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INTUITIONISTIC FUZZY IDEALS OF M- Γ GROUPS

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ABSTRACT

In this paper we define and derive lemmas and theorems of Intuitionistic Fuzzy Ideals of M- Γ Groups analogous to Fuzzy Ideals.

Keywords: *Intuitionistic fuzzy rings, Near ring, Fuzzy M- Γ sub group, Intuitionistic fuzzy ideals, of Intuitionistic Fuzzy Ideals of M- Γ Groups.*

INTRODUCTION

In 1986 Atanassov [1] introduced the notion of an Intuitionistic Fuzzy set as a generalization of Zadeh [9] fuzzy set. Atanassov [1] also described different operations of Intuitionistic Fuzzy set and their properties elaborately. Clay [3] discussed various properties and applications of near rings. The theories and applications of Fuzzy sets and Fuzzy logics were put forth by Klir and Yuan [9]. Characteristics and properties of Intuitionistic Fuzzy Ideals of Near Rings were discussed in 2004 by Ma and Zhan [10]. Based on the introductory study of M- Γ Groups, various results of Fuzzy Ideals of M- Γ Groups were defined, discussed and derived by Satyanarayana and Prasad [14]. An analogous definition, theorems and properties of Intuitionistic Fuzzy Ideals of M- Γ Groups are explored.

PRELIMINARIES

Definition 2.1-Crisp Sets [19]: The Crisp set is defined to classify the individuals in the Universe in two groups: Members and Non- Members

Definition 2.2–Fuzzy Sets [19]: A fuzzy set has objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one.

Definition 2.3–Fuzzy Subsets [19]: Let S be any non-empty set, A mapping μ from S to [0,1] is called a Fuzzy sub set of S.

Definition 2.4-Intuitionistic Fuzzy sets [4, 9]: Let X be any non-empty set, a fuzzy subset A of X is of the form A = $\{<x, \mu_A(x), \gamma_A(x)>/x \epsilon X\}$ where the functions $\mu_A : X \rightarrow [0.1]$ and $\gamma_A : X \rightarrow [0.1]$ denote the degrees of membership and non-membership of the element $x \epsilon X$ to A respectively and satisfy $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \epsilon X$. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition 2.5 [9]: Let A be any fuzzy set in X. Then for any $\alpha \in [0, 1]$, α cut of A, denoted by ${}^{\alpha}A$, is defined as ${}^{\alpha}A = \{x: x \in X \text{ such that } \mu_A(x) \geq \alpha\}$ and the strong α cut of A denoted by ${}^{\alpha+}A$, is defined as ${}^{\alpha+}A = \{x: x \in X \text{ such that } \mu_A(x) > \alpha\}$

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Definition 2.6 [9]: Let A be any fuzzy set in X, then denote $_{\alpha}A$ by $_{\alpha}A(x) = \alpha.\ {}^{\alpha}A(x)$. Also $_{\alpha+}A(x) = \alpha.\ {}^{\alpha+}A(x)$; where $_{\alpha}A$ and $_{\alpha+}A$ are fuzzy sets.

Definition 2.7 [9]: Let A be any fuzzy set in X. Then the level set of A denoted by ^(A), is defined as

$$\wedge(A) = \{\alpha \mid A(x) = \alpha : x \in X\}$$

Definition 2.8 [14]: By a near-ring we mean a non-empty set R with two binary operations "+" and " · " satisfying the following axioms:

(i) (R, +) is a group
(ii) (R, ·) is a semigroup
(iii) $x \cdot (y + z) = x \cdot y + x \cdot z$ for all x, y, z ∈ R.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word " near-ring " instead of " left near-ring ". We denote xy instead of x · y.

Definition 2.9 [14]: An ideal of a near-ring R is a subset I of R such that

(i) (I, +) is a normal subgroup of (R, +)
(ii) $RI \subseteq I$
(iii) $(x + i)y - xy \in I$ for all $i \in I$ and $x, y \in R$.

Definition 2.10 [8]: A fuzzy set μ in a near-ring R is called a fuzzy ideal of R if it satisfies:

(i) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$
(ii) $\mu(y + x - y) \geq \mu(x)$
(iii) $\mu(xy) \geq \mu(y)$
(iv) $\mu((x + z)y - xy) \geq \mu(z)$ for all x, y, z ∈ R.

