuitionistic fuzzy set (IFS) has been introduced by Atanassov [1], as a generalization concept of fuzzy set (FS) introduced by Zadeh [15]. IFSs are characterized by two functions expressing the degree of membership and the degree of non-membership. Atanassov and Gargov [2] introduced the concept of IVIFS, which is a generalization of the IFS. The fundamental characteristic of the IVFS and IVIFS is that the values of its membership and non-membership function are intervals rather than exact numbers. In the application of IVIFSs, the similarity measures play an important role in the analysis and research of medical diagnosis [7, 11] and pattern recognition problems [3, 6, 8, 10, 13]. The similarity measure is an essential tool to compare and determine degree of similarity between IVIFSs.

In [5], Chen presented some similarity measures between vague set and between elements. Hong and Kim [9] presented a similarity measure between IFSs to overcome the drawback of Chen's similarity measure [5]. Li and Cheng [10] presented similarity measure between IFS and applied them to pattern recognition problem. Burillo and Bustince [3] introduced the notions of entropy of IVFSs and IFSs to measure the degree of intuitionism of an IVFS or IFS. Szmidt and Kacprzyk [12] defined a similarity measure using a distance measure which involves both similarity and dissimilarity. Expanding upon this work, Szmidt and Kacprzyk [12] considered a family of similarity measures and compared with some existing similarity measures. Hong and Kim [9], Hung and Yang [8], Xu [14] defined independently some similarity measures based on different distance measures for IFSs. On the relationship between entropies and similarity measures of IFSs, Zeng and Guo [16] proved that some similarity measures and entropies of IVFSs can be deduced by normalized distances of IVFSs based on their axiomatic definitions. Zeng and Li [17] showed that similarity measures and entropies of IVFSs can be transformed by each other. Zhang and yu [18] put forward some entropy formulas of IFSs according to the relationship between entropies and similarity measures of IFSs.

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Liu [10] proposed a set of axiomatic requirements for entropy measures of IVIFSs, which extends Szmidt and Kacprzyk's axioms formulated for entropy of IFS [12]. Xu [13] generalized some formulas of similarity measures of IFSs to IVIFSs which are based on the distance measures of IVIFSs. Wei [13] propose an entropy measure for IVIFSs, which generalizes three entropy measures and also apply the similarity measure to solve problems on pattern recognitions, multi-criteria fuzzy decision making and medical diagnosis. However, the existing similarity measures [4, 7-9, 12-14, 16-18] between IFSs and IVIFSs have the drawbacks that they get unreasonable results in some situations and they cannot get correct classification results for dealing with the pattern recognition problems and medical diagnosis problems.

In this paper, we propose a new similarity measure of IVIFS based on the mid points of the transformed triangular fuzzy numbers and apply the proposed similarity measure to deal with pattern recognition and medical diagnosis problems. First we propose a new similarity measure between IVIFSs and prove some basic properties. Then, existing similarity measure between IFSs and IVIFSs are analyzed, compared and summarized by their counter-intuitive examples. The focus of this study is on apparent weakness of the existing similarity measures, and the conditions of reasons they do not work.

The paper is organized as follows. In section 2, we briefly review the basic definitions. In section 3, the existing similarity measures are discussed. In section 4, similarity measure between two IVIFSs is defined with example and some basic properties are studied. In section 5, by numerical examples we give the comparative analysis between the proposed measure and the existing similarity measure. In section 6, an application of proposed similarity measure between IVIFS in medical diagnosis problem. In section 7, the applications of the similarity measures in pattern recognition are discussed. Finally, the conclusions are discussed in section 8.

2. PRELIMINARIES

This section briefly reviews some concepts of intuitionistic fuzzy sets [1], interval-valued fuzzy sets [2] and basic properties of similarity measures between IVIFS.

Definition 2.1[15]: Let *X* be a nonempty set. A **fuzzy set (FS)** *A* drawn from *X* is defined as $A = \{x, \mu_A(x) : x \in X\}$, where $\mu_A(x) : X \to [0,1]$ is the membership function of the fuzzy set *A*.

Definition 2.2 [1]: Let *X* be a nonempty set. An **Intuitionistic fuzzy set (IFS)** *A* in *X* is an object having the form $A = \{x, \mu_A(x), \nu_A(x) : x \in X\}$, where the functions $\mu_A(x), \nu_A(x) : X \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set *A*, which is a subset of *X*, and for every element $x \in X, 0 \le \mu_A(x) + \nu_A(x) \le 1$. For each IFS A in X, the amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of indeterminacy (hesitation part), which may provide to membership value, non-membership value or both.

