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# SECURE EDGE DOMINATION AND VERTEX EDGE DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

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# ABSTRACT

In this paper, we introduce the concept of secure edge domination and inverse secure edge domination of an intuitionistic fuzzy graph (IFG). The relationship between edge domination number  $\gamma'_{se}(G)$  and inverse secure edge domination number  $\gamma^{-1}_{se}(G)$  is studied. Also, we introduce a total secure edge dominating set, inverse total secure edge dominating set and vertex edge dominating set in intuitionistic fuzzy graph with suitable illustrations.

# 2010 Mathematics Subject Classification: 05C72, 03E72, 03F55.

**Keywords:** Intuitionistic Fuzzy Graphs: secure edge dominating set, inverse secure edge dominating set, total secure edge dominating set, secure edge domination number, inverse secure edge domination number, vertex edge dominating set.

# 1. INTRODUCTION

The study of dominating set in graph was introduced by Ore and Berge in 1962. The edge domination was introduced by S.Arumugam and Velammal [1]. The edge dominating set has many application in resource allocation, network routing and encoding theory problems [2]. A. Somasundaram and S.Somasundaram [9] introduced the concept of domination in fuzzy graphs and obtained several bounds for domination number. V.R.Kulli [5] introduced the concept of secure edge domination in fuzzy graphs. Vinod Kumar and Geetharamani [10] introduced the concept of vertex edge domination in fuzzy graph and analyzed its operation on fuzzy graphs. M.G.Karunambigai *et al.* [4] introduced secure domination set, secure total dominating set, 2-secure dominating set and its domination number in intuitionistic fuzzy graphs. Research work of several investigation [3, 5, 7, 8] have motivated us to develop the secure edge domination and vertex in intuitionistic fuzzy graphs.

This paper organized as follows. The basic definition and theorems needed for this study discussed in section 2. Section 3 deals with secure edge domination and inverse secure edge domination in IFG. In section 4 we have introduced the concept of vertex edge domination in intuitionistic fuzzy graphs.

# 2. PRELIMINARIES

**Definition 2.1:** [6] An Intuitionistic Fuzzy Graph (IFG) is of the form G = (V, E) where

- (*i*).  $V = \{v_1, v_2, ..., v_n\}$  such that  $\mu_1: V \to [0,1]$  and  $\gamma_1: V \to [0,1]$ , denotes the degree of membership and non-membership of the element  $v_j \in V$  respectively and  $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1, \forall v_i \in V$ ,
- (ii).  $E \subseteq V \times V$  where  $\mu_1: V \times V \rightarrow [0,1]$  and  $\gamma_1: V \times V \rightarrow [0,1]$  are such that  $\mu_2(v_i, v_j) \leq \min(\mu_1(v_i), \mu_1(v_j))$  and  $\gamma_2(v_i, v_j) \leq \max(\gamma_1(v_i), \gamma_1(v_j))$

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denotes the degree of membership and non-membership of an edge  $(v_i, v_j) \in E$  respectively, where,  $0 \le \mu_2(v_i, v_j) + \gamma_1(v_i, v_j) \le 1$ , for every  $(v_i, v_j) \in E$ .

For each intuitionistic fuzzy graph G, the degree of hesitance (hesitation degree) of a vertex  $v_i \in V$  in G is  $\pi_1(v_i) = 1 - \mu_1(v_i) - \nu_1(v_i)$  and the degree of hesitance (hesitation degree) of an edge  $e_{ij} = (v_i, v_j) \in E$  in G is  $\pi_2(e_{ij}) = 1 - \mu_2(e_{ij}) - \nu_2(e_{ij})$ .

**Definition 2.2:** [7] An IFG, G = (V, E) is said to be complete IFG if  $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$  and  $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$  for every  $v_i, v_j \in V$ .

**Definition 2.3:** [7] An IFG, G = (V, E) is said to be strong IFG if  $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$  and  $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$  for every  $(v_i, v_i) \in E$ .