Definition 2.11 [7]: A fuzzy set μ in a near-ring R is an anti-fuzzy ideal of R if it satisfies:

(i) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$
(ii) $\mu(y + x - y) \leq \mu(x)$
(iii) $\mu(xy) \leq \mu(y)$
(iv) $\mu((x + z)y - xy) \leq \mu(z)$ for all x, y, z ∈ R.

Definition 2.12 [12]: Let *R* be a ring. An IFS $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in R\}$ of *R* is said to be **intuitionistic fuzzy subring** of *R* (In short IFSR) of *R* if

(i) $\mu_A(x - y) \geq \text{Min}\{\mu A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \geq \text{Min}\{\mu A(x), \mu_A(y)\}$
(iii) $\gamma_A(x - y) \leq \text{Max}\{\gamma_A(x), \gamma_A(y)\}$
(iv) $\gamma_A(xy) \leq \text{Max}\{\gamma_A(x), \gamma_A(y)\}$, for all $x, y \in R$

Definition 2.13 [12]: Let *R* be a ring. An IFSR *A* of *R* is said to be intuitionistic fuzzy normal subring (in short IFNSR) of *R* if

(i) $\mu_A(xy) = \mu_A(yx)$
(ii) $\gamma_A(xy) = \gamma_A(yx)$, for all $x, y \in R$

Definition 2.14 [15]: An IFS $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in R\}$ of a ring *R* said to be

(a) **intuitionistic fuzzy Left Ideal** of R (In short IFLI) of R if

(i) $\mu_A(x - y) \geq \text{Min}\{\mu A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \geq \mu_A(y)$
(iii) $\gamma_A(x - y) \leq \text{Max}\{\gamma_A(x), \gamma_A(y)\}$
(iv) $\gamma_A(xy) \leq \gamma_A(y)$, for all $x, y \in R$

(b) **intuitionistic fuzzy Right Ideal** of *R* (In short IFRI) of *R* if

(i) $\mu_A(x - y) \geq \text{Min}\{\mu A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \geq \mu_A(x)$
(iii) $\gamma_A(x - y) \leq \text{Max}\{\gamma_A(x), \gamma_A(y)\}$
(iv) $\gamma_A(xy) \leq \gamma_A(x)$, for all $x, y \in R$

(c) **intuitionistic fuzzy Ideal** of R (In short IFI) of R if

(i) $\mu_A (x - y) \geq \text{Min } \{\mu A (x), \mu_A (y)\}$

(ii) $\mu_A (xy) \geq \text{Max } \{A(x), A(y)\}$

(iii) $\gamma_A (x - y) \leq \text{Max } \{\gamma_A (x), \gamma_A (y)\}$

(iv) $\gamma_A (xy) \leq \text{Min } \{\gamma_A (x), \gamma_A (y)\}$, for all $x, y \in R$

Theorem 2.15 [15]: If $A = \{< x, \mu_A (x), A(x) >: x \in R\}$ be IFSR of ring R, then

(i) $\mu_A (0) \geq \mu_A (x)$ and $\gamma A (0) \leq \gamma A(x)$

(ii) $\mu_A (- x) = \mu_A (x)$ and $\gamma A (- x) = \gamma A(x)$, for all $x, y \in R$

(iii) If R is ring with unity 1, then $\mu A (1) \leq \mu A(x)$ and $\gamma_A (1) \geq \gamma_A (x)$, for all $x \in R$

Definition 2.16 [11]: An intuitionistic fuzzy groupoid, denoted by ((X, I, I), F), is an IFS (X, I, I) together with an intuitionistic fuzzy binary operation Fdefined over it. An intuitionistic fuzzy semigroup is an intuitionistic fuzzy groupoid that is associative. An intuitionistic fuzzy monoid is an intuitionistic fuzzy semigroup that admits an identity. An intuitionistic fuzzy group is an intuitionistic fuzzy monoid in which each intuitionistic fuzzy element has an inverse. An intuitionistic fuzzy group ((G, I, I), F) is called an abelian (commutative) intuitionistic fuzzy group if and only if for all (x, I, I), (y, I, I) ∈ ((G, I, I), F) (x, I, I) F (y, I, I) = (y, I, I) F (x, I, I).

