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SOLUTION OF FIRST ORDER FUZZY DIFFERENTIAL EQUATION IN INTUITIONISTIC FUZZY ENVIRONMENT USING INVERSE LAPLACE TRANSFORM TECHNIQUE

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ABSTRACT

In this paper we have represent a new method of solving intuitionistic fuzzy differential equations. we have discussed the solution of the fuzzy differential equations considering the constant coefficient and the initial value as triangular intuitionistic fuzzy number. Numerical examples are also presented to understand the proposed method.

Keywords: *Differential equation, Intuitionistic fuzzy number, Intuitionistic fuzzy solution, Laplace transform.*

1. INTRODUCTION

Fuzzy differential equation plays an important role in modelling the real life situation in which the data are imprecise in nature. Chang and Zadeh [5] first introduced the concept of fuzzy derivative. The fuzzy differential was first introduced by Kandel and Byatt [7] in 1987. It was applied in many field of sciences [3, 4, 6, 8] such as biology, physics, engineering etc. In fuzzy diferential equation we use linear fuzzy numbers such as triangular fuzzy numbers, trapezoidal fuzzy numbers etc. In these numbers we only know the membership function of an element of fuzzy set. Here the non membership value is only the complement of membership value and indeterminacy does not occur here. Atanassov [1] extended the concept of fuzzy set theory by introducing the concept of intuitionistic fuzzy set theory. Here we deal with both the membership and non-membership functions . The indeterminacy is measured as a function of them. Many researchers have used intuitionistic fuzzy numbers in their papers [9, 14, 15]. G.S. Mahapatra and T.K.Roy have taken an electric network model of dark room in their paper in 2009. The imprecise reliability of the system was computed by considering each component of the electric network model system as triangular intuitionistic fuzzy numbers. A.K. Shaw and T. K. Roy computed the reliability of the electric network model considering the component of the system as trapezoidal intuitionistic fuzzy numbers in their paper in 2013. In another paper G.S. Mahapatra and T.K.Roy presented the starting failure of an automobile system by intuitionistic fuzzy system.

Each components of failure was represented by trapezoidal intuitionistic fuzzy number of the system failure model to compute the imprecise failure. Although the intuitionistic fuzzy differential equations has better acceptance than the fuzzy differential equations there are only few papers where these concepts are applied [1, 11, 15].

In our paper we have considered a linear first order intuitionistic fuzzy differential equation with intuitionistic fuzzy number as constant coefficient as well as the initial condition. There are some papers where they have discussed about the solution of intuitionistic fuzzy differential equations [12,13] But in our paper we have used inverse Laplace transform to compute the exponential of coefficient matrix and then using the intuitionistic fuzzy initial conditions we have found the solution of the intuitionistic fuzzy differential equation.

2. PRELIMINARIES

We first introduce some concept on fuzzy and intuitionistic fuzzy set theory

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Definition 2.1: [2] Let X be a crisp set. Then

$$\tilde{A} = \{\langle x, \mu_A(x)\rangle | x\epsilon X\} \tag{1}$$

is called a fuzzy set, where $\mu_A(x)$ is the membership function of A . $\mu_{\tilde{A}}: X \longrightarrow [0,1]$ and $\mu_{\tilde{A}}(x)$ the membership degree of the element x to A which is a single value belonging to the unit closed interval [0 , 1].

Attanassov (1986) [16] generalizes Zadeh's fuzzy set with the concept of intuitionistic fuzzy set (IFS) as defined bellow:

Definition 2.2: An intuitionistic fuzzy set (IFS) is an object having the following form:

$$A = \{\langle x , \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | \ x\epsilon X\} \tag{2}$$

Which is characterized by a membership function

$$\mu_{\tilde{A}}: X \longrightarrow [0,1], x \,\epsilon\, X \ \longrightarrow \mu_{\tilde{A}}(x) \,\epsilon\, [0 ,1] \tag{3}$$

And a non membership function

$$\nu_{\tilde{A}}: X \longrightarrow [0,1], x \,\epsilon\, X \ \longrightarrow \nu_{\tilde{A}}(x) \,\epsilon\, [0 ,1] \tag{4}$$

the two function also satisfy the following condition :

$$0 \le \ \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \ \le 1 \ , \ \forall x\epsilon X \tag{5}$$

Where $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ define the degree of membership and non membership of an element x of the universal set X.