Definition 2.3 [2]: Let D[0, 1] be the set of all closed subintervals of the interval [0, 1] and $X(\neq \phi)$ be a given set. Following Atanassov and Gargov [2], an IVIFS A in X is an expression given by $A = \{x, \mu_A(x), \nu_A(x) : x \in X\}$, where $\mu_A(x) : X \to D[0,1]$, $\nu_A(x) : X \to D[0,1]$, with the condition $0 \le \sup(\mu_A(x)) + \sup(\nu_A(x)) \le 1$ for any $x \in X$.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of belongingness and the degree of non-belongingness of the element *x* to the set A. Thus for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals and their lower and upper end points are, respectively, denoted by $\mu_A^L(x)$, $\mu_A^U(x)$, $\nu_A^L(x)$ and $\nu_A^U(x)$. We can denote by $A = \{x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)] : x \in X\}$, where $0 \le \mu_A^U(x) + \nu_A^U(x) \le 1, \mu_A^L(x) \ge 0, \nu_A^L(x) \ge 0$. For each element *x* we can compute the unknown degree (hesitancy degree) of an intuitionsitic fuzzy interval of $x \in X$ in the set *A* defined as follows: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) = [1 - \mu_A^U - \nu_A^U, 1 - \mu_A^L - \nu_A^L]$

Definition 2.4[13]: Let A, B and C be interval valued intuitionistic fuzzy sets defined in the universe of discourse X. A similarity measure S between the interval valued intuitionistic fuzzy sets should satisfy the following properties:

(1) $S(A,B) \in [0,1]$
(2) $S(A,B) = S(B,A)$
(3) $S(A,B) = 1$ if and only if $A=B$
(4) If $A \subseteq B \subseteq C$, then $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$

3. EXISTING SIMILARITY MEASURE IN THE LITERATURE

In this section, we review the existing similarity measures available in the literature.

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universe of discourse. For two IVIFSs $A = \{(x_i, [\mu_A^L(x_i), \mu_A^U(x_i)],$ and $[\nu_A^L(x_i), \nu_A^U(x_i)]) / 1 \leq i \leq n\}$ $B = \{(x_i, [\mu_B^L(x_i), \mu_B^U(x_i)], [\nu_B^L(x_i), \nu_B^U(x_i)]) / 1 \leq i \leq n\}$. Then Chen [5], Li and Cheng [10], Hong and Kim [9], Hung and Yang [8] presented similarity measures between IFSs and Xu [14], wei [13] proposed similarity measure between IVIFSs, respectively, as follows:

Chen [5], Li and Cheng [10], Hong and Kim [9], Hung and Yang [8] proposed independently the following similarity measures between the IFSs A and B:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{n} \left| \mu_A(x_i) - \nu_A(x_i) - (\mu_B(x_i) - \nu_B(x_i)) \right|}{2n} \tag{1}$$

$$S_{DC}(A,B) = 1 - \sqrt[p]{\frac{\sum_{i=1}^{n} \left| \psi_A(x_i) - \psi_B(x_i) \right|^p}{n}}, \tag{2}$$

where p is a parameter $1 \leq p < \infty$, for each i,

$$\psi_A(x_i) = \frac{\mu_A(x_i) + 1 - \nu_A(x_i)}{2}, \psi_B(x_i) = \frac{\mu_B(x_i) + 1 - \nu_B(x_i)}{2}$$

$$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| \nu_A(x_i) - \nu_B(x_i) \right|}{2n} \tag{3}$$

$$S_{HY}^1(A,B) = 1 - d_H(A,B),\ S_{HY}^2(A,B) = \frac{1 - d_H(A,B)}{1 + d_H(A,B)},\ S_{HY}^3(A,B) = \frac{e^{-d_H(A,B)} - e^{-1}}{1 - e^{-1}}, \tag{4}$$

where $d_H(A,B) = \frac{1}{n} \sum_{i=1}^{n} \max\left\{ \left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| \nu_A(x_i) - \nu_B(x_i) \right| \right\}$