Definition 2.17 [6]: A fuzzy set μ of G is called a fuzzy MΓ-subgroup of G if it satisfies the following two conditions:

(i) $\mu (x - y) \geq \min\{\mu(x), \mu(y)\}$ and

(ii) $\mu(a\alpha y) \geq \mu(y)$ for all $x, y \in G$, $a \in M$, and $\alpha \in \Gamma$.

In this section, M denotes a gamma near ring, and G stands for an MΓ-group.

Definition 2.18 [14]: Let M be a Γ-near ring. An additive group G is said to be a Γ-near ring module (or MΓ-module) if there exists a mapping $M \times \Gamma \times G \to G$ (denote the image of (m, α, g) by $m\alpha g$ for $m \in M$, $\alpha \in \Gamma$, $g \in G$) satisfying the conditions

(i) $(m_1 + m_2)\, \alpha_1\, g = m_1\alpha_1\, g + m_2\alpha_1\, g$ and

(ii) $(m_1\, \alpha_1\, m_2)\, \alpha_2\, g = m_1\alpha_1\, (m_2\, \alpha_2\, g)$, for all $m_1, m_2 \in M$, $\alpha_1, \alpha_2 \in \Gamma$, and $g \in G$.

Definition 2.19 [14]:

(i) An additive subgroup H of G is said to be a MΓ-subgroup if $m\alpha h \in H$ for all $m \in M$, $\alpha \in \Gamma$, and $h \in H$. (Note that (0) and G are the trivial MΓ-subgroups.)

(ii) A normal subgroup H of G is said to be a submodule (or ideal) of G if $m\alpha\, (g + h) - m\alpha g \in H$ for $m \in M$, $\alpha \in \Gamma$, $g \in G$, and $h \in H$.

(iii) For two MΓ-modules G1 and G2, a group homomorphism θ: G1→ G2 is said to be a module homomorphism (or Γ-module homomorphism) if $\theta(m\alpha g) = m\alpha(\theta g)$ for $m \in M$, $\alpha \in \Gamma$, and $g \in G1$.

INTUITIONISTIC FUZZY IDEALS OF M- Γ GROUPS:

Definition 3.1: An intuitionistic fuzzy set A (μ_A, γ_A) of G is called an intuitionistic fuzzy MΓ subgroup of G if it satisfies the following two conditions.

a. $\mu (x - y) \geq \min \{\mu (x), \mu (y)\}$
   $\gamma (x - y) \leq \max \{\gamma (x), \gamma (y)\}$

b. $\mu (a\, \alpha\, y) \geq \mu (y)$
   $\gamma (a\, \alpha\, y) \leq \gamma (y)$ for all $x, y \in G$, $a \in M$ and $\alpha \in \Gamma$.

Where M denotes a gamma near ring and G stands for an MΓ group.

Definition 3.2: If G is said to be an intuitionistic fuzzy ideal if M: G→ [0, 1] and γ: G→ [0, 1] satisfying the following properties.

i. $\mu (x + y) \geq \min \{\mu(x), \mu(y)\}$

ii. $\mu (x + y - x) \geq \mu(y)$

iii. $\mu (x) = \mu (-x)$

iv. $\mu (n\alpha\, (a + x) - n\, \alpha\, a) \geq \mu(x)$

v. $\gamma (x + y) \leq \max \{\gamma(x), \gamma(y)\}$

vi. $\gamma (x + y - x) \leq \gamma(y)$

vii. $\gamma (x) = \gamma (- x)$

viii. $\gamma (n\, \alpha\, (a + x) - n\, \alpha\, a) \leq \gamma (x)$ for all $n \in M$, $\alpha \in \Gamma$, $a, x, y \in G$.

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Remark 3.3: If μ and γ satisfies (i), (ii), (iii), (v), (vi), (vii) then A (μ_A, γ_A) is the intuitionistic fuzzy normal MΓ subgroup of G.

