

## INTUITIONISTIC FUZZY CONTRA $\gamma^*$ GENERALIZED OPEN MAPPINGS

RIYA V. M<sup>1</sup> & JAYANTHI D<sup>2</sup>

<sup>1</sup>Research scholar of Mathematics,  
 Avinashilingam University, Coimbatore, Tamil Nadu, India.

<sup>2</sup>Assistant Professor of Mathematics,  
 Avinashilingam University, Coimbatore, Tamil Nadu, India.

E-mail: [riyavm@gmail.com](mailto:riyavm@gmail.com)<sup>1</sup> & [jayanthimathss@gmail.com](mailto:jayanthimathss@gmail.com)<sup>2</sup>

### ABSTRACT

*In this paper, we have introduced the notion of intuitionistic fuzzy contra  $\gamma^*$  generalized closed mappings and intuitionistic fuzzy contra  $\gamma^*$  generalized open mappings. Furthermore we have provided some properties of intuitionistic fuzzy contra  $\gamma^*$  generalized closed mappings and discussed some fascinating theorems.*

**Keywords:** Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy  $\gamma^*$  generalized closed sets, intuitionistic fuzzy contra  $\gamma^*$  generalized closed mappings, intuitionistic fuzzy contra  $\gamma^*$  generalized open mappings and intuitionistic fuzzy almost contra  $\gamma^*$  generalized open mappings.

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## 1. INTRODUCTION

Atanassov [1] introduced intuitionistic fuzzy sets. Coker [2] introduced intuitionistic fuzzy topological spaces. Seak Jong Lee and Eun Pyo Lee [8] have introduced intuitionistic fuzzy closed mappings in intuitionistic fuzzy topological spaces. Riya V. M and D. Jayanthi [7] introduced intuitionistic fuzzy  $\gamma^*$  generalized closed mappings and  $\gamma^*$  generalized open mappings. In this paper we have introduced intuitionistic fuzzy contra  $\gamma^*$  generalized open mappings, intuitionistic fuzzy almost contra  $\gamma^*$  generalized open mappings, intuitionistic fuzzy contra  $M\gamma^*$  generalized open mappings and investigated some of their properties. Also we have provided some characterization of intuitionistic fuzzy contra  $\gamma^*$  generalized open mappings and intuitionistic fuzzy almost contra  $\gamma^*$  generalized open mappings.

## 2. PRELIMINARIES

**Definition 2.1:** [1] An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions  $\mu_A: X \rightarrow [0,1]$  and  $\nu_A: X \rightarrow [0,1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set A, respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by  $A = \langle x, \mu_A, \nu_A \rangle$  instead of denoting  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ .

**Definition 2.2:** [1] Let A and B be two IFSs of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \} \text{ and}$$

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}. \text{ Then,}$$

- (a)  $A \subseteq B$  if and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ ,
- (b)  $A = B$  if and only if  $A \subseteq B$  and  $A \supseteq B$ ,
- (c)  $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$ ,
- (d)  $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$ ,
- (e)  $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$ .

The intuitionistic fuzzy sets  $0_- = \langle x, 0, 1 \rangle$  and  $1_- = \langle x, 1, 0 \rangle$  are respectively the empty set and the whole set of X.

**Definition 2.3:** [2] An intuitionistic fuzzy topology (IFT in short) on  $X$  is a family  $\tau$  of IFSs in  $X$  satisfying the following axioms:

- i)  $0_-, 1_- \in \tau$ ,
- ii)  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ ,
- iii)  $\cup G_i \in \tau$  for any family  $\{G_i; i \in J\} \in \tau$ .

In this case the pair  $(X, \tau)$  is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

**Definition 2.4:** [3] Let  $(X, \tau)$  be an IFTS and  $A = \langle x, \mu_A, \nu_A \rangle$  be an IFS in  $X$ . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\begin{aligned} \text{int}(A) &= \cup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}, \\ \text{cl}(A) &= \cap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}. \end{aligned}$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\text{cl}(A^c) = (\text{int}(A))^c$  and  $\text{int}(A^c) = (\text{cl}(A))^c$ .

**Definition 2.5:** [8] Two IFSs  $A$  and  $B$  are said to be  $q$ -coincident ( $A_q B$  in short) if and only if there exists an element  $x \in X$  such that  $\mu_A(x) > \nu_B(x)$  or  $\nu_A(x) < \mu_B(x)$ .

**Definition 2.6:** [8] Two IFSs  $A$  and  $B$  are said to be not  $q$ -coincident ( $A_q B$  in short) if and only if  $A \subseteq B^c$ .

**Definition 2.7:** [3] An intuitionistic fuzzy point (IFP for short), written as  $p_{(\alpha, \beta)}$ , is defined to be an IFS of  $X$  given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise} \end{cases}$$

An IFP  $p_{(\alpha, \beta)}$  is said to belong to a set  $A$  if  $\alpha \leq \mu_A$  and  $\beta \geq \nu_A$ .

