Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 185-189
Available online through www.ijma.info ISSN 2229 – 5046

CUT SETS, DISTANCE, AND SIMILARITY MEASURES ON TYPE-2 INTUITIONISTIC FUZZY SET

M. CLEMENT JOE ANAND[1]
Assistant Professor (S.G.), Department of Mathematics,
Hindustan Institute of Technology & Science (Deemed to be University),
Padur, Chennai-603103, Tamil Nadu, India.

JANANI BHARATRAJ[2]
Research Scholar, Department of Mathematics,
Hindustan Institute of Technology & Science (Deemed to be University),
Padur, Chennai-603103, Tamil Nadu, India.

E-mail: [1]arjoemi@yahoo.com & [2]jananichari@gmail.com

ABSTRACT

Intuitionistic Fuzzy Sets are substantial extensions of fuzzy sets which plays a key factor in describing and providing ease of solving higher complexities in engineering and science. However, certain ambiguous situations cannot be addressed by using fuzzy sets and intuitionistic fuzzy sets. The extension of fuzzy set, namely, Type-2 Fuzzy Sets and Interval Type-2 Fuzzy Sets paved the way for implementing methods and techniques for unanswered problems. In this research, an attempt has been made to represent Type-2 Intuitionistic Fuzzy Set in a refined manner. The concept of cut sets has been modified to suit the requirement of a type-2 intuitionistic fuzzy set and distance formulae namely Euclidean, Normalized Euclidean, Chebyshev, Normalized Chebyshev, Manhattan, Normalized Manhattan, Minkowski, Canberra and Sorensen distances have been redefined for a Type-2 Intuitionistic Fuzzy Set (T2IFS). Also, basic operators and a cosine similarity measure have been described for a T2IFS.

Keywords: *Cosine similarity, Cut-planes, Distance Formulae, Intuitionistic Fuzzy Set, Type-2 Fuzzy Set.*

Mathematics Subject Classification Code: *03E72.*

1. INTRODUCTION

Fuzzy Sets as introduced by Zadeh (1965) has seen a wide spread application in all field of science, engineering, and technology. The inclusion of degree of non-membership and degree of hesitancy further deepened the usability of fuzzy sets. Atanassov (1986) introduced the other two degrees and named the extension as Intuitionistic Fuzzy Set, which dealt with hesitant values of variables. Furthermore, Fuzzy Sets were developed into type-2 fuzzy sets which addressed problems that were little cumbersome while solving using standard fuzzy sets. The book on “Uncertain Rule-based Fuzzy Logic Systems and other researches” by Mendel (2001), provides discussions regarding theoretical and practical application part of type-2 fuzzy sets. Mendel et. al. (2006) have also studied advanced operations in type-2 fuzzy sets. Having seen the success of extensions of fuzzy sets, the question on the structure of Type-2 Intuitionistic Fuzzy Sets arises. Various researchers from across the globe have tried their hand on describing Type-2 Intuitionistic Fuzzy Sets and studied basic operations on T2IFS. Type-2 intuitionistic fuzzy sets were first studied by Zhao and Xiao (2012). A clear finding of T2IFS was given in their research paper which concluded that the T2IFS are basically generalizations of fuzzy sets, interval fuzzy sets, type-2 fuzzy sets and interval-valued intuitionistic fuzzy sets. Few basic operations on type-2 relations have been discussed and an approximate reasoning has been described based on T2IFS. Cuong et al. (2012) described T2IFS using logical operators like OR, AND, NEGATION, and described their properties. Nghiem et al. (2013) proposed a method for image thresholding based on T2IFS. Jana (2016) studied some arithmetic operations on type-2 trapezoidal intuitionistic fuzzy sets and the concept was applied to solve transportation problem. Singh and Garg (2017) gave some distance measures on T2IFS, with applications to multicriteria decision making problems. Based on the review of literature taken, it was found that the research on t2ifs is still in the nascent stage. The theory of cut sets has a vital role to play while describing fuzzy and intuitionistic fuzzy sets. While the researchers have

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

contributed to distance metrics and logical operators, the role of cut sets in defuzzification techniques and more importantly, the identity of the T2IFS which forms the basis for defining degrees of membership, non-membership and hesitancy have not been considered. Also, the distance metrics which play a major role in ranking and decision-making problems have been studied without considering the hesitancy degree. In this present study, an analysis of cut sets mapped into a cut-plane has been described. Also, two basic operators and modal operators have been described in the study. Furthermore, distance metrics like Euclidean, Normalized Euclidean, Chebyshev, Normalized Chebyshev, Manhattan, Normalized Manhattan, Minkowski, Canberra and Sorensen, between two type-2 IFS and cosine similarity measure have been defined as an icing to the study of T2IFS.