**Definition 2.4:** [8] An intuitionistic fuzzy graph G = (V, E) is said to be a  $(K_1, K_2)$  -regular if  $d_G(v_i) = (K_1, K_2)$  for all  $v_i \in V$  and also G is said to be a regular intuitionistic fuzzy graph of degree  $(K_1, K_2)$ .

**Definition 2.5:** [8] Let *u* be a vertex in an IFG G = (V, E) then  $N(u) = \{v : v \in V \text{ and } (u, v) \text{ is strong edge} \}$  is called neibourhood of *u*.

**Definition 2.6:** [8] An edge (u, v) is said to be a strong edge if  $\mu_2(u, v) \ge \mu_2^{\infty}(u, v)$  and  $\nu_2(u, v) \ge \nu_2^{\infty}(u, v)$ .

**Definition 2.7:** [8] If  $v_i, v_j \in V \subseteq G$ , the  $\mu$  -strength of connectedness between  $v_i$  and  $v_j$  is  $\mu_2^{\infty}(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) | k = 1, 2, ...n\}$  and  $\nu$  -strength of connectedness between  $v_i$  and  $v_j$  is  $v_2^{\infty}(v_i, v_j) = \inf\{v_2^k(v_i, v_j) | k = 1, 2, ...n\}$ . If u, v are connected by means of paths of length k then  $\mu_2^k(u, v)$  is defined as  $\sup\{\mu_2(u, v_1) \land \mu_2(v_1, v_2) \land \mu_2(v_2, v_3) \dots \land \mu_2(v_{k-1}, v) | (u, v_1, v_2 \dots v_{k-1}, v \in V)\}$  and  $v_2^k(u, v)$  is defined as  $\inf\{v_2(u, v_1) \lor v_2(v_1, v_2) \lor v_2(v_2, v_3) \dots \lor v_2(v_{k-1}, v) | (u, v_1, v_2 \dots v_{k-1}, v \in V)\}$ .

**Definition 2.8:** [8] An edge (u, v) is said to be a strong edge if  $\mu_2(u, v) \ge \mu_2^{\infty}(u, v)$  and  $\nu_2(u, v) \ge \nu_2^{\infty}(u, v)$ .

**Definition 2.9:** [8] Let G = (V, E) be an IFG on V. Let  $u, v \in V$ , we say that u dominates v in G if there exists a strong edge between them.

**Definition 2.10:** [8] A subset S of V is called dominating set in G if for every  $v \in V - S$ , there exists  $u \in S$  such that u dominates v.

**Definition 2.11:** [8] A dominating set S of an IFG is said to be minimal dominating set if no proper subset of S is a dominating set.

**Definition 2.12:** [8] Minimum cardinality among all minimal dominating set is called vertex domination number of G and is denoted by  $\gamma(G)$ .

**Definition 2.13:** [8] Let G = (V, E) be an IFG, then the vertex cardinality of V is defined by  $|V| = \sum_{v_i \in V} \left(\frac{1 + \mu_1(v_i) - \nu_1(v_i)}{2}\right) \text{ for all } v_i \in V.$ 

**Definition 2.14:** [8] Let G = (V, E) be an IFG, then the vertex cardinality of V is defined by

$$|\mathsf{E}| = \sum_{\mathsf{v}_i,\mathsf{v}_j \in \mathsf{E}} \left( \frac{1 + \mu_2(\mathsf{v}_i,\mathsf{v}_j) - \nu_2(\mathsf{v}_i,\mathsf{v}_j)}{2} \right) \text{ for all } (\mathsf{v}_i,\mathsf{v}_j) \in \mathsf{E}.$$

**Definition 2.15:** [3] Let G = (V, E) be an IFG. Let  $e_i$  and  $e_j$  be two adjacent edges of G. We say that  $e_i$  dominates  $e_j$  if  $e_i$  is a strong edge in G.

**Definition 2.16:** [3] A subset *S* of *E* is called a edge dominating set in G if for every  $e_j \in E - S$ , there exists  $e_i \in S$  such that  $e_i$  dominates  $e_j$ .