**Definition 2.7:** Let  $(X, \tau)$  be an IFTS and IFS  $A = \{(x, \mu_B(x), \nu_B(x)) : x \in X\}$  is said to be an

- (a) intuitionistic fuzzy semi closed set [4] (IFSCS in short) if  $\text{int}(\text{cl}(A)) \subseteq A$ ,
- (b) intuitionistic fuzzy  $\alpha$ -closed set [4] (IF $\alpha$ CS in short) if  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ ,
- (c) intuitionistic fuzzy pre-closed set [4] (IFPCS in short) if  $\text{cl}(\text{int}(A)) \subseteq A$ ,
- (d) intuitionistic fuzzy regular closed set [4] (IFRCS in short) if  $\text{cl}(\text{int}(A)) = A$ ,
- (e) intuitionistic fuzzy generalized closed set [8] (IFGCS in short) if  $\text{cl}(A) \subseteq U$ , whenever  $A \subseteq U$ , and  $U$  in an IFOS,
- (f) intuitionistic fuzzy  $\gamma$  closed set [5] (IF $\gamma$ CS in short) if  $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A$

An IFS  $A$  is called intuitionistic fuzzy semi open set, intuitionistic fuzzy  $\alpha$  open set, intuitionistic fuzzy pre open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set and intuitionistic fuzzy  $\gamma$  open set (IFSOS, IF $\alpha$ OS, IFPOS, IFROS, IFGOS and IF $\gamma$ OS) if the complement  $A^c$  is an IFSCS, IF $\alpha$ CS, IFPCS, IFRCS, IFGCS and IF $\gamma$ CS respectively.

In the following diagram, we provide the relations between various types of intuitionistic fuzzy closedness (openness).

$$\begin{array}{ccccccc} \text{regular closed} & \Rightarrow & \text{closed} & \Rightarrow & \alpha \text{ closed} & \Rightarrow & \text{pre-closed} \\ & & \Downarrow & & \Downarrow & & \\ & & \text{g closed} & & \text{semi closed} & & \end{array}$$

The reverse implications are not true in general [4].

**Definition 2.8:** [5] Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then the  $\gamma$ -interior and  $\gamma$ -closure of  $A$  are defined as

$$\begin{aligned} \gamma \text{int}(A) &= \cup \{G / G \text{ is an IF}\gamma\text{OS in } X \text{ and } G \subseteq A\} \\ \gamma \text{cl}(A) &= \cap \{K / K \text{ is an IF}\gamma\text{CS in } X \text{ and } A \subseteq K\} \end{aligned}$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\gamma \text{cl}(A^c) = (\gamma \text{int}(A))^c$  and  $\gamma \text{int}(A^c) = (\gamma \text{cl}(A))^c$ .

**Corollary 2.9:** [3] Let  $A, A_i (i \in J)$  be intuitionistic fuzzy sets in  $X$  and  $B, B_j (j \in K)$  be intuitionistic fuzzy sets in  $Y$  and  $f: X \rightarrow Y$  be a function. Then

- a)  $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

- b)  $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
- c)  $A \subseteq f^{-1}(f(A))$  [If  $f$  is injective, then  $A = f^{-1}(f(A))$ ]
- d)  $f(f^{-1}(B)) \subseteq B$  [If  $f$  is surjective, then  $B = f(f^{-1}(B))$ ]
- e)  $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
- f)  $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
- g)  $f^{-1}(0_{\sim}) = 0_{\sim}$
- h)  $f^{-1}(1_{\sim}) = 1_{\sim}$
- i)  $f^{-1}(B^c) = (f^{-1}(B))^c$

**Definition 2.10:** [6] An IFS  $A$  of an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy  $\gamma^*$  generalized closed set (briefly  $IF\gamma^*GCS$ ) if  $cl(int(A)) \cap int(cl(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ .

**Result 2.11:** [6] Every IFRCS, IFCS, IFSCS, IFPCS,  $IF\gamma^*CS$ , IFGCS is an  $IF\gamma^*GCS$  but the converses need not be true in general.

**Definition 2.12:** [6] If every  $IF\gamma^*GCS$  in  $(X, \tau)$  is an  $IF\gamma^*CS$  in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy  $\gamma^* T_{1/2}$  ( $IF\gamma^*T_{1/2}$  in short) space.

**Definition 2.13:** [6] If every  $IF\gamma^*GCS$  in  $(X, \tau)$  is an IFCS in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy  $\gamma^*c T_{1/2}$  ( $IF\gamma^*cT_{1/2}$  in short) space.

**Definition 2.14:** [6] If every  $IF\gamma^*GCS$  in  $(X, \tau)$  is an IFGCS in  $(X, \tau)$ , then the space can be called as an intuitionistic fuzzy  $\gamma^*gT_{1/2}$  ( $IF\gamma^*gT_{1/2}$  in short) space.