2. TYPE-2 INTUITIONISTIC FUZZY SETS OF A CERTAIN LEVEL

In this section, T2IFS is defined and cut sets for T2IFS are discussed. α-cuts play an important role in bridging fuzzy sets with crisp sets. Similarly, (α,β)-cuts connect intuitionistic fuzzy sets with crisp sets. One of the main aims of introducing cut sets apart from defuzzification, is to identify the membership, non-membership, and hesitancy degrees. These cut sets help in ranking of fuzzy sets and intuitionistic fuzzy sets in decision making problems. Also, cut sets are very important in defining total ordering on the class of intuitionistic fuzzy numbers. Hence it becomes mandatory to define cut sets for T2IFS.

Definition 2.1: Let X be a non-empty set. Then, $\tilde{F}$, a subset of X is said to be a Type-2 Intuitionistic Fuzzy Set if $\tilde{F}: X \to Map(E,[0,1]) \times Map(E,[0,1])$ where $E = \{(u,v)\epsilon[0,1]\times[0,1]: u+v \le 1\}$. The degrees of membership, non-membership and hesitancy are given as follows.

$$\mu_{\tilde{F}}: X \times X \to [0,1]\times[0,1], \nu_{\tilde{F}}: X \times X \to [0,1]\times[0,1], \pi_{\tilde{F}}: X \times X \to [0,1]\times[0,1]$$

Such that $\mu_{\tilde{F}}(u,v) + \nu_{\tilde{F}}(u,v) + \pi_{\tilde{F}}(u,v) = (1,1)$ with $\mu_{\tilde{F}}(u) + \nu_{\tilde{F}}(u) + \pi_{\tilde{F}}(u) = 1$ and

$$\mu_{\tilde{F}}(v) + \nu_{\tilde{F}}(v) + \pi_{\tilde{F}}(v) = 1.$$

2,1. Cut Sets on T2IFS

The theory of cut sets has played a major role in bridging the gap between fuzzy sets and crisp sets. Researchers have worked on describing single cut set to multi-level cut sets for fuzzy sets and intuitionistic fuzzy sets and have been successful in proving decomposition and representation theorems. Many applications related to medical diagnosis have also been achieved. In case of Type-2 Intuitionistic Fuzzy Sets, we cannot obtain a single straight line as a cut set. Here, the cut sets transform into cut-planes which decomposes the type-2 intuitionistic fuzzy set to an intuitionistic fuzzy set. The theory of cut-planes is given as follows.

Definition 2.2: Single cut-plane for T2IFS.
Let $\tilde{F}$ be a T2IFS and let $\alpha,\beta,\gamma,\delta\epsilon[0,1]$ such that $(\alpha,\gamma)\epsilon[0,1]\times[0,1]$ and $(\beta,\delta)\epsilon[0,1]\times[0,1]$. Then the single cut-plane for the membership and non-membership functions is given by $N^{T2IFS}_{(\alpha,\gamma),(\beta,\delta)} = \{(f_\mu(u,v), g_\nu(u,v))/f_\mu(u,v) \ge (\alpha,\gamma) \text{ and } g_\nu(u,v) \le (\beta,\delta)\}$. Clearly, each set is an intuitionistic fuzzy set and the cut-plane decomposes T2IFS to IFS.