Definition 3.4: For any family {Ai ⁄ i J} of intuitionistic fuzzy of a set S, we define the intersection of the intuitionistic fuzzy sets Ai for i ϵ J as

$\mu(x) = \min\{\mu_i(x) / x \in Ai\}$

$\gamma(x) = \min\{\gamma_i(x) / x \in Ai\}$

Ai (μ_{Ai}, γ_{Ai}) is an intuitionistic fuzzy subset of a set S.

Result 3.5: If A (μ_A, γ_A) is an intuitionistic fuzzy ideal of MΓ group G then

a. $\mu(g - g^1) \geq \min\{\mu(g), \mu(g^1)\}$
   $\gamma(g - g^1) \leq \max\{\gamma(g), \gamma(g^1)\}$

b. $\mu(g + g^1) = \mu(g^1 + g)$
   $\gamma(g + g^1) = \gamma(g^1 + g)$ for all g, g1 ϵ G.

Proposition 3.6: If A (μ_A, γ_A) is an intuitionistic fuzzy of the MΓ group G for x, y ϵ G with $\mu(x) > \mu(y)$ and $\gamma(x) > \gamma(y)$ then, $\mu(x + y) = \mu(y)$ and $\gamma(x + y) = \gamma(x)$.

Moreover, if x, y ϵ G, $\mu(x) \neq \mu(y)$ and $\gamma(x) \neq \gamma(y)$ then,

$\mu(x + y) = \min\{\mu(x), \mu(y)\}$

$\gamma(x + y) = \max\{\gamma(x), \gamma(y)\}$

Proof: Suppose $\mu(x) > \mu(y)$ and $\gamma(x) > \gamma(y)$

By definition,

$$\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$$
$$= \mu(y) \text{ (since } \mu(x) > \mu(y))$$
$$\Rightarrow \mu(x + y) \geq \mu(y) \qquad (1)$$

$$\mu(y) = \mu(y + x - x)$$
$$\geq \min\{\mu(y + x), \mu(-x)\} \text{ [since } \mu(-x) = \mu(x)]$$
$$= \min\{\mu(y + x), \mu(x)\} \qquad (2)$$
$$= \mu(x) \text{ if } \mu(y + x) > \mu(x)$$
$$\Rightarrow \mu(y) \geq \mu(x) \text{ which is a contradiction.}$$

Therefore, $\mu(y + x) < \mu(x)$.

Substituting in **2**

$$\mu(y) \geq \mu(x + y) \qquad (3)$$

1 and 3 $\Rightarrow \mu(x + y) = \mu(y)$

Consider $\gamma(x + y) \leq \max\{\gamma(x), \gamma(y)\}$

$\leq \gamma(x)$ [since $\gamma(x) > \gamma(y)$]

i.e. $\gamma(x + y) \leq \gamma(x)$ (4)

Consider $\gamma(x) = \gamma(x + y - y)$

$$\leq \max\{\gamma(x + y), \gamma(-y)\}$$
$$= \max\{\gamma(x + y), \gamma(y)\} \text{ [since } \gamma(-y) = \gamma(y)] \qquad (5)$$

If $\gamma(x + y) < \gamma(y)$

$\Rightarrow \gamma(x) \leq \gamma(y)$ which is a contradiction to the given data.

Therefore, $\gamma(x + y) > \gamma(y)$

5 $\Rightarrow \gamma(x) \leq \gamma(x + y)$ (6)

4 and **6** $\Rightarrow \gamma\, x) = \gamma(x + y)$

This completes the preposition

Theorem 3.7: Let M be a zero symmetric Γ near ring and G be a MΓ group. If A (μ_A, γ_A) is an intuitionistic fuzzy ideal of G then μ (m α x) $\geq \mu$ (x) for all x ϵ G, m ϵ M and $\alpha \epsilon \Gamma$ and γ (m α x) $\leq \gamma$ (x)

Proof: Consider μ (m α x) = μ (m α (x + 0))
$= \mu$ (m α (0 + x – 0))
$= \mu$ (m α (0 + x) – m α)
$\geq \mu$ (x)
Since M is a zero symmetric Γ near ring.

Similarly, γ (m α x) = γ (m α (x + 0))
$= \gamma$ (m α (0 + x) – 0)
$= \gamma$ (m α (0 + x) – m α 0)
$\leq \gamma$ (x)
Thus, the theorem is proved.