**Definition 2.15:** [7] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\gamma^*$  generalized closed mapping ( $IF\gamma^*G$  closed mapping for short) if  $f(V)$   $IF\gamma^*GCS$  in  $Y$  for every IFCS  $V$  of  $X$ .

**Definition 2.16:** [7] A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy  $\gamma^*$  generalized open mapping ( $IF\gamma^*G$  open mapping for short) if  $f(V)$   $IF\gamma^*GOS$  in  $Y$  for every IFOS  $V$  of  $X$ .

### 3. Intuitionistic fuzzy contra $\gamma^*$ generalized open mappings

In this section we have introduced intuitionistic fuzzy contra  $\gamma^*$  generalized open mappings and investigated some of their properties.

**Definition 3.1:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy contra  $\gamma^*$  generalized open( $IF$  contra  $\gamma^*G$  open for short) mapping if  $f(V)$  is an  $IF\gamma^*GCS$  in  $(Y, \sigma)$  for every IFOS  $V$  of  $(X, \tau)$ .

**Example 3.2:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$ ,  $G_2 = \langle y, (0.5_u, 0.4_v), (0.5_u, 0.6_v) \rangle$ . Then  $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$  and  $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ . The IFS  $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$  is an IFOS in  $X$ . Then  $f(G_1) = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v) \rangle$  is an  $IF\gamma^*GCS$  in  $Y$  as  $f(G_1) \subseteq G_2$  and  $cl(int(f(G_1))) \cap int(cl(f(G_1))) = 0_{\sim} \subseteq G_2$ , where  $G_2$  is an IFOS in  $Y$ . Hence  $f(G_1)$  is an  $IF\gamma^*GCS$  in  $Y$ . Therefore  $f$  is an  $IF$  contra  $\gamma^*G$  open mapping.

**Definition 3.3:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called an intuitionistic fuzzy contra  $\gamma^*$  generalized closed( $IF$  contra  $\gamma^*G$  closed for short) mapping if  $f(V)$  is an  $IF\gamma^*GOS$  in  $(Y, \sigma)$  for every IFCS  $V$  of  $(X, \tau)$ .

**Theorem 3.4:** If  $f: X \rightarrow Y$  is an  $IF$  contra  $\gamma^*G$  closed mapping and  $Y$  is an  $IF\gamma^*gT_{1/2}$  space, then  $f(A)$  is an IFGOS in  $Y$  for every IFCS  $A$  in  $X$ .

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $IF$  contra  $\gamma^*G$  closed mapping and let  $A$  be an IFCS in  $X$ . Then by hypothesis  $f(A)$  is an  $IF\gamma^*GOS$  in  $Y$ . Since  $Y$  is an  $IF\gamma^*gT_{1/2}$  space,  $f(A)$  is an IFGOS in  $Y$ .

**Theorem 3.5:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping, suppose that one of the following properties hold:

- i)  $f(cl(B)) \subseteq int(\gamma cl(f(B)))$  for each IFS  $B$  in  $X$
- ii)  $cl(\gamma int(f(B))) \subseteq f(int(B))$  for each IFS  $B$  in  $X$
- iii)  $f^{-1}(cl(\gamma int(A))) \subseteq int(f^{-1}(A))$  for each IFS  $A$  in  $Y$
- iv)  $f^{-1}(cl(A)) \subseteq int(f^{-1}(A))$  for each IFOS  $A$  in  $Y$ .

Then  $f$  is an IF contra  $\gamma^*$ G open mapping.

**Proof:**

(i)  $\Rightarrow$  (ii): is obvious by taking complement in (i).

(ii)  $\Rightarrow$  (iii): Let  $A \subseteq Y$ . Put  $B = f^{-1}(A)$  in  $X$ . This implies  $A = f(f^{-1}(A)) = f(B)$  in  $Y$ . Now  $\text{cl}(\gamma\text{int}(A)) = \text{cl}(\gamma\text{int}(f(B))) \subseteq f(\text{int}(B))$  by (ii). Therefore  $f^{-1}(\text{cl}(\gamma\text{int}(A))) \subseteq f^{-1}(f(\text{int}(B))) = \text{int}(B) = \text{int}(f^{-1}(A))$ .

(iii)  $\Rightarrow$  (iv): Let  $A \subseteq Y$  be an IF $\gamma$ OS. Then  $\gamma(\text{int}(A)) = A$ . By hypothesis,  $f^{-1}(\text{cl}(\gamma\text{int}(A))) \subseteq \text{int}(f^{-1}(A))$ . Therefore  $f^{-1}(\text{cl}(A)) = f^{-1}(\text{cl}(\gamma\text{int}(A))) \subseteq \text{int}(f^{-1}(A))$ .