Definition 2.3: Interval Cut-Planes with matrix representation of values.
Given a T2IFS $\tilde{F}$, we can define boundaries for the membership function and non-membership function by confining them between two planes respectively. Such a cut-plane can be called Interval Cut-Plane, given by;

$$I = \{(f_\mu(u,v), g_\nu(u,v))/(\varepsilon,\zeta) \ge f_\mu(u,v) \ge (\alpha,\gamma) \text{ and } (\eta,\tau) \le g_\nu(u,v) \le (\beta,\delta)\}$$

Thus, the planes get sandwiched between two planes and required results can be obtained using Interval Cut-Planes.
The values obtained using this technique can be represented in the form of matrix, which enables users to access multiple values at the same time. [1,2]

3. OPERATORS ON TYPE-2 INTUITIONISTIC FUZZY SET

Operators play a major role in manipulating any set. They mold the fuzzy sets into a user-friendly set and allow various other operations on them. The following operators have been modified to suit the needs of a Type-2 Intuitionistic Fuzzy Set.

Operator $D_{(\alpha,\gamma)}$: Given a T2IFS $\tilde{F}$, an operator $D_{(\alpha,\gamma)}$ is defined as,

$$D_{(\alpha,\gamma)} = [\mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v), \nu_{\tilde{F}}(u,v) + (1-\alpha,1-\gamma).\pi_{\tilde{F}}(u,v)]$$

Upon simplifying the above operator, it is evident that $D_{(\alpha,\gamma)}$ is also a T2IFS.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Operator $F_{(\alpha,\gamma),(\beta,\delta)}$ ***:*** Given a T2IFS $\tilde{F}$, an operator $F_{(\alpha,\gamma),(\beta,\delta)}$ is defined as,

$$F_{(\alpha,\gamma),(\beta,\delta)} = [[\mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v), \nu_{\tilde{F}}(u,v) + (\beta,\delta).\pi_{\tilde{F}}(u,v)]$$

Theorem 3.1: For every T2IFS $\tilde{F}$ and for every $\alpha, \beta, \gamma, \delta \epsilon [0,1]$, such that $\alpha + \gamma \leq 1, \beta + \delta \leq 1$

i) The Operator $F_{(\alpha,\gamma),(\beta,\delta)}$ is a T2IFS.

ii) $D_{(\alpha,\gamma)} = F_{(\alpha,\gamma),(1-\alpha,1-\gamma)}$

Proof:

(i) Consider $F_{(\alpha,\gamma),(\beta,\delta)} = [[\mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v), \nu_{\tilde{F}}(u,v) + (\beta,\delta).\pi_{\tilde{F}}(u,v)]$

$$F_{(\alpha,\gamma),(\beta,\delta)} = \mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v) + \nu_{\tilde{F}}(u,v) + (\beta,\delta).\pi_{\tilde{F}}(u,v)$$

$$F_{(\alpha,\gamma),(\beta,\delta)} = \mu_{\tilde{F}}(u,v) + (\alpha.\pi_{\tilde{F}}(u,v), \gamma.\pi_{\tilde{F}}(u,v)) + \nu_{\tilde{F}}(u,v) + (\beta.\pi_{\tilde{F}}(u,v), \delta.\pi_{\tilde{F}}(u,v))$$

$$F_{(\alpha,\gamma),(\beta,\delta)} = (1,1).$$

Thus $F_{(\alpha,\gamma),(\beta,\delta)}$ is a T2IFS.

(ii) Consider $F_{(\alpha,\gamma),(\beta,\delta)} = [[\mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v), \nu_{\tilde{F}}(u,v) + (\beta,\delta).\pi_{\tilde{F}}(u,v)]$

Replace $(\beta,\delta) = (1-\alpha, 1-\gamma)$.

$$F_{(\alpha,\gamma),(1-\alpha,1-\gamma)} = [[\mu_{\tilde{F}}(u,v) + (\alpha,\gamma).\pi_{\tilde{F}}(u,v), \nu_{\tilde{F}}(u,v) + (1-\alpha,1-\gamma).\pi_{\tilde{F}}(u,v)]$$

$$F_{(\alpha,\gamma),(1-\alpha,1-\gamma)} = D_{(\alpha,\gamma)}.$$

Hence proved.

4. DISTANCES OVER TYPE-2 INTUITIONISTIC FUZZY SET

The study of T2IFS is never complete without the distance measures. The distance measures under any number system helps in ranking of alternatives thereby helping in decision making scenarios. In this section, we have defined Minkowski distance, Canberra distance and Sorensen distance apart from Euclidean and Manhattan distances. The following definitions include the degree of hesitancy which many researchers do not consider while calculating distances. The inclusion of hesitancy gives a wholesome outlook of the distance measures.