Xu [14] proposed a similarity measures, based on the distance measure, between A and B as follows:

$$S_{X1}(A,B) = 1 - \left[\frac{1}{4n} \sum_{i=1}^{n} \begin{pmatrix} \left| \mu_A^L(x_i) - \mu_B^L(x_i) \right|^p + \left| \mu_A^U(x_i) - \mu_B^U(x_i) \right|^p + \left| \nu_A^L(x_i) - \nu_B^L(x_i) \right|^p \\ + \left| \nu_A^U(x_i) - \nu_B^U(x_i) \right|^p \end{pmatrix} \right]^{1/p}, p > 0 \tag{5}$$

$$S_{X2}(A,B) = 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \max \begin{Bmatrix} \left| \mu_A^L(x_i) - \mu_B^L(x_i) \right|^p, \left| \mu_A^U(x_i) - \mu_B^U(x_i) \right|^p, \left| \nu_A^L(x_i) - \nu_B^L(x_i) \right|^p \\ , \left| \nu_A^U(x_i) - \nu_B^U(x_i) \right|^p \end{Bmatrix} \right]^{1/p}, p > 0 \tag{6}$$

Wei [13] presented the similarity measure between IVIFS using the entropy measure

$$S_W(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2-\min\{\mu_i^L,\ \nu_i^L\}-\min\{\mu_i^U,\ \nu_i^U\}}{2+\max\{\mu_i^L,\ \nu_i^L\}+\max\{\mu_i^U,\ \nu_i^U\}} \tag{7}$$

where $\mu_i^L=\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|$, $\mu_i^U=\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|$, $\nu_i^L=\left|\nu_A^L(x_i)-\nu_B^L(x_i)\right|$, $\nu_i^U=\left|\nu_A^U(x_i)-\nu_B^U(x_i)\right|$

4. A NEW SIMILARITY MEASURE BETWEEN INTERVAL-VALUED INTUITIONISTIC FUZZY SET

In this section, we define a new similarity measure between IVIFS based on the midpoint of the transformed triangular fuzzy numbers. Relevant properties of similarity measures are also studied.

Let $A=\{(x_i,[\mu_A^L(x_i),\mu_A^U(x_i)],[\nu_A^L(x_i),\nu_A^U(x_i)])/1\le i\le n\}$ and $B=\{(x_i,[\mu_B^L(x_i),\mu_B^U(x_i)],[\nu_B^L(x_i),\nu_B^U(x_i)])/1\le i\le n\}$ be two interval valued intuitionistic fuzzy sets in the universe of discourse X, where $X=\{x_1,x_2,...x_n\}$. Let $(\mu_A^L(x_i),1-\nu_A^L(x_i)],[\mu_A^U(x_i),1-\nu_A^U(x_i)])$ and $(\mu_B^L(x_i),1-\nu_B^L(x_i)],[\mu_B^U(x_i),1-\nu_B^U(x_i)])$ denote the intuitionistic fuzzy values of element x_i belonging to the interval valued intuitionistic fuzzy sets A and B, where $1\le i\le n$.

Let A_{x_i} and B_{x_i} be two triangular fuzzy numbers transformed from the intuitionistic fuzzy values, as shown in Fig. 1. $\psi_A(x_i)$ and $\psi_B(x_i)$ are the mid points of the transformed triangular fuzzy numbers A_{x_i} and B_{x_i}. Based on Fig. 1, the degree of similarity $S(A_{x_i},B_{x_i})$ between the intuitionistic fuzzy values $(\mu_A^L(x_i),1-\nu_A^L(x_i)],[\mu_A^U(x_i),1-\nu_A^U(x_i)])$ and $(\mu_B^L(x_i),1-\nu_B^L(x_i)],[\mu_B^U(x_i),1-\nu_B^U(x_i)])$ is calculated as follows:

$$S(A_{x_i},B_{x_i})=1-\frac{1}{2}\left(\left|\psi_A^L(x_i)-\psi_B^L(x_i)\right|+\left|\psi_A^U(x_i)-\psi_B^U(x_i)\right|\right)\left(1-\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\right)$$
$$-\left|\pi_A(x_i)-\pi_B(x_i)\right|\left(\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\right) \tag{8}$$