Suppose (iv) holds: Let  $A$  be an IFOS in  $X$ . Then  $f(A)$  is an IFS in  $Y$  and  $\gamma\text{int}(f(A))$  is an IF $\gamma$ OS in  $Y$ . Hence by hypothesis,  $f^{-1}(\text{cl}(\gamma\text{int}(f(A)))) \subseteq \text{int}(f^{-1}(\gamma\text{int}(f(A)))) \subseteq \text{int}(f^{-1}(f(A))) = \text{int}(A) = A$ . Therefore  $\text{cl}(\gamma\text{int}(f(A))) = f(f^{-1}(\text{cl}(\gamma\text{int}(f(A)))) \subseteq f(A)$ . Now  $\text{cl}(\text{int}(f(A))) \subseteq \text{cl}(\gamma\text{int}(f(A))) \subseteq f(A)$ . This implies  $f(A)$  is an IFPCS in  $Y$  and hence an IF $\gamma^*$ GCS in  $Y$  [6]. Thus  $f$  is an IF contra  $\gamma^*$ G open mapping.

**Theorem 3.6:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping. Suppose that one of the following properties are hold:

- i)  $f^{-1}(\gamma\text{cl}(A)) \subseteq \text{int}(f^{-1}(A))$  for each IFS  $A$  in  $Y$
- ii)  $\gamma\text{cl}(f(B)) \subseteq f(\text{int}(B))$  for each IFS  $B$  in  $X$
- iii)  $f(\text{cl}(B)) \subseteq \gamma\text{int}(f(B))$  for each IFS  $B$  in  $X$

Then  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Proof:**

(i)  $\Rightarrow$  (ii): Let  $B \subseteq X$ . Then  $f(B)$  is an IFS in  $Y$ . By hypothesis,  $f^{-1}(\gamma\text{cl}(f(B))) \subseteq \text{int}(f^{-1}(f(B))) = \text{int}(B)$ . Now  $\gamma\text{cl}(f(B)) = f(f^{-1}(\gamma\text{cl}(f(B)))) \subseteq f(\text{int}(B))$ .

(ii)  $\Rightarrow$  (iii): is obvious by taking complement in (ii).

Suppose (iii) holds: Let  $A$  be an IFCS in  $X$ . Then  $\text{cl}(A) = A$  and  $f(A)$  is an IFS in  $Y$ . Now  $f(A) = f(\text{cl}(A)) \subseteq \gamma\text{int}(f(A)) \subseteq f(A)$ , by hypothesis. This implies  $f(A)$  is an IF $\gamma$ OS in  $Y$  and hence IF $\gamma^*$ GOS in  $Y$ . Therefore  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Theorem 3.7:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective mapping. Then  $f$  is an IF contra  $\gamma^*$ G closed mapping if  $\text{cl}(f^{-1}(A)) \subseteq f^{-1}(\gamma\text{int}(A))$  for every IFS  $A$  in  $Y$ .

**Proof:** Let  $A$  be an IFCS in  $X$ . Then  $\text{cl}(A) = A$  and  $f(A)$  is an IFS in  $Y$ . By hypothesis  $\text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(\gamma\text{int}(f(A)))$ . Since  $f$  is bijective,  $f^{-1}(f(A)) = A$ . Therefore  $A = \text{cl}(A) = \text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(\gamma\text{int}(f(A)))$ . Now  $f(A) \subseteq f(f^{-1}(\gamma\text{int}(f(A)))) = \gamma\text{int}(f(A)) \subseteq f(A)$ . Hence  $f(A)$  is an IF $\gamma$ OS in  $Y$  and hence an IF $\gamma^*$ GOS in  $Y$ . Thus  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Theorem 3.8:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a bijective mapping where  $Y$  is an IF  $\gamma^*T_{1/2}$  space, then the following are equivalent:

- i)  $f$  is an IF contra  $\gamma^*$ G closed mapping
- ii) for each IFP  $p_{(\alpha, \beta)} \in Y$  and for each IFCS  $B$  containing  $f^{-1}(p_{(\alpha, \beta)})$ , there exists an IF $\gamma$ OS  $A$  in  $Y$  and  $p_{(\alpha, \beta)} \in A$  such that  $A \subseteq f(B)$
- iii) For each IFP  $p_{(\alpha, \beta)} \in Y$  and for each IFCS  $B$  containing  $f^{-1}(p_{(\alpha, \beta)})$ , there exists an IF $\gamma$ OS  $A$  in  $Y$  and  $p_{(\alpha, \beta)} \in A$  such that  $f^{-1}(A) \subseteq B$

**Proof:**

(i)  $\Rightarrow$  (ii): Let  $B$  be an IFCS in  $X$ . Let  $p_{(\alpha, \beta)}$  be an IFP in  $Y$  such that  $f^{-1}(p_{(\alpha, \beta)}) \in B$ . Then  $p_{(\alpha, \beta)} \in f(f^{-1}(p_{(\alpha, \beta)})) \in f(B)$ . By hypothesis  $f(B)$  is an IF $\gamma^*$ GOS in  $Y$ . Since  $Y$  is an IF $\gamma^*T_{1/2}$  space,  $f(B)$  is an IF $\gamma$ OS in  $Y$ . Now let  $A = \gamma\text{int}(f(B)) \subseteq f(B)$ . Therefore  $A \subseteq f(B)$ .