**Definition 2.17:** [3] An edge dominating set S of an IFG is said to be minimal edge dominating set if no proper subset of S is an edge dominating set.

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Definition 2.18: [3] Minimum cardinality among all minimal edge dominating set is called edge domination number of G and is denoted by  $\gamma'(G)$ 

**Definition 2.19:** [3] The strong neighbourhood of an edge  $e_i$  in an IFG G is  $Ns(e_i) = \{e_i \in E(G) | e_i \text{ is strong edge and adjacent to } e_i \text{ in } G\}$ 

Theorem 2.1: [3] Let P(Kn) be the number of edges in minimum edge dominating set of Kn then  $P(K_n) = \frac{n}{2}$ , when n is even and  $P(K_n) = \frac{n-1}{2}$ , when n is odd.

Note 1: Combination of  $\frac{n}{2}\left(\frac{n-1}{2}\right)$  edges when n is even(odd) in  $K_n$  forms an edge dominating set.

**Theorem 2.2:** [3] Let G = (V, E) be an IFG without isolated edges. If D is minimum edge dominating set then S - Dis an edge dominating set, where S is the set of all strong edges in G.

### 3. SECURE EDGE DOMINATION AND INVERSE SECURE EDGE DOMINATION IN INTUITIONISTIC **FUZZY GRAPHS**

**Definition 3.1:** Let G = (V, E) be an IFG. An edge dominating set F of E is a secure edge dominating set if for every edge  $e \in E - F$ , there exists an edge  $f \in F$ , which is adjacent to e such that  $\{(F - \{f\}) \cup \{e\}\}$  is an edge dominating set.

**Definition 3.2:** A secure edge dominating set F of an IFG is said to be minimal secure edge dominating set if no proper subset of F is a secure edge dominating set.

Definition 3.3: Minimum cardinality among all minimal secure edge dominating set is called secure edge domination number of G and is denoted by  $\gamma'_{se}(G)$ .

**Example 3.1:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4, v_5\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), v_5\}$  $(v_3, v_5), (v_4, v_5), (v_5, v_1)$ .

Here  $\{e_5, e_6\}, \{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_5\}, \{e_1, e_2, e_6\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_1, e_3, e_6\}, \{e_1, e_4, e_5\}, \{e_1, e_4, e_6\}, \{e_1, e_3, e_6\}, \{e_1, e_4, e_6\}, \{e_1, e_6, e_6\}, \{e_1$  $\{e_2, e_3, e_4\}, \{e_2, e_3, e_5\}, \{e_2, e_4, e_6\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_3, e_4, e_6\}$  are minimal secure edge dominating sets of G and  $\gamma'_{se}(G) = 0.9.$ 

**Definition 3.4:** A total edge dominating set F of an edge set E of an IFG G = (V, E) is said to be a secure total edge dominating set of G if for every  $e \in E - F$  there exists an edge  $f \in F$ , such that e and f are adjacent and  $\{(F - \{f\}) \cup \{e\}\}\$  is a total edge dominating set of G.

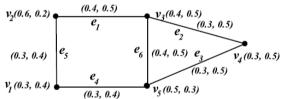
Definition 3.5: A secure total edge dominating set F of an IFG is said to be minimal secure total edge dominating set if no proper subset of F is a secure total edge dominating set.

Definition 3.6: Minimum cardinality among all minimal secure total edge dominating set is called secure total edge domination number of *G* and is denoted by  $\gamma'_{ste}(G)$ .

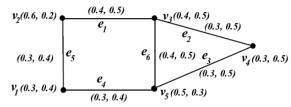
**Example 3.2:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4, v_5\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), v_5\}$  $(v_3, v_5), (v_4, v_5), (v_5, v_1)$ 

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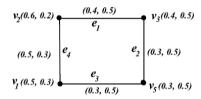
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Here  $\{e_1, e_2, e_3\}$ ,  $\{e_1, e_2, e_5\}$ ,  $\{e_1, e_2, e_6\}$ ,  $\{e_1, e_3, e_6\}$ ,  $\{e_1, e_4, e_5\}$ ,  $\{e_1, e_4, e_6\}$ ,  $\{e_1, e_5, e_6\}$ ,  $\{e_2, e_3, e_4\}$ ,  $\{e_2, e_4, e_6\}$ ,  $\{e_4, e_5, e_6\}$ ,  $\{e_3, e_4, e_5\}$ ,  $\{e_3, e_4, e_5\}$ ,  $\{e_3, e_4, e_6\}$  are minimal secure total edge dominating sets of G and  $\gamma'_{ste}(G) = 1.35$ .