where $S(A_{x_i},B_{x_i})\in[0,1]$, $\left(\left|\psi_A^L(x_i)-\psi_B^L(x_i)\right|+\left|\psi_A^U(x_i)-\psi_B^U(x_i)\right|\right)$ denote the distance between the mid points $\psi_A^L(x_i)=\dfrac{\mu_A^L(x_i)+1-\nu_A^L(x_i)}{2}$, $\psi_A^U(x_i)=\dfrac{\mu_A^U(x_i)+1-\nu_A^U(x_i)}{2}$, $\psi_B^L(x_i)=\dfrac{\mu_B^L(x_i)+1-\nu_B^L(x_i)}{2}$ and $\psi_B^U(x_i)=\dfrac{\mu_B^U(x_i)+1-\nu_B^U(x_i)}{2}$ of the transformed triangular fuzzy numbers A_{x_i} and B_{x_i}. $\left|\pi_A(x_i)-\pi_B(x_i)\right|$ denote the degree of indeterminacy of element x_i belonging to the interval valued intuitionistic fuzzy sets A and B, respectively, $\pi_A(x_i)=1-\frac{1}{2}\left(\mu_A^L(x_i)+\mu_A^U(x_i)+\nu_A^L(x_i)+\nu_A^U(x_i)\right)$, $\pi_B(x_i)=1-\frac{1}{2}\left(\mu_B^L(x_i)+\mu_B^U(x_i)+\nu_B^L(x_i)+\nu_B^U(x_i)\right)$, $0\le\frac{\pi_A(x_i)+\pi_B(x_i)}{2}\le 1$ and $1\le i\le n$.

The proposed similarity measure $S_I(A,B)$ between the interval-valued intuitionistic fuzzy sets A and B is

$$S_I(A,B)=w_1\times S(A_{x_1},B_{x_1})+w_2\times S(A_{x_2},B_{x_2})+.....+w_n\times S(A_{x_n},B_{x_n})$$
$$=\sum_{i=1}^{n}(w_i\times S(A_{x_i},B_{x_i})) \tag{9}$$

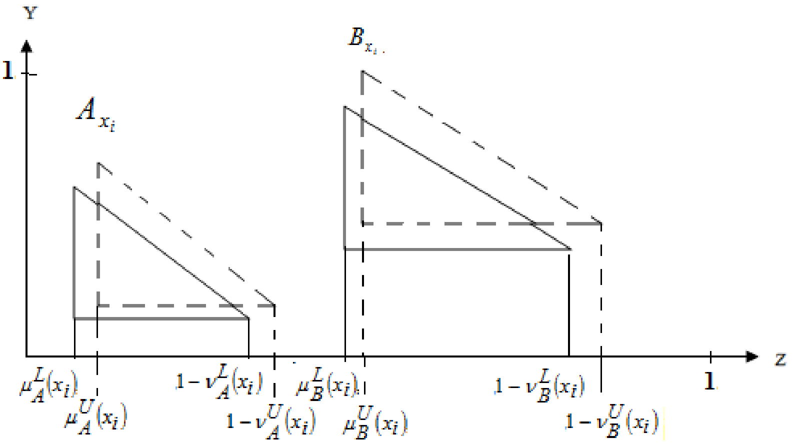

Figure-1: The transformed triangular fuzzy numbers A_{x_i} and B_{x_i} of the interval intuitionistic fuzzy values $(\mu_A^L(x_i),1-\nu_A^L(x_i)],[\mu_A^U(x_i),1-\nu_A^U(x_i)])$ and $(\mu_B^L(x_i),1-\nu_B^L(x_i)],[\mu_B^U(x_i),1-\nu_B^U(x_i)])$ respectively.

where $S_I(A,B)\in[0,1]$, w_i is the weight of element $x_i \in X, w_i \in [0,1], \sum_{i=1}^{n} w_i = 1$, A_{x_i} and B_{x_i} are the triangular fuzzy numbers of intuitionistic fuzzy values $([\mu_A^L(x_i),1-\nu_A^L(x_i)],[\mu_A^U(x_i),1-\nu_A^U(x_i)])$ and $([\mu_B^L(x_i),1-\nu_B^L(x_i)],[\mu_B^U(x_i),1-\nu_B^U(x_i)])$ of element x_i belonging to the IVIFS A and B respectively. $S(A_{x_i},B_{x_i})$ is the similarity degree between A_{x_i} and B_{x_i}, $1\le i\le n$, number of elements x_i in IVIFSs A and B. The larger the value of $S_I(A,B)$, the higher the degree of similarity between the intuitionistic fuzzy sets A and B.