(ii)  $\Rightarrow$  (iii): Let  $B$  be an IFCS in  $X$ . Let  $p_{(\alpha, \beta)}$  be an IFP in  $Y$  such that  $f^{-1}(p_{(\alpha, \beta)}) \in B$ . Then  $p_{(\alpha, \beta)} \in f(f^{-1}(p_{(\alpha, \beta)})) \in f(B)$ . By hypothesis  $f(B)$  is an IF $\gamma$ OS in  $Y$  and  $A \subseteq f(B)$ . This implies  $f^{-1}(A) \subseteq f^{-1}(f(B)) \subseteq B$ .

(iii)  $\Rightarrow$  (i): Let  $B$  be an IFCS in  $X$  and let  $p_{(\alpha, \beta)} \in Y$ . Let  $f^{-1}(p_{(\alpha, \beta)}) \in B$ . By hypothesis there exists an IF $\gamma$ OS  $A$  in  $Y$  such that  $p_{(\alpha, \beta)} \in A$  and  $f^{-1}(A) \subseteq B$ . This implies  $p_{(\alpha, \beta)} \in A \subseteq f(f^{-1}(A)) \subseteq f(B)$ . That is  $p_{(\alpha, \beta)} \in f(B)$ . Since  $A$  is an IF $\gamma$ OS,  $A = \gamma\text{int}(A) \subseteq \gamma\text{int}(f(B))$ . Therefore  $p_{(\alpha, \beta)} \in \gamma\text{int}(f(B))$ . But  $f(B) = \cup \{ p_{(\alpha, \beta)} / p_{(\alpha, \beta)} \in f(B) \subseteq \gamma\text{int}(f(B)) \subseteq f(B)$ . Hence  $f(B)$  is an IF $\gamma$ OS in  $Y$  and hence  $f(B)$  is an IF $\gamma^*$ GOS in  $Y$ . Thus  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Remark 3.9:** The composition of two IF contra  $\gamma^*$ G open mapping is not an IF contra  $\gamma^*$ G open mapping in general as seen in the following example.

**Example 3.10:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $Z = \{p, q\}$ . Then  $\tau = \{0, G_1, 1\}$ ,  $\sigma = \{0, G_2, 1\}$  and  $\delta = \{0, G_3, G_4, 1\}$  are IFTs on  $X$ ,  $Y$  and  $Z$  respectively, where  $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b) \rangle$ ,  $G_2 = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.7_v) \rangle$ ,  $G_3 = \langle z, (0.6_p, 0.8_q), (0.2_p, 0.2_q) \rangle$  and  $G_4 = \langle z, (0.5_p, 0.6_q), (0.5_p, 0.4_q) \rangle$ . Then  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \delta)$  are IFTSs. Now define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$  and  $g: (Y, \sigma) \rightarrow (Z, \delta)$  by  $g(u) = p$  and  $g(v) = q$ . Here  $f$  and  $g$  are IF contra  $\gamma^*$ G open mappings but their composition  $g \circ f: (X, \tau) \rightarrow (Z, \delta)$  defined by  $g(f(a)) = p$  and  $g(f(b)) = q$  is not an IF contra  $\gamma^*$ G open mapping, since  $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b) \rangle$  is an IFOS in  $X$  but  $g(f(G_1)) = \langle z, (0.5_p, 0.8_q), (0.2_p, 0.2_q) \rangle$  is not an IF $\gamma^*$ GCS in  $Z$  as  $g(f(G_1)) \subseteq G_3$  but  $\text{cl}(\text{int}(g(f(G_1)))) \cap \text{int}(\text{cl}(g(f(G_1)))) = 1 \notin G_3$ .

**Theorem 3.11:** For a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$ , where  $Y$  is an IF $\gamma^*$ T<sub>1/2</sub> space, the following are equivalent:

- $f$  is an IF contra  $\gamma^*$ G closed mapping
- For every IFCS  $A$  in  $X$  and for every IFP  $p_{(\alpha, \beta)} \in Y$ , if  $f^{-1}(p_{(\alpha, \beta)}) \subseteq A$  then  $p_{(\alpha, \beta)} \in \text{int}(f(A))$

**Proof:**

**(i)  $\Rightarrow$  (ii):** Let  $f$  be an IF contra  $\gamma^*$ G closed mapping. Let  $A \subseteq X$  be an IFCS and let  $p_{(\alpha, \beta)} \in Y$ . Also let  $f^{-1}(p_{(\alpha, \beta)}) \subseteq A$  then  $p_{(\alpha, \beta)} \in f(A)$ . By hypothesis  $f(A)$  is an IF $\gamma^*$ GOS in  $Y$ . Since  $Y$  is an IF $\gamma^*$ cT<sub>1/2</sub>space,  $f(A)$  is an IFOS in  $Y$ . Hence  $\text{int}(f(A)) = f(A)$ . This implies  $p_{(\alpha, \beta)} \in \text{int}(f(A))$ .