**Definition 3.7:** Let *F* be a minimal edge dominating set which has the minimum cardinality. Then  $F' \subseteq E - F$  is said to be an inverse edge dominating set of *G* with respect to *F* if *F'* is an edge dominating set. The inverse edge domination number  $\gamma^{-1}_{o}(G)$  is the minimum cardinality of all inverse edge dominating set *F'* of *G*.

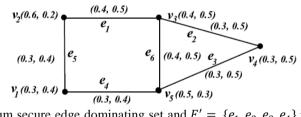
**Example 3.3:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_3, v_1)\}$ .



Here  $\{e_1, e_2\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}$  are minimal edge dominating sets of G. Here  $F = \{e_2, e_3\}$  and  $F' = \{e_1, e_4\}$  is an edge dominating set and  $\gamma_e^{-1}(G) = 1.05$ .

**Definition 3.8:** Let G = (V, E) be an IFG. Let *F* be a minimal secure edge dominating set which has the minimum cardinality. Then  $F' \subseteq E - F$  is said to be an inverse secure edge dominating set of G with respect to *F* if *F'* is a secure edge dominating set. The inverse secure edge domination number  $\gamma_{se}^{-1}(G)$  is the minimum cardinality of an inverse secure edge dominating set of *G*.

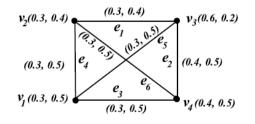
**Example 3.4:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4, v_5\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_3, v_5), (v_4, v_5), (v_5, v_1)\}$ .



Here  $F = \{e_5, e_6\}$  is a minimum secure edge dominating set and  $F' = \{e_1, e_2, e_3, e_4\}$  is a secure edge dominating sets of G and  $\gamma_{se}^{-1}(G) = 1.25$ .

**Definition 3.9:** Let G = (V, E) be an IFG. Let F be a minimal secure total edge dominating set which has the minimum cardinality. Then  $F' \subseteq E - F$  is said to be an inverse secure total edge dominating set of G with respect to F if F' is a secure total edge dominating set. The inverse secure total edge domination number  $\gamma^{-1}_{ste}(G)$  is the minimum cardinality of an inverse secure total edge dominating set of G.

**Example 3.5:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_3, v_1), (v_2, v_4), (v_3, v_1)\}$ .



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Here  $F = \{e_3, e_4, e_6\}$  and  $F' = \{e_1, e_2, e_5\}$  is a secure total edge dominating sets of G and  $\gamma_{ste}^{-1}(G) = 1.30$ .

Theorem 3.1: If F is a total edge dominating set of a complete IFG G then F is also a secure edge dominating set of G.

**Proof:** Given that *F* is a total edge dominating set of a complete IFG *G* then *F* is an edge dominating set of *G*. Then by theorem 2.1, *F* contains minimum  $\frac{n}{2}$  edges when *n* is even and minimum  $\frac{n-1}{2}$  edges when *n* is odd. Since *G* is complete, all edges are strong edges. Now for every  $\in E - F$ , there exists  $f \in F$  such that *e* and *f* are adjacent and  $\{(F - \{f\}) \cup \{e\}\}$  contains at least  $\frac{n}{2}$  edges when *n* is even and atleast  $\frac{n-1}{2}$  edges when *n* is odd. Any combination of  $\frac{n}{2}(\frac{n-1}{2})$  edges when *n* is even (odd) in  $K_n$  forms a edge dominating set by Note.1. Therefore  $\{(F - \{f\}) \cup \{e\}\}$  is an edge dominating set of *G*.