Remark 3.8: If A (μ_A, γ_A) is an intuitionistic fuzzy ideal then
a. μ (0) $\geq \mu$ (g) and
γ (1) $\leq \gamma$ (g) for all g ϵ G.
b. μ (0) = $\sup_{g \epsilon G} \mu$ (g)
γ (1) = $\sup_{g \epsilon G} \gamma$ (g).

As A (μ_A, γ_A) is an intuitionistic fuzzy, the image of A under μ, μ (A) is a subset of [0, μ (0)] and the image of A under γ, γ (A) is a subset of [γ(1), 1].

Examples of Intuitionistic Fuzzy Ideal 3.9:
a. Every constant intuitionistic fuzzy set A (μ_A, γ_A) where μ_A: A → [0, 1] and,
b. γ_A : A → [0 , 1] as μ_A (x) = 0.6 and γ_A (x) = 0.2 for all x ϵ A is an intuitionistic fuzzy ideal of G.
c. Take N = Z with usual operations '+' and '.'. Then (N, +,.) forms a near ring. Let Γ = {.} and M = N. then M is a near ring with G = Z. Then (G, +) is a group of integers with usual addition. Now G is a N- group and a MΓ group.

Let μ: A → [0, 1] and γ: A → [0, 1] be defined by
μ (x) = {0.5 if x = 4n for some n ϵ z
0.3 if x = 2n for some n ϵ z and x $\neq$ 4m for some m ϵ z
0 if x is odd
1 if x = 0 and otherwise.

γ (x) = {0.3 if x = 4n for some n ϵ z
0.4 if x = 2n for some n ϵ z and x $\neq$ 4m for some m ϵ z
1 if x is odd
0 if x = 0 and otherwise.

Here $0 \leq \mu$ (x) + γ (x) ≤ 1 for all x ϵ A Then A satisfies all the axioms of intuitionistic fuzzy ideal of G.

Lemma 3.10: If A (μ_A, γ_A) is an intuitionistic fuzzy of the MΓ group G and if μ (x – y) = μ (0) and γ (x – y) = γ (1) then μ (x) = μ (y) and γ (x) = γ (y) for all x, y ϵ G.

Proof: Assume μ (x – y) = μ (0)

Consider μ (x) = μ (x – y + y)
$\geq$ min {μ (x – y), μ (y)}
= min {μ (0), μ (y)} by ***** our assumption.
$= \mu$ (y)
μ (x) $\geq \mu$ (y) -----------> **I**

On the other hand,
μ (y) = μ (y – x + x)
$\geq$ min {μ (y – x), μ (x)} [μ (y – x) = μ (- (x – y))
$= \mu$ (x – y)]
= min {μ (0), μ (x)}
$= \mu$ (x) [μ (0) = 1, γ (1) = 0]
μ (y) $\geq \mu$ (x) ----------> **II**
I & II ⟹ μ (x) = μ (y)

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Assume γ (x – y) = γ (1)
= γ (x – y + y)

Consider, γ (x) = γ (x - y + y)
$\leq$ max {γ (x – y), γ (y)}
$\leq$ max {γ (1), γ (y)} by our assumption
= γ (y)
γ (x) $\leq$ γ (y) ----------> **III**

On the other hand,
γ (y) = γ (y – x + x)
$\leq$ max {γ (y – x), γ (x)} [γ (y – x) = γ (- (x – y))
= max {γ (1), γ (x)} = γ (x – y)]
= γ (x)
γ (y) $\leq$ γ (x) ------------> **IV**

III & IV $\Longrightarrow$ γ (x) = γ (y)
The converse of the above lemma 3.10 is not true. That is, we can find an MΓ group G and a intuitionistic fuzzy ideal H of G such that μ (x) = μ (y) and γ (x) = γ (y) but μ (x – y) $\neq$ 0 and γ (x – y) $\neq$ 0. For this consider the following example:

Consider the example of intuitionistic fuzzy ideal.

Let x = 3, y = 7 then by μ, γ defined
μ (3) = 0 and μ (7) = 0 $\Longrightarrow$ μ (x) = μ (y)

But μ (x – y) = μ (3 – 7) = μ (- 4) = 0.5 $\neq$ 1 = μ (0)
And γ (3) = 1 and γ (7) = 1 $\Longrightarrow$ γ (x) = γ (y)

But γ (x – y) = γ (3 – 7) = 0.3 $\neq$ 0 = γ (1).