This condition is known as intuitionistic condition for each IFS A in X. If $\pi_A(x) = 1 - \ \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \ \forall x\epsilon X$ then $\pi_A(x)$ is called the indeterminacy degree of x to $\tilde{A}$.Fuzzy sets is also a special types of intuitionistic fuzzy sets where the indeterminacy degree for all $x\epsilon X$ is zero.

Definition 2.3: Let α , β be two fixed numbers such that $\alpha, \beta \,\epsilon [0, 1]$ and $\alpha + \beta \le 1$ then the (α, β)-cut is defined as

$$A_{\alpha,\beta} = \begin{cases} \langle x , \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | \ x\epsilon X \\ \mu_{\tilde{A}}(x) \ge \alpha , \nu_{\tilde{A}}(x) \le \ \beta \end{cases} \tag{6}$$

Where (α, β)-cut denoted by $A_{\alpha,\beta}$ is defined as the crisp set of elements x which belonging to A atleast to the degree α and which does not belong to A at most to the degree β.

For TIFN $\tilde{A}^i = \langle a, l_a, r_a; w_a, u_a\rangle$ the (α , β)-cut is given by $A_{\alpha,\beta} = \min\{A_\alpha, A_\beta\}$ (7)

Definition 2.4: An intuitionistic fuzzy number $\tilde{A}^i = \{\langle x , \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | \ x\epsilon R\}$ is defined as follows:

(i) $\tilde{A}^i$ is an intuitionistic fuzzy subset of the real line R.
(ii) $\tilde{A}^i$ is normal i.e. there exists an $x\epsilon R$ such that $\mu_{\tilde{A}^i}(x) = 1$.
(iii) $\tilde{A}^i$ is convex for the membership function $\mu_{\tilde{A}^i}(x)$ i.e. $\mu_{\tilde{A}^i}(\lambda x + (1-\lambda)y) \ge \min\{\mu_{\tilde{A}^i}(x), \mu_{\tilde{A}^i}(y)\}$ $\forall x, y \in R , \lambda \,\epsilon\, [0 , 1]$
(iv) $\tilde{A}^i$ is concave for the non membership function $\vartheta_{\tilde{A}^i}(x)$
i.e. $\vartheta_{\tilde{A}^i}(\lambda x + (1-\lambda)y) \le \min\{\vartheta_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(y)\}$ $\forall x, y \in R , \lambda \,\epsilon\, [0 , 1]$

Definition 2.5: A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^i = \langle a , l_a, r_a; w_a, u_a\rangle$ is a subset of intuitionistic fuzzy sets (IFS) in R and its membership and non membership function are given by

$$\mu_{\tilde{a}^i}(x) = \begin{cases} \frac{(x-a+l_a)w_a}{l_a} & ; a - \ l_a \le x \le a \\ \frac{(a+r_a-x)w_a}{r_a} & ; a \le x \ \le a + r_a \\ 0 & ; \ otherwise \end{cases} \tag{8}$$

and

$$\vartheta_{\tilde{a}^i}(x) = \begin{cases} \frac{(a-x)+u_a(x-a+l_a)}{l_a} & ; a - l \le x \le a \\ \frac{(x-a)+u_a(a+r_a-x)}{r_a} & ; a \le x \ \le a + r \\ 1 & ; \ otherwise \end{cases} \tag{9}$$

where l_a and r_a are left spreads right spreads respectively. w_a and u_a represents the maximum degree of membership and minimum degree of non membership respectively with the condition

$$0 \ \le w_a \le 1 \ ; \ 0 \ \le u_a \le 1 \ \text{ and } \ 0 \ \le w_a + u_a \le 1$$

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3. SOLUTION PROCEDURE FOR INTUITIONISTIC FUZZY DIFFERENTIAL EQUATION

Let us consider an intuitionistic fuzzy differential equation with constant coefficient and the initial value as TIFN

$$\frac{d\tilde{x}^i(t)}{dt} = \tilde{a}^i \tilde{x}^i(t) \tag{10}$$

With the initial condition $\tilde{x}^i(0) = \tilde{c}^i$ (11)