Hence F is a secure edge dominating set of G.

Note 2: An IFG need not contain an inverse edge dominating set.

**Example 3.6:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_3, v_1)\}$ .

$$\begin{array}{c} v_2(0.4, \ 0.6) & (0.3, \ 0.6) \\ (0.3, \ 0.4) & e_4 & e_2 \\ v_1(0.3, \ 0.5) & e_3 \\ (0.3, \ 0.6) & v_4(0.4, \ 0.6) \end{array}$$

Here  $\{(v_1, v_2), (v_3, v_4)\}$  are strong edges.

Let  $F = \{(v_1, v_2), (v_3, v_4)\}$  be a minimal edge dominating set. Then  $E - F = \{(v_2, v_3), (v_4, v_1)\}$  is not an edge dominating set, since  $\{(v_2, v_3), (v_4, v_1)\}$  are not strong edges.

Theorem 3.2: A strong IFG G contains an inverse edge dominating set.

**Proof:** Let F be a minimal edge dominating set of an IFG G. Since G is strong, all edges are strong edges.

Then by Theorem 2.2, if F is a minimum edge dominating set of an IFG G then E - F is also an edge dominating set of an IFG G.

Thus E - F is an edge dominating set.

Hence every strong IFG G contains inverse edge dominating set.

**Theorem 3.3:** For any IFG G,  $\gamma'(G) \leq \gamma'_{se}(G) \leq \gamma^{-1}_{se}(G)$ 

**Proof:** By definition, every secure edge dominating set is an edge dominating set. But minimum secure edge dominating set need not to be minimum edge dominating set and the cardinality of minimum secure edge dominating set will always exceeds or equals the cardinality of minimum edge dominating set.

ie,  $\gamma'_{se}(G) \ge \gamma'(G)$ 

By definition, inverse secure edge dominating set is obtained by a minimal secure edge dominating set of G. Therefore, the minimum cardinality of inverse secure edge dominating set will always exceeds or equals the minimum cardinality of secure edge dominating set.

ie, 
$$\gamma^{-1}_{se}(G) \ge \gamma'_{se}(G)$$

By definition, every inverse secure edge dominating set is a secure edge dominating set and every secure edge dominating set is an edge dominating set. Hence every minimum inverse secure edge dominating set is an edge dominating set. Therefore, the minimum cardinality of inverse secure edge dominating set will always exceeds or equals the minimum cardinality of edge dominating set.

ie,  $\gamma^{-1}{}_{se}(G) \ge \gamma'(G)$ Hence  $\gamma'(G) \le \gamma'{}_{se}(G) \le \gamma^{-1}{}_{se}(G)$ .

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International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India. Secure Edge Domination and Vertex Edge Domination in Intuitionistic Fuzzy Graphs / IJMA- 9(1), Jan.-2018, (Special Issue)

**Theorem 3.4:** For an IFG  $K_n$ ,  $\gamma'_{se}(K_n) \leq \gamma'(K_n)$ , when n is even.

**Proof:** Let *F* be an edge dominating set of a complete IFG  $K_n$ .

By Theorem 2.1,  $P(K_n) = \frac{n}{2}$ , when n is even.

Therefore F contains  $\frac{n}{2}$  edges and  $\gamma'(K_n)$  is the minimum cardinality of  $\frac{n}{2}$  edges.

Since *G* is complete IFG, all edges are strong edges. Now for every edge  $e \in E - F$  is adjacent to edge  $f \in F$  such that  $\{(F - \{f\}) \cup \{e\}\}$  contains  $\frac{n}{2}$  edges and is an edge dominating set by Note.1. Which implies every secure edge dominating set of  $K_n$  has  $\frac{n}{2}$  edges. Hence  $\gamma'_{se}(K_n) \leq \gamma'(K_n)$ .