**(ii)  $\Rightarrow$  (i):** Let  $A \subseteq X$  be an IFCS then  $f(A)$  is an IFS in  $Y$ . Let  $p_{(\alpha, \beta)} \in Y$  and let  $f^{-1}(p_{(\alpha, \beta)}) \subseteq A$  then  $p_{(\alpha, \beta)} \in f(A)$ . By hypothesis this implies  $p_{(\alpha, \beta)} \in \text{int}(f(A))$ . That is  $f(A) \subseteq \text{int}(f(A))$ . But  $\text{int}(f(A)) \subseteq f(A)$ . Therefore  $\text{int}(f(A)) = f(A)$ . Thus  $f(A)$  is an IFOS in  $Y$  and hence an IF $\gamma^*$ GOS in  $Y$ . This implies  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Theorem 3.12:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF contra  $\gamma^*$ G open mapping, where  $Y$  is an IF $\gamma^*$ T<sub>1/2</sub> space, then the following conditions hold:

- $\gamma\text{cl}(f(B)) \subseteq f(\text{int}(\gamma\text{cl}(B)))$  for each IFOS  $B$  in  $X$
- $f(\text{cl}(\gamma\text{int}(B))) \subseteq \gamma\text{int}(f(B))$  for each IFCS  $B$  in  $X$

**Proof:** Let  $B \subseteq X$  be an IFOS. Then  $\text{int}(B) = B$ . By hypothesis  $f(B)$  is an IF $\gamma^*$ GCS in  $Y$ . Since  $Y$  is an IF $\gamma^*$ T<sub>1/2</sub> space,  $f(B)$  is an IF $\gamma$ CS in  $Y$ . This implies  $\gamma\text{cl}(f(B)) = f(B) = f(\text{int}(B)) \subseteq f(\text{int}(\gamma\text{cl}(B)))$ .

(ii) can be proved by taking complement in (i).

**Theorem 3.13:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is an IF contra  $\gamma^*$ G closed mapping, where  $Y$  is an IF $\gamma^*$ T<sub>1/2</sub> space if and only if  $f(\gamma\text{cl}(B)) \subseteq \gamma\text{int}(f(\text{cl}(B)))$  for every IFS  $B$  in  $X$ .

**Proof:**

**Necessity:** Let  $B \subseteq X$  be an IFS. Then  $\text{cl}(B)$  is an IFCS in  $X$ . By hypothesis,  $f(\text{cl}(B))$  is an IF $\gamma^*$ GOS in  $Y$ . Since  $Y$  is an IF $\gamma^*$ T<sub>1/2</sub> space,  $f(\text{cl}(B))$  is an IF $\gamma$ OS in  $Y$ . Therefore  $f(\gamma\text{cl}(B)) \subseteq f(\text{cl}(B)) = \gamma\text{int}(f(\text{cl}(B)))$ .

**Sufficiency:** Let  $B \subseteq X$  be an IFCS. Then  $\text{cl}(B) = B$ . By hypothesis,  $f(\gamma\text{cl}(B)) \subseteq \gamma\text{int}(f(\text{cl}(B))) = \gamma\text{int}(f(B))$ . But  $\gamma\text{cl}(B) = B$ . Therefore  $f(B) = f(\gamma\text{cl}(B)) \subseteq \gamma\text{int}(f(B)) \subseteq f(B)$ . This implies  $f(B)$  is an IF $\gamma$ OS in  $Y$  and hence an IF $\gamma^*$ GOS in  $Y$ . Hence  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Theorem 3.14:** Let  $f: X \rightarrow Y$  be a bijective mapping. Then the following conditions are equivalent if  $Y$  is an IF $\gamma^*$ cT<sub>1/2</sub> space:

- $f$  is an IF contra  $\gamma^*$ G closed mapping
- $f$  is an IF contra  $\gamma^*$ G open mapping
- $\text{int}(\text{cl}(f(A))) \subseteq f(A)$  for every IFOS  $A$  in  $X$ .

**Proof:**

**(i)  $\Leftrightarrow$  (ii):** is obviously true.

**(ii)  $\Rightarrow$  (iii):** Let  $A$  be an IFOS in  $X$ . Then  $f(A)$  is an IF $\gamma^*$ GCS in  $Y$ . Since  $Y$  is an IF $\gamma^*$ cT<sub>1/2</sub> space,  $f(A)$  is an IFCS in  $Y$ . Therefore  $\text{cl}(f(A)) = f(A)$ . This implies  $\text{int}(\text{cl}(f(A))) = \text{int}(f(A)) \subseteq f(A)$ .