Theorem 3.11: Let G be an MΓ group an A (μA, γ_A) be an intuitionistic fuzzy subset of G. then the following conditions are equivalent.

1. A is an intuitionistic fuzzy ideal of the MΓ group G and
2. U (μ_A, t) and L (γ_A , s) are ideals of the MΓ group G for all s, t ϵ [0, 1].

Proof:
1 $\Longrightarrow$ 2: Let s, t ϵ [0, 1]

Let x, y ϵ U (μ_A, t) $\Longrightarrow$ μ_A (x) > t and μ_A (y) > t.
μ_A (x – y) $\geq$ min {μA (x), μ_A (- y)}
= min {μA (x), μ_A (y)}
= min {t, t} = t.
$\Longrightarrow$ μ_A (x – y) $\geq$ t
$\Longrightarrow$ x – y ϵ U (μA, t).
$\Longrightarrow$ μ_A (t) is a subgroup of (G, t)

Let g ϵ G.

Consider, μ_A (g + x – g) $\geq$ μ_A (x) $\geq$ t
$\Longrightarrow$ g + x – g ϵ U (μ_A, t)
$\Longrightarrow$ U (μ_A, t) is a normal subgroup of (G, +).

Let m ϵ M and α ϵ Γ.

Consider, μ_A (m α (g + x) – m α g) $\geq$ μ_A (x) $\geq$ t
$\Longrightarrow$ m α (g + x – m α g ϵ U (μ_A, t)
$\Longrightarrow$ U (μ_A, t) is an intuitionistic fuzzy of the MΓ group G.

Similarly, L (γ_A, t) is an intuitionistic fuzzy of the MΓ group G.

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Let x, y ϵ L (γ_A, s) ⟹ γ_A (x) < s and γ_A (y) < s.

Therefore, γ_A (x – y) ≤ max {γ_A (x), γ_A (-y)}
= max {γ_A (x), γ_A (y)}
= max {s, s} = s
⟹ γ_A (x – y) ≤ s
⟹ x – y ϵ L (γ_A, s).
⟹ L (γ_A, s) is a subgroup of (G, +)

Let g ϵ G. Consider
γ_A (g + x – g) ≤ γ_A (x) ≤ s
⟹ g + x – g ϵ L (γ_A, s)
⟹ L (γ_A, s) is a normal subgroup of (G , +).

Let m ϵ M and α ϵ Γ.

Consider γ_A (m α (g + x) – m α g) ≤ γ_A (x) ≤s.
⟹ m α (g + x) – m α g ϵ L (γ_A, s)
⟹ L (γ_A, s) is an intuitionistic fuzzy ideal of the MΓ group G.

2 ⟹ 1 Assume U (μ_A, t) and L (γ_A, t) are the ideals of the MΓ group of the group G.

Let x, y ϵ U (μ_A, t) ⟹ μ_A (x) ≥ t and μ_A (y) ≥ t

With μ_A (x – y) ≥ as U_A (μA, t) is an intuitionistic fuzzy ideal.

As x ϵ U (μA, t)
⟹ g + x – g ϵ U (μA / t) for all g ϵ G. Since U (μA (t)) is a normal subgroup.
⟹ μ_A (g + x – g) ≥ t.

To prove A is intuitionistic fuzzy let us check all the conditions of definition 2.

(*) Let x, y ϵ U (μ_A, t) such that
μ_A (x – y) < min {μA (x), μ_A (y)}

Take t = [μ (x – y) + min {μ_A (x), μ_A (y)}] / 2

Then 0 ≤ μ (x – y) < t < min {μA (x), μ (y)} ≤ 1
⟹ for x, y ϵ U (μ_A, t) which is a contradiction to U (μ_A, t) is an ideal.