The proposed similarity measure between IVIFS satisfies the following properties:

(P1) $0\le S(A,B)\le 1$

(P2) $s(A,B)=1$ if and only if $A=B$

(P3) $S(A,B)=S(A,B)$

5. COMPARISON WITH SOME EXISTING SIMILARITY MEASURE

In this section, we review some of the drawbacks of the existing similarity measures.

(1) Let A = {<x, 0, 0>}, B = {<x, 0.5, 0.5>} and C={<x, 0.499, 0.501>}be three IFSs. Intuitively, one can see that the IFS C is more similar to the IFS B than to IFS A. Using the Similarity measures Eqns. (1) and (2), however, we get the S_C(A,B) = S_{DC}(A,B) = 1, S_C(B,C) = S_{DC}(B,C) =0.999. Hence S_C(A,B) > S_C(B,C) , S_{DC}(A,B) > S_{DC}(B,C) , which is not so reasonable as we expect. Now, using our similarity measure given in Eq. (9), we have S_I(A,B)=0.5 and S_I(B,C) =0.9999, which means B is much more similar to C than to A. Therefore, the new similarity measure is more reasonable than (1) and (2).

(2) Let A= {<x, 0.3, 0.5>}, B = {<x, 0.1, 0.5>} and C={<x, 0.4, 0.6>} be three IFSs. Using similarity measure (3) we have S_H(A, B) = S_H(A, C) = 0.9, which indicates that S_H could not distinguish which one between B and C is more similar to A. However, using the similarity measure (9), we get that S_I(A,B) = 0.905 and S_I(A,C) = 0.98, so that we can say the IFS A is more similar to C than to B.

(3) Let A = {<x, 0.3, 0.5>}, B={<x, 0.4, 0.6>} and C={<x, 0.3, 0.6>} be three IFSs. Using similarity measure (4) we have S_{HY}(A,B) = S_{HY}(A,C), which indicate that S_{HY} could not distinguish reasonable result. However, using the similarity measure (9), we get that S_I(A,B) = 0.9213 and S_I(A,C) = 0.98 , which means A is a little more similar to C than to B, consistent with the intuition.

(4) Let A= {<x, [0.2, 0.3], [0.4, 0.6]>}, B = {<x, [0.3, 0.4], [0.3, 0.5]>} and C={<x, [0.3, 0.4], [0.4, 0.6]>} be three IVIFSs. Using Eqns. (5) and (6) S_{X1}(A,B) = S_{X1}(A,C) = S_{X2}(A,B) = S_{X2}(A,C)=0.9, which is not reasonable. Using Eqn. (7) we have S_W(A,B) = 0.8182 and S_W(A,C) = 0.9091, which is not intuitively consistent. Now we calculate the similarity measures S(A,B) and S(A,C) by Eqn. (9), we have S_I(A,B) = 1 and S_I(A,C) = 0.98 , which indicates that

IVIFS A is more similar to IVIFS B than to IVIFS C and is consistent with intuition. Therefore, the similarity measure Eqn. (9) is illustrated to be more reasonable than Eqns. (5) , (6) and (7) in some cases.

(5) Let A = {<x, 0.3, 0.5>}, B={<x, 0.1, 0.5>} and C={<x, 0.4, 0.59>} be three IFSs. Using similarity measure (7) we have $S_W(A,B) = 0.8333$ and $S_W(A,C) = 0.8272$, which indicate that $S_W(A,B) > S_W(A,C)$ i.e., A is a more similar to B than to C. However, using the similarity measure (9), we get that $S_I(A,B) = 0.905$ and $S_I(A,C) = 0.9778$, which means A is a little more similar to C than to B, consistent with the intuition.

Hence compared with above similarity measures, the new similarity measure defined by the eqn. (9) is more effective to differentiate IFSs in general cases.