Let $\widetilde{F_1}$ and $\widetilde{F_2}$ be two Type-2 Intuitionstic Fuzzy Sets. Then,

- The Euclidean distance is given by,

$$d(\widetilde{F_1}, \widetilde{F_2}) = \sqrt{(u_1 - u_2)^2 + (v_1 - v_2)^2 + (M)^2 + (C)^2 + (J)^2 + (A)^2 + (N)^2 + (I)^2}$$

where, M $= \mu_{\widetilde{F_1}}(u_1) - \mu_{\widetilde{F_2}}(u_2)$, $C = \mu_{\widetilde{F_1}}(v_1) - \mu_{\widetilde{F_2}}(v_2)$, $J = \nu_{\widetilde{F_1}}(u_1) - \nu_{\widetilde{F_2}}(u_2)$,
$A = \nu_{\widetilde{F_1}}(v_1) - \nu_{\widetilde{F_2}}(v_2)$.
$N = \pi_{\widetilde{F_1}}(u_1) - \pi_{\widetilde{F_2}}(u_2)$ and $I = \pi_{\widetilde{F_1}}(v_1) - \pi_{\widetilde{F_2}}(v_2)$ respectively.

- The Normalized Euclidean distance is given by,

$$d_4(\widetilde{F_1}, \widetilde{F_2}) = \Bigg[\frac{1}{4n}\sum_{i=1}^{n} |u_1(x_i) - u_2(x_i)|^2 + |v_1(x_i) - v_2(x_i)|^2 + |\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))|^2 + |\mu_{\widetilde{F_1}}(v_1(x_i)) - \mu_{\widetilde{F_2}}(v_2(x_i))|^2 + |\nu_{\widetilde{F_1}}(u_1(x_i)) - \nu_{\widetilde{F_2}}(u_2(x_i))|^2 + |\nu_{\widetilde{F_1}}(v_1(x_i)) - \nu_{\widetilde{F_2}}(v_2(x_i))|^2 + |\pi_{\widetilde{F_1}}(u_1(x_i)) - \pi_{\widetilde{F_2}}(u_2(x_i))|^2 + |\pi_{\widetilde{F_1}}(v_1(x_i)) - \pi_{\widetilde{F_2}}(v_2(x_i))|^2\Bigg]^{\frac{1}{2}}$$

- The Chebyshev distance between two T2IFS is given by,

$$D_{(\widetilde{F_1},\widetilde{F_2})} = max\big(|u_1 - u_2|, |v_1 - v_2|, |\mu_{\widetilde{F_1}}(u_1) - \mu_{\widetilde{F_2}}(u_2)|, |\mu_{\widetilde{F_1}}(v_1) - \mu_{\widetilde{F_2}}(v_2)|, |\nu_{\widetilde{F_1}}(u_1 - \nu_{\widetilde{F_2}}(u_2)|, |\nu_{\widetilde{F_1}}(v_1) - \nu_{\widetilde{F_2}}(v_2)|, |\pi_{\widetilde{F_1}}(u_1) - \pi_{\widetilde{F_2}}(u_2)|, |\pi_{\widetilde{F_1}}(v_1) - \pi_{\widetilde{F_2}}(v_2)|\big)$$

- The Hamming distance between two T2IFS is given by,

$$d_1(\widetilde{F_1}, \widetilde{F_2}) = \frac{1}{4}\sum_{i=1}^{n} |u_1(x_i) - u_2(x_i)| + |v_1(x_i) - v_2(x_i)| + |\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))| + |\mu_{\widetilde{F_1}}(v_1(x_i)) - \mu_{\widetilde{F_2}}(v_2(x_i))| + |\nu_{\widetilde{F_1}}(u_1(x_i)) - \nu_{\widetilde{F_2}}(u_2(x_i))| + |\nu_{\widetilde{F_1}}(v_1(x_i)) - \nu_{\widetilde{F_2}}(v_2(x_i))| + |\pi_{\widetilde{F_1}}(u_1(x_i)) - \pi_{\widetilde{F_2}}(u_2(x_i))| + |\pi_{\widetilde{F_1}}(v_1(x_i)) - \pi_{\widetilde{F_2}}(v_2(x_i))|$$