**Theorem 3.5:** For any IFG G,  $\gamma'_{se}(G) + \gamma^{-1}_{se}(G) \leq S(G)$ .

**Proof:** The proof is obvious.

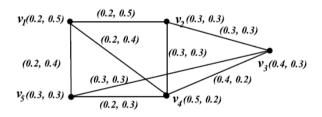
#### 4. VERTEX EDGE DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

**Definition 4.1:** Let G = (V, E) be an IFG. A set  $S \subseteq V$  is said to be vertex edge dominating set if for every strong edge  $e_{ij} \in E$ , there exists a vertex  $v_i \in S$  which dominates  $e_{ij}$ .

**Definition 4.2:** A vertex edge dominating set D of an IFG G is called minimal vertex edge dominating set if no proper subset of D is a vertex edge dominating set.

**Definition 4.3:** Minimum cardinality among all minimal vertex edge dominating set is called vertex edge domination number and is denoted by  $\gamma_{VE}(G)$ .

**Example 4.1:** Consider an IFG, G = (V, E), such that  $V = \{v_1, v_2, v_3, v_4, v_5\}$  and  $E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_3, v_5), (v_4, v_5), (v_5, v_1), (v_1, v_4), (v_5, v_3)\}$ .



Here  $\{v_1, v_3, v_4\}, \{v_1, v_2, v_3, v_5\}, \{v_2, v_4, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}, \{v_2, v_3, v_5\}$  are minimal vertex edge dominating sets and  $\gamma_{VE}(G) = 1.55$ .

**Theorem 4.1:** Vertex edge domination number in a complete IFG is the sum of the minimum cardinality of (n-1) vertices.

**Proof:** Let  $P(K_n)$  be the number of vertices in vertex edge dominating set of  $K_n$ . It can be easily verified that  $P(K_2) = 1$  and  $\gamma_{VE}(K_2)$  is a cardinality of a vertex.

When n = 3, consider  $K_3$ . Then clearly  $P(K_3)$  has two vertices in vertex edge dominating set of  $K_3$  and and  $\gamma_{VE}(K_3)$  is a sum of cardinality of the two vertices. Continuing this manner,  $P(K_n)$  has (n - 1) vertices in vertex edge dominating set of  $K_n$  and  $\gamma_{VE}(K_n)$  is a sum of cardinality of the (n - 1) vertices.

Hence  $\gamma_{VE}(G) = |v_1| + |v_2| + \dots + |v_{n-1}|.$ 

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#### Secure Edge Domination and Vertex Edge Domination in Intuitionistic Fuzzy Graphs / IJMA- 9(1), Jan.-2018, (Special Issue)

**Theorem 4.2:** If G = (V, E) is a complete IFG then  $\gamma_{VE}(G) = |V| - \{|v|\}$ , where v is a vertex which has maximum cardinality.

**Proof:** Let G = (V, E) be a complete IFG. Since G is complete it has  $\frac{n-1}{2}$  edges and all edges are strong edges.

Every vertex is incident with (n - 1) vertices. Let  $\{v_i\}$  be a vertex having the minimum cardinality value of G and there are (n - 1) edges incident on it.

Consider a vertex induced subgraph  $G - \{v_i\}$ . Clearly  $G - \{v_i\}$  is also a complete IFG.

Again we take a vertex having the minimum cardinality value in  $-\{v_i\}$ . Continuing this process to cover all the edges.

We need to take (n-1) vertices.

By theorem 4.1, Vertex edge domination number in a complete IFG is the sum of the minimum cardinality of (n - 1) vertices.

ie.,  $\gamma_{VE}(G) = |v_1| + |v_2| + \dots + |v_{n-1}|.$ 

Now  $|\mathbf{V}| = |v_1| + |v_2| + \dots + |v_n|$ .

 $|\mathbf{V}| - \{|v|\} = |v_1| + |v_2| + \dots + |v_{n-1}|.$ 

 $\therefore \gamma_{VE}(G) = |V| - \{|v|\}$  where v is a vertex which has maximum cardinality.

Note 3: If G = (V, E) is a complete IFG with  $(\mu_1, \nu_1)$  as constant function then  $\gamma_{VE}(G) = n |V|$ .

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