6. APPLICATION OF SIMILARITY MEASURES TO MEDICAL DIAGNOSIS

In this section, we present an example to illustrate medical diagnosis process. For medical diagnosis of headache (adopted from [11]), the example uses the patient's degree $\langle M_Q(p,s), N_Q(p,s)\rangle$, and conformability degree $\langle M_R(s,d), N_R(s,d)\rangle$. Let us consider patient P_1. P_1's symptoms are (M5, M8, M12, M15, M18, M19) of migraine, (T3, T6, T10) of tension headache and (C4, C11) of cluster headache. P_1 simultaneously has symptoms M5, M8 and M15 (the symptoms are displayed in the composite symptom M22), therefore, the symptoms of migraine are represented in (M12, M18, M19, M22) whose weight $w = (1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9, 1/9)$

Table-1: Patient P_1's degrees: $\langle M_Q(p_1,s), N_Q(p_1,s)\rangle$

Symptom	M12	M18	M19	M22	T3	T6	T10	C4	C11
M_Q	[0.5, 0.6]	[0.5, 0.6]	[0.4, 0.6]	[0.7, 0.8]	[0.6, 0.7]	[0.5, 0.7]	[0.4, 0.6]	[0.5, 0.6]	[0.5, 0.7]
N_Q	[0.2, 0.3]	[0.1, 0.3]	[0.1, 0.2]	[0.1, 0.2]	[0.1, 0.2]	[0.2, 0.3]	[0.2, 0.4]	[0.1, 0.2]	[0.2, 0.3]

Table-2: Conformability degrees: $\langle M_R(s,d), N_R(s,d)\rangle$

Symptoms	**Migraine**		**Tension**		**Cluster**	
	$\mathbf{M_R}$	$\mathbf{N_R}$	$\mathbf{M_R}$	$\mathbf{N_R}$	$\mathbf{M_R}$	$\mathbf{N_R}$
M12	[0.6, 0.7]	[0.1, 0.2]	[0.2, 0.3]	[0.5, 0.6]	[0.1, 0.3]	[0.4, 0.6]
M18	[0.6, 0.7]	[0.2, 0.3]	[0.2, 0.4]	[0.4, 0.6]	[0.4, 0.6]	[0.1, 0.2]
M19	[0.5, 0.6]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.7]	[0.3, 0.4]	[0.3, 0.5]
M22	[0.7, 0.8]	[0.1, 0.2]	[0.1, 0.2]	[0.6, 0.8]	[0.1, 0.2]	[0.7, 0.8]
T3	[0.3, 0.4]	[0.4, 0.5]	[0.6, 0.7]	[0.1, 0.2]	[0.2, 0.3]	[0.5, 0.6]
T6	[0.2, 0.4]	[0.4, 0.6]	[0.6, 0.7]	[0.1, 0.3]	[0.1, 0.3]	[0.5, 0.6]
T10	[0.2, 0.3]	[0.4, 0.5]	[0.5, 0.6]	[0.2, 0.3]	[0.1, 0.2]	[0.4, 0.6]
C4	[0.5, 0.6]	[0.2, 0.3]	[0.1, 0.2]	[0.6, 0.7]	[0.6, 0.7]	[0.1, 0.2]
C11	[0.2, 0.4]	[0.3, 0.5]	[0.3, 0.4]	[0.2, 0.3]	[0.5, 0.7]	[0.1, 0.3]

Table-3: Similarity measure for P_1's symptoms

T	**Migraine**	**Tension**	**Cluster**
P_1	0.8799	0.7858	0.7803

The largest similarity value gives a proper diagnosis for the patient. As a result from Table 3, we can diagnose that patient P_1 suffers preferentially from migraine.

7. APPLICATION OF SIMILARITY MEASURES TO PATTERN RECOGNITIONS

In this section, a pattern recognition problem about the classification of building materials (adopted from [13]) is used to illustrate the proposed similarity measure. Assume that there are four classes of building material, which are represented by the IVIFS $A = \{(x_i, [\mu_A^L(x_i), \mu_A^U(x_i)], [\nu_A^L(x_i), \nu_A^U(x_i)]) / i = 1,2,3,4\}$ in the feature space $X = \{x_1, x_2, \ldots x_{12}\}$ whose weight vector is $w = \{0.1, 0.05, 0.08, 0.06, 0.03, 0.07, 0.09, 0.12, 0.15, 0.07, 0.13, 0.05\}^T$, and there is an unknown building material B:

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$$A_1 = \{< x_1, [0.1,0.2], [0.5,0.6] >, < x_2, [0.1,0.2], [0.7,0.8] >, < x_3, [0.5,0.6], [0.3,0.4] >, < x_4, [0.8,0.9], [0,0.1] >,$$
$$< x_5, [0.4,0.5], [0.3,0.4] >, < x_6, [0,0.1], [0.8,0.9] >, < x_7, [0.3,0.4], [0.5,0.6] >, < x_8, [1.0,1.0], [0,0] >,$$
$$< x_9, [0.2,0.3], [0.6,0.7] >, < x_{10}, [0.4,0.5], [0.4,0.5] >, < x_{11}, [0.7,0.8], [0.1,0.2] >, < x_{12}, [0.4,0.5], [0.4,0.5] >\}$$

$$A_2 = \{< x_1, [0.5,0.6], [0.3,0.4] >, < x_2, [0.6,0.7], [0.1,0.2] >, < x_3, [1.0,1.0], [0,0] >, < x_4, [0.1,0.2], [0.6,0.7] >,$$
$$< x_5, [0,0.1], [0.8,0.9] >, < x_6, [0.7,0.8], [0.1,0.2] >, < x_7, [0.5,0.6], [0.3,0.4] >, < x_8, [0.6,0.7], [0.2,0.3] >,$$
$$< x_9, [1.0,1.0], [0,0] >, < x_{10}, [0.1,0.2], [0.7,0.8] >, < x_{11}, [0,0.1], [0.8,0.9] >, < x_{12}, [0.7,0.8], [0.1,0.2] >\}$$

$$A_3 = \{< x_1, [0.4,0.5], [0.3,0.4] >, < x_2, [0.6,0.7], [0.2,0.3] >, < x_3, [0.9,1.0], [0,0] >, < x_4, [0,0.1], [0.8,0.9] >,$$
$$< x_5, [0,0.1], [0.8,0.9] >, < x_6, [0.6,0.7], [0.2,0.3] >, < x_7, [0.1,0.2], [0.7,0.8] >, < x_8, [0.2,0.3], [0.6,0.7] >,$$
$$< x_9, [0.5,0.6], [0.2,0.4] >, < x_{10}, [1.0,1.0], [0,0] >, < x_{11}, [0.3,0.4], [0.4,0.5] >, < x_{12}, [0,0.1], [0.8,0.9] >\}$$

$$A_4 = \{< x_1, [1.0,1.0], [0,0] >, < x_2, [1.0,1.0], [0,0] >, < x_3, [0.8,0.9], [0,0.1] >, < x_4, [0.7,0.8], [0.1,0.2] >,$$
$$< x_5, [0,0.1], [0.7,0.9] >, < x_6, [0,0.1], [0.8,0.9] >, < x_7, [0.1,0.2], [0.7,0.8] >, < x_8, [0.1,0.2], [0.7,0.8] >,$$
$$< x_9, [0.4,0.5], [0.3,0.4] >, < x_{10}, [1.0,1.0], [0,0] >, < x_{11}, [0.3,0.4], [0.4,0.5] >, < x_{12}, [0,0.1], [0.8,0.9] >\}$$

$$B = \{< x_1, [0.9,1.0], [0,0] >, < x_2, [0.9,1.0], [0,0] >, < x_3, [0.7,0.8], [0.1,0.2] >, < x_4, [0.6,0.7], [0.1,0.2] >,$$
$$< x_5, [0,0.1], [0.8,0.9] >, < x_6, [0.1,0.2], [0.7,0.8] >, < x_7, [0.1,0.2], [0.7,0.8] >, < x_8, [0.1,0.2], [0.7,0.8] >,$$
$$< x_9, [0.4,0.5], [0.3,0.4] >, < x_{10}, [1.0,1.0], [0,0] >, < x_{11}, [0.3,0.4], [0.4,0.5] >, < x_{12}, [0,0.1], [0.7,0.9] >\}$$

Our goal is to find which class the unknown pattern B belongs to.

We first calculate the degree of similarity between A and B by Eq. (9), and obtain
$S_I(A_1, B) = 0.6297$, $S_I(A_2, B) = 0.6026$, $S_I(A_3, B) = 0.8464$, $S_I(A_4, B) = 0.9770$

From the results above, we know that the degree of similarity between A_4 and B is the largest one, and thus the unknown pattern B should belong to the pattern A_4 .

8. CONCLUSIONS

In this paper, we have defined similarity measure between IVIFS based on the midpoint of the transformed triangular fuzzy numbers and has been applied to make a diagnosis of headache in medical diagnosis problem. Finally we give an example to show the possibilities of applications of similarity measure between two IVIFS sets in pattern recognition problem. Thus we can use the method to solve the problems that contain uncertainty such as problems in pattern recognition, medical diagnosis, game theory, coding theory and so on.

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