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

- The Normalized Hamming distance is given by

$$d_2(\widetilde{F_1},\widetilde{F_2}) = \frac{1}{4n}\sum_{i=1}^{n} |u_1(x_i) - u_2(x_i)| + |v_1(x_i) - v_2(x_i)| + |\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))| + |\mu_{\widetilde{F_1}}(v_1(x_i)) - \mu_{\widetilde{F_2}}(v_2(x_i))| + |\nu_{\widetilde{F_1}}(u_1(x_i)) - \nu_{\widetilde{F_2}}(u_2(x_i))| + |\nu_{\widetilde{F_1}}(v_1(x_i)) - \nu_{\widetilde{F_2}}(v_2(x_i))| + |\pi_{\widetilde{F_1}}(u_1(x_i)) - \pi_{\widetilde{F_2}}(u_2(x_i))| + |\pi_{\widetilde{F_1}}(v_1(x_i)) - \pi_{\widetilde{F_2}}(v_2(x_i))|$$

- The Minkowski distance is given by,

$$d_3(\widetilde{F_1},\widetilde{F_2}) = \left[\sum_{i=1}^{n} |u_1(x_i) - u_2(x_i)|^p + |v_1(x_i) - v_2(x_i)|^p + |\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))|^p + |\mu_{\widetilde{F_1}}(v_1(x_i)) - \mu_{\widetilde{F_2}}(v_2(x_i))|^p + |\nu_{\widetilde{F_1}}(u_1(x_i)) - \nu_{\widetilde{F_2}}(u_2(x_i))|^p + |\nu_{\widetilde{F_1}}(v_1(x_i)) - \nu_{\widetilde{F_2}}(v_2(x_i))|^p + |\pi_{\widetilde{F_1}}(u_1(x_i)) - \pi_{\widetilde{F_2}}(u_2(x_i))|^p + |\pi_{\widetilde{F_1}}(v_1(x_i)) - \pi_{\widetilde{F_2}}(v_2(x_i))|^p\right]^{\frac{1}{p}}$$

- The Canberra distance is given by,

$$d_5(\widetilde{F_1},\widetilde{F_2}) = \left[\sum_{i=1}^{n} \frac{|u_1(x_i) - u_2(x_i)|}{|u_1(x_i)| + |u_2(x_i)|} + \frac{|v_1(x_i) - v_2(x_i)|}{|v_1(x_i)| + |v_2(x_i)|} + \frac{|\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))|}{|\mu_{\widetilde{F_1}}(u_1(x_i))| + |\mu_{\widetilde{F_2}}(u_2(x_i))|} + \frac{|\mu_{\widetilde{F_1}}(u_1(x_i)) - \mu_{\widetilde{F_2}}(u_2(x_i))|}{|\mu_{\widetilde{F_1}}(u_1(x_i))| + |\mu_{\widetilde{F_2}}(u_2(x_i))|} + \frac{|\nu_{\widetilde{F_1}}(u_1(x_i)) - \nu_{\widetilde{F_2}}(u_2(x_i))|}{|\nu_{\widetilde{F_1}}(u_1(x_i)| + |\nu_{\widetilde{F_2}}(u_2(x_i))|} + \frac{|\nu_{\widetilde{F_1}}(v_1(x_i)) - \nu_{\widetilde{F_2}}(v_2(x_i))|}{|\nu_{\widetilde{F_1}}(v_1(x_i))| + |\nu_{\widetilde{F_2}}(v_2(x_i))|} + \frac{|\pi_{\widetilde{F_1}}(u_1(x_i)) - \pi_{\widetilde{F_2}}(u_2(x_i))|}{|\pi_{\widetilde{F_1}}(u_1(x_i))| + |\pi_{\widetilde{F_2}}(u_2(x_i))|} + \frac{|\pi_{\widetilde{F_1}}(v_1(x_i)) - \pi_{\widetilde{F_2}}(v_2(x_i))|}{|\pi_{\widetilde{F_1}}(v_1(x_i))| + |\pi_{\widetilde{F_2}}(v_2(x_i))|}\right]$$