Therefore μ_A (x - y) ≥ t
⟹ μ_A (x + y) = μ_A (x – (- y))
= min {μ_A (x), μ_A (- y)}
= min {μ_A (x), μ_A (y)}
≥ min {t, t} = t
⟹ μ_A (x + y) ≥ t -------------> **(i)**

(**) Let x, y ϵ G such that μ_A (y + x – y) < μ_A (x)

Take t = [μ_A (y + x – y) + α_A (x)] / 2
⟹ 0 ≤ μ_A (y + x – y) < t < μ_A (x) ≤ 1
⟹ for x, y ϵ U (μA, t), y + x – y does not belong to U (μ_A, t) which is a contradiction to U (μA, t) is an ideal
⟹ μ_A (y + x – y) ≥ μ (x) ------------> **(ii)**

(***) Let y ϵ G and x ϵ U (μA, t) such that
μ_A (xy) < μ_A (x)

Take t = [μ_A (xy) + μ_A (x)] / 2
⟹ 0 ≤ μ_A (x) < t < μ_A (xy) ≤ 1
⟹ for x ϵ U (μ_A, t), xy does not belongs to U (μ_A, t) which is a contradiction to U (μA, t) is an ideal.

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Therefore μ_A (xy) > μ_A (x) ------------> **(iii)**

(****) Let x, y, z ϵ G such that

μ_A [(x + z) y – xy] < μ_A (z)

Take t = {μ_A [(x + z) y – xy] + μ_A (z)} / 2

⟹ $0 \leq \mu_A$ [(x + z) y – xy] < t < μ_A (z) ≤ 1

⟹ for z ϵ U (μ_A, t) but (x + z) y – xy does not belongs to U (μ_A, t) which is a contradiction to U (μ_A, t) is an ideal.

⟹ μ_A [(x + z) z – xy] ≥ μ_A (z) ----------> **(iv)**

From **(i), (ii), (iii), (iv)** μ_A satisfies first four conditions of definition 2.

Similarly, γ_A satisfies the last four conditions of definition 2.

Hence A is an intuitionistic fuzzy ideal of the M Γ group.

Proposition 3.12: Let H be an intuitionistic fuzzy ideal of the MΓ group G and A (α_1, β_1) and B (α_2, β_2) are the two-level ideals of H. Then the following two conditions are equivalent.

a. A (α_1, β_1) = B (α_2, β_2) and
b. There is no x ϵ G such that $\alpha_1 < \mu$ (x) < α_2 and $\beta_1 > \gamma$ (x) > β_2.

Proof:

a ⟹ b

let us assume there exists an element x ϵ G such that

$\alpha_1 < \mu$ (x) < α_2 and $\beta_1 > \gamma$ (x) > β_2

⟹ μ (x) > α_1 and γ (x) < β_1 and μ (x) < α_2 and γ (x) > β_2

⟹ x ϵ A (α_1 , β_1) and x does not belongs to B (α_2 , β_2) which is a contradiction to (i). Hence our assumption is wrong and so there is no x ϵ G such that α1 < μ (x) < α_2 and $\beta_1 > \gamma$ (x) > β_2.

b ⟹ a

Let x ϵ B (α_2, β_2)

⟹ μ (x) > α_2 and γ (x) < β_2

Since $\alpha_2 > \alpha_1$ and $\beta_2 < \beta_1$ we have μ (x) > α_1 and γ (x) < β_1

⟹ x ϵ B (α_1, β_1)

Hence B (α_2, β_2) ⊂ B (α_1, β_1) ---------> **I**

Let x ϵ B (α_1, β_1) ⟹ μ (x) > α_1 and γ (x) < β_1

By our assumption there is no x such that

$\alpha_1 < \mu$ (x) < α_2 and $\beta_1 > \gamma$ (x) > β_2

⟹ μ (x) does not less than α_2 and γ (x) does not greater than β_2.

⟹ μ (x) > α_2 and γ (x) < β_2

⟹ x ϵ B (α_2, β_2)

And so, on B (α_1, β_1) ⊂ B (α_2, β_2) ----------> **II**

From **I & II**

B (α_1, β_1) = B (α_2, β_2) and hence (ii) ⟹ (i)

Hence the Proof is completed.

CONCLUSION

We defined Intuitionistic Fuzzy Ideals of M- Γ Groups by extending its Fuzzy Ideals and discussed equivalent conditions on Intuitionistic Fuzzy Γ Groups. We have also explained the properties and illustrated an example to prove Intuitionistic Fuzzy Ideals of M- Γ Groups.

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