**(iii)  $\Rightarrow$  (i):** Let  $A$  be an IFCS in  $X$ . Then its complement  $A^c$  is an IFOS in  $X$ . By hypothesis,  $\text{int}(\text{cl}(f(A^c))) \subseteq f(A^c)$ . Hence  $f(A^c)$  is an IFSCS in  $Y$ . Since every IFSCS [4] is an IF $\gamma^*$ GCS,  $f(A^c)$  is an IF $\gamma^*$ GCS in  $Y$ . Therefore  $f(A)$  is an IF $\gamma^*$ GOS in  $Y$ . Hence  $f$  is an IF contra  $\gamma^*$ G closed mapping.

**Definition 3.15:** A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be an intuitionistic fuzzy almost contra  $\gamma^*$  generalized open mapping (IF almost contra  $\gamma^*$ G open mapping for short) if  $f(A)$  is an IF $\gamma^*$ GCS in  $Y$  for every IFROS  $A$  in  $X$ .

**Example 3.16:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$ ,  $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  be IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ .

Now the IFS  $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$  is an IFROS in  $X$ , since  $\text{int}(\text{cl}(G_1)) = \text{int}(G_1^c) = G_1$ . We have  $f(G_1) = \langle y, (0.5_u, 0.5_v), (0.5_u, 0.5_v) \rangle$  is an IF $\gamma^*$ GCS as  $f(G_1) \subseteq G_2$  and  $\text{int}(\text{cl}(f(G_1))) \cap \text{cl}(\text{int}(f(G_1))) = 0_- \cap 1_- = 0_- \subseteq G_2$ . Thus  $f$  is an IF almost contra  $\gamma^*$ G open mapping.

**Theorem 3.17:** Every IF contra  $\gamma^*$ G open mapping is an IF almost contra  $\gamma^*$ G open mapping but not conversely in general.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an IF contra  $\gamma^*$ G open mapping. Let  $A$  be an IFROS in  $X$ . Since every IFROS is an IFOS,  $A$  is an IFOS in  $X$ . Then  $f(A)$  is an IF $\gamma^*$ GCS in  $Y$ , by hypothesis. Therefore  $f$  is an IF almost contra  $\gamma^*$ G open mapping.

**Example 3.18:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b) \rangle$ ,  $G_2 = \langle x, (0.2_a, 0.2_b), (0.5_a, 0.6_b) \rangle$ ,  $G_3 = \langle y, (0.6_u, 0.8_v), (0.2_u, 0.2_v) \rangle$  and  $G_4 = \langle y, (0.5_u, 0.7_v), (0.2_u, 0.2_v) \rangle$ . Then  $\tau = \{0_-, G_1, G_2, 1_-\}$  and  $\sigma = \{0_-, G_3, G_4, 1_-\}$  be IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ .

Now  $G_2 = \langle x, (0.2_a, 0.2_b), (0.5_a, 0.6_b) \rangle$  is an IFROS in  $X$ , Since  $\text{int}(\text{cl}(G_2)) = \text{int}(G_2^c) = G_2$ . We have  $f(G_2) = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.6_v) \rangle \subseteq G_3, G_4$ . As  $\text{int}(\text{cl}(f(G_2))) \cap \text{cl}(\text{int}(f(G_2))) = 1_- \cap 0_- = 0_- \subseteq G_3, G_4$ ,  $f$  is an IF almost contra  $\gamma^*$ G open mapping, but not an IF contra  $\gamma^*$ G open mapping, as  $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b) \rangle$  is an IFOS in  $X$  and  $f(G_1) \subseteq G_3$  but  $\text{int}(\text{cl}(f(G_1))) \cap \text{cl}(\text{int}(f(G_1))) = 1_- \cap 1_- = 1_- \not\subseteq G_3$ .

**Definition 3.19:** A mapping  $f: X \rightarrow Y$  is said to be an intuitionistic fuzzy contra  $M\gamma^*$  generalized open mapping (IF contra  $M\gamma^*$ G open mapping) if  $f(A)$  is an IF $\gamma^*$ GCS in  $Y$  for every IF $\gamma^*$ GOS  $A$  in  $X$ .