- The Sorensen Distance is given by,

$$d_6(\widetilde{F_1},\widetilde{F_2}) = \frac{\sum_{i=1}^{n} |u_1(x_i) - u_2(x_i| + |v_1(x_i) - v_2(x_i)| + |M| + |C| + |J| + |A| + |N| + |I||}{\sum_{i=1}^{n} |u_1(x_i) + (x_i| + |v_1(x_i) + (x_i)| + |M1| + |C1| + |J1| + |A1| + |N1| + |I1|}$$

Here, M $= \mu_{\widetilde{F_1}}(u_1) - \mu_{\widetilde{F_2}}(u_2),\ C = \mu_{\widetilde{F_1}}(v_1) - \mu_{\widetilde{F_2}}(v_2),\ J = \nu_{\widetilde{F_1}}(u_1) - \nu_{\widetilde{F_2}}(u_2)$,
$A = \nu_{\widetilde{F_1}}(v_1) - \nu_{\widetilde{F_2}}(v_2),\ N = \pi_{\widetilde{F_1}}(u_1) - \pi_{\widetilde{F_2}}(u_2)$ and $I = \pi_{\widetilde{F_1}}(v_1) - \pi_{\widetilde{F_2}}(v_2)$ respectively and,
M1 $= \mu_{\widetilde{F_1}}(u_1) + \mu_{\widetilde{F_2}}(u_2), C1 = \mu_{\widetilde{F_1}}(v_1) + \mu_{\widetilde{F_2}}(v_2), J1 = \nu_{\widetilde{F_1}}(u_1) + \nu_{\widetilde{F_2}}(u_2)$,
$A1 = \nu_{\widetilde{F_1}}(v_1) + \nu_{\widetilde{F_2}}(v_2), N1 = \pi_{\widetilde{F_1}}(u_1) + \pi_{\widetilde{F_2}}(u_2)$ and $I1 = \pi_{\widetilde{F_1}}(v_1) + \pi_{\widetilde{F_2}}(v_2)$ respectively.

- The Cosine Similarity Measure on two T2IFS is given by,

$$C_{T2IFS}(\widetilde{F_1},\widetilde{F_2}) = \frac{1}{n}\sum_{i=1}^{n}\frac{S}{P}$$

where,

$$S = u_1(x_i).u_2(x_i) + v_1(x_i).v_2(x_i) + \mu_{\widetilde{F_1}}(u_1(x_i)).\mu_{\widetilde{F_2}}(u_2(x_i)) + \mu_{\widetilde{F_1}}(v_1(x_i)).\mu_{\widetilde{F_2}}(v_2(x_i)) + \nu_{\widetilde{F_1}}(u_1(x_i)).\nu_{\widetilde{F_2}}(u_2(x_i)) + \nu_{\widetilde{F_1}}(v_1(x_i)).\nu_{\widetilde{F_2}}(v_2(x_i)) + \pi_{\widetilde{F_1}}(u_1(x_i)).\pi_{\widetilde{F_2}}(u_2(x_i)) + \pi_{\widetilde{F_1}}(v_1(x_i)).\pi_{\widetilde{F_2}}(v_2(x_i))$$

$$P = \sqrt{{u_1}^2(x_i) + {v_1}^2(x_i)} \times \sqrt{{u_2}^2(x_i) + {v_2}^2(x_i)} \times \sqrt{\mu_{\widetilde{F_1}}(u_1(x_i))^2 + \nu_{\widetilde{F_1}}(u_1(x_i))^2 + \pi_{\widetilde{F_1}}(u_1(x_i))^2} \times \sqrt{\mu_{\widetilde{F_2}}(u_2(x_i))^2 + \nu_{\widetilde{F_2}}(u_2(x_i))^2 + \pi_{\widetilde{F_2}}(u_2(x_i))^2} \times \sqrt{\mu_{\widetilde{F_1}}(v_1(x_i))^2 + \nu_{\widetilde{F_1}}(v_1(x_i))^2 + \pi_{\widetilde{F_1}}(v_1(x_i))^2} \times \sqrt{\mu_{\widetilde{F_2}}(v_2(x_i))^2 + \nu_{\widetilde{F_2}}(v_2(x_i))^2 + \pi_{\widetilde{F_2}}(v_2(x_i))^2}$$