**Example 3.20:** Let  $X = \{a, b\}$  and  $Y = \{u, v\}$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively, where  $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b) \rangle$  and  $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v) \rangle$ . Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ .

$$\text{IF}\gamma^*\text{GO}(X) = \{0_-, 1_-, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / 0 \leq \mu_a + \nu_a \leq 1, 0 \leq \mu_b + \nu_b \leq 1\}$$

$$\text{IF}\gamma^*\text{GC}(Y) = \{0_-, 1_-, \mu_u \in [0, 1], \mu_v \in [0, 1], \nu_u \in [0, 1], \nu_v \in [0, 1] / 0 \leq \mu_u + \nu_u \leq 1, 0 \leq \mu_v + \nu_v \leq 1\}$$

We have every IF $\gamma^*$ GOS in  $X$  is an IF $\gamma^*$ GCS in  $Y$ . Therefore  $f$  is an IF contra  $M\gamma^*$ G open mapping.

**Theorem 3.21:** Let  $f: X \rightarrow Y$  be a bijective mapping. Then the following are equivalent.

- (i)  $f$  is an IF contra  $M\gamma^*$ GOM
- (ii)  $f(A)$  is an IF $\gamma^*$ GOS in  $Y$  for every IF $\gamma^*$ GCS  $A$  in  $X$

**Proof:** Proof is obvious for a bijective mapping as  $f(A^c) = (f(A))^c$ .

**Theorem 3.22:** Every IF contra  $M\gamma^*$ G open mapping is an IF contra  $\gamma^*$ G open mapping but not conversely in general.

**Proof:** Let  $f: X \rightarrow Y$  be an IF contra  $M\gamma^*$ G open mapping. Let  $A \subseteq X$  be an IFOS. Then  $A$  is an IF $\gamma^*$ GOS in  $X$ . By hypothesis,  $f(A)$  is an IF $\gamma^*$ GCS in  $Y$ . Hence  $f$  is an IF contra  $\gamma^*$ G open mapping.

**Example 3.23:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$ ,  $G_2 = \langle y, (0.6_u, 0.8_v), (0.4_u, 0.1_v) \rangle$  and  $G_3 = \langle y, (0.3_u, 0.3_v), (0.5_u, 0.4_v) \rangle$ . Then  $\tau = \{0_-, G_1, 1_-\}$  and  $\sigma = \{0_-, G_2, G_3, 1_-\}$  are IFTs on  $X$  and  $Y$  respectively. Define a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = u$  and  $f(b) = v$ .

Now  $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  is an IFOS in  $X$ . We have  $f(G_1) = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v) \rangle$  is an IF $\gamma^*$ GCS. Hence  $f$  is an IF contra  $\gamma^*$ G open mapping.

Now let  $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$  which is an IF $\gamma^*$ GOS in  $X$ . But it is not an IF $\gamma^*$ GCS in  $Y$ , since  $f(G_1) \subseteq G_2$  but  $\text{int}(\text{cl}(f(G_1))) \cap \text{cl}(\text{int}(f(G_1))) = 1_- \cap G_3^c = G_3^c \not\subseteq G_2$ . Hence  $f$  is not an IF contra  $M\gamma^*$ G open mapping.

**Theorem 3.24:**

- (i) (i) If  $f : X \rightarrow Y$  is an IF  $\gamma^*$ G open mapping and  $g : Y \rightarrow Z$  is an IF contra  $\gamma^*$ G open mapping, then  $g \circ f : X \rightarrow Z$  is an IF contra  $\gamma^*$ G open mapping.
- (ii) If  $f : X \rightarrow Y$  is an IF contra  $\gamma^*$ G open mapping and  $g : Y \rightarrow Z$  is an IF contra  $M\gamma^*$ G open mapping, then  $g \circ f : X \rightarrow Z$  is an IF  $\gamma^*$ G open mapping.
- (iii) If  $f : X \rightarrow Y$  is an IF almost contra  $\gamma^*$ G open mapping and  $g : Y \rightarrow Z$  is an IF contra  $M\gamma^*$ G open mapping, then  $g \circ f : X \rightarrow Z$  is an IF almost  $\gamma^*$ G open mapping.

**Proof:**

- (i) Let A be an IFOS in X. Then  $f(A)$  is an IF $\gamma^*$ GOS in Y. Therefore  $g(f(A))$  is an IF $\gamma^*$ GCS in Z. Hence  $g \circ f$  is an IF contra  $\gamma^*$ G open mapping.
- (ii) Let A be an IFOS in X. Then  $f(A)$  is an IF $\gamma^*$ GCS in Y, since f is an IF contra  $\gamma^*$ GOM. Since g is an IF contra  $\gamma^*$ G open mapping,  $g(f(A))$  is an IF $\gamma^*$ GOS in Z. Therefore  $g \circ f$  is an IF $\gamma^*$ G open mapping.
- (iii) Let A be an IFROS in X. Then  $f(A)$  is an IF $\gamma^*$ GCS in Y, since f is an IF almost contra  $\gamma^*$ G open mapping. Since g is an IF contra  $\gamma^*$ G open mapping,  $g(f(A))$  is an IF $\gamma^*$ GOS in Z. Therefore  $g \circ f$  is an IF almost  $\gamma^*$ G open mapping.

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