5. CONCLUSION

The present works is aimed at developing more operators and study their properties when an intuitionistic fuzzy set is extended into a type-2 intuitionistic fuzzy set. In this paper, we have successfully extended cut sets into cut-planes and described interval cut-planes too. We have formulated the distance formulae including Euclidean, Normalized Euclidean, Chebyshev, Normalized Chebyshev, Manhattan, Normalized Manhattan, Minkowski, Canberra and Sorensen distances have been redefined for a Type-2 Intuitionistic Fuzzy Set (T2IFS) using the degree of hesitancy.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Further, cosine similarity measure has been formulated for type-2 intuitionistic fuzzy set. This research also paves the way for more operators to be redefined for T2IFS. Using this as a stepping stone, decomposition and representation theorems can be developed.

6. ACKNOWLEDGMENT

The authors sincerely thank the anonymous reviewer for his/her suggestions in improving the content of the research work.

REFERENCES

1. Anand, M.C.J. and Bharatraj, J., A New Alpha, Beta, Gamma, Delta -cuts in Trapezoidal Fuzzy Number and its Approach in Decomposition Theorem, Global Journal of Pure and Applied Mathematics, 12(2016), 875-877.
2. Anand, M.C.J. and Bharatraj, J., Cut Sets on Trapezoidal Fuzzy Number and Intuitionistic Fuzzy Number: A new Perspective, International Journal of Pure and Applied Mathematics, 112(2017), 749-762.
3. Anand, M.C.J. and Bharatraj, J., Theory of Triangular Fuzzy Number, Proceedings of National Conference on Advanced Trends in Mathematics, (2017).
4. Anand, M.C.J. and Lathamaheswari, M., A New Generalized Normal Type-2 Trapezoidal Fuzzy Number, Global Journal of Pure and Applied Mathematics, 12(2016), 565-574.
5. Atanassov, K.T., Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1986), 87-96.
6. Atanassov, K. T., New Operations defined on intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 61(1994), 137-142.
7. Atanassov, K.T., Intuitionistic Fuzzy Sets, Springer-Verlag, Heidelberg, (1999).
8. Atanassov, K.T. On Intuitionistic Fuzzy Sets Theory, Springer, Berlin, Heidelberg, (2012).
9. Atanassov, K.T., Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(2006), 87-96.
10. Coung, B.C., Anh, T.H. and Hai, B.D., Some Operations on Type-2 Intuitionistic Fuzzy sets, Journal of Computer Science and Cybernetics, 28(2012), 274-283.
11. Jana, D.K., Novel Arithmetic Operations on Type-2 Intuitionistic Fuzzy and its application to transportation problem, Pacific Science Review A: Natural Science and Engineering, 18(2016), 178-189.
12. Karnik, N. and Mendel, J.M., Operations on type-2 fuzzy sets, Fuzzy Sets and Systems, 122(2001), 327-348.
13. Mendel, J.M., Uncertain rule-based fuzzy logic systems, Upper Saddle River, NJ, Prentice-hall, (2001).
14. Mendel, J.M., Rohn, R.I. and Liu, F., Interval Type-2 fuzzy logic systems made simple, IEEE Transactions on Fuzzy Systems, 14(2006), 808-821.
15. Mendel, J.M., Advances in type-2 fuzzy sets and systems, Information Science, 177(2007), 84-110.
16. Nghiem, T.V., Nguyen, D.D., and Ngo, L.T., Intuitionistic Type-2 Fuzzy Set approach to Image Thresholding, SoCPaR, (2013).
17. Singh, S. and Garg, H., Distance measures between type-2 intuitionistic fuzzy sets and their applications to mcdm process, Application Intelligence, 46(2017), 788-799.
18. Ye, J., Cosine Similarity measures for Intuitionistic fuzzy sets and their applications, Mathematical and Computer Modelling, 53(2011), 91-97.
19. Zadeh, L.A., Fuzzy Sets, Information and Control, 8(1965), 338-353.
20. Zhao, T. and Xiao J., Type-2 intuitionistic fuzzy sets, Control Theory & Applications, 29(2012), 1215-1222.

Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.