Now taking the (α , β)-cut on both sides of the equation (10) it can be written as

$$\frac{d}{dt}([x_l(\alpha), x_r(\alpha)]; [x_l(\beta), x_r(\beta)]) = ([a_l(\alpha), a_r(\alpha)]; [a_l(\beta), a_r(\beta)])([x_l(\alpha), x_r(\alpha)]; [x_l(\beta), x_r(\beta)]) \tag{12}$$

Which satisfy the initial conditions

$$\tilde{x}_l^i(0\,,\alpha) = \tilde{c}_l^i(\,\alpha) \tag{13}$$
$$\tilde{x}_r^i(0\,,\alpha) = \tilde{c}_r^i(\,\alpha) \tag{14}$$
$$\tilde{x}_l^i(0\,,\beta) = \tilde{c}_l^i(\beta) \tag{15}$$
$$\tilde{x}_r^i(0,\beta) = \tilde{c}_r^i(\beta) \tag{16}$$

When (α, β)-cut is positive in matrix form equation (10) can be written as

$$\frac{d}{dt}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} = \begin{pmatrix} a_l(\alpha) & 0 & 0 & 0 \\ 0 & a_r(\alpha) & 0 & 0 \\ 0 & 0 & a_l(\beta) & 0 \\ 0 & 0 & 0 & a_r(\beta) \end{pmatrix}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} \tag{17}$$

When (α, β)-cut is negative in matrix form equation (6) can be written as

$$\frac{d}{dt}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} = \begin{pmatrix} 0 & a_r(\alpha) & 0 & 0 \\ a_l(\alpha) & 0 & 0 & 0 \\ 0 & 0 & 0 & a_r(\beta) \\ 0 & 0 & a_l(\beta) & 0 \end{pmatrix}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} \tag{18}$$

For a particular value of α the coefficient matrix say A becomes a constant matrix of order 4×4 .Now to solve the system of equations (17) or (18) the main job is the computation of the matrix e^{At}. If we choose the function

$$f(t) = \begin{cases} e^{at} & , \quad t > 0 \\ 0 & , \; otherwise \end{cases} \tag{19}$$

Then the Laplace transform of f(t) is $F(s) = \frac{1}{s-a}\; ; s \in R$

Therefore Laplace transform of e^{At} is $(sI_n - A)^{-1}$ where $A \in M_n(R)$ and $t \in R$. Hence we can compute the e^{At} as the inverse Laplace transform of $(sI_n - A)^{-1}$.The solution of the system (8) is given by

$$\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} = e^{At}\begin{pmatrix} c_l(\alpha) \\ c_r(\alpha) \\ c_l(\beta) \\ c_r(\beta) \end{pmatrix} \tag{20}$$

where the elements of the matrix can be found by using the initial conditions (13) – (16).

4. NUMERICAL EXAMPLE

Let us consider the IFD equation

$$\frac{d\tilde{x}^i}{dt} = \tilde{a}^i \tilde{x}^i \tag{21}$$

with the initial condition

$$\tilde{x}_0^i = \tilde{c}_0^i \tag{22}$$

where $\tilde{a}^i = \langle -3,1,1; 0.6,0.3\rangle$ and $\tilde{c}_0^i = \langle 3,2,2; 0.5,0.3\rangle$ Then α- cut of $\tilde{a}^i$ is given by $\tilde{a}_\alpha^i = \left[-4 + \frac{5\alpha}{3}, -2 - \frac{5\alpha}{3}\right]$ and the β- cut of $\tilde{a}^i$is given by $\tilde{a}_\beta^i = \left[-4 + \frac{10(1-\beta)}{7}, -2 - \frac{10(1-\beta)\alpha}{7}\right]$.

Hence the (α,β)-cut is $[\frac{-22}{7}, \frac{-20}{7}]$ for particular value of α=0.3 and β=0.4.Now the differential equation (17) can be written in matrix form as

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$$\frac{d}{dt}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} = \begin{pmatrix} 0 & \frac{-20}{7} & 0 & 0 \\ \frac{-22}{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-20}{7} \\ 0 & 0 & \frac{-22}{7} & 0 \end{pmatrix}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} \tag{30}$$

$$(sI_4 - A)^{-1} = \begin{pmatrix} \frac{s}{s^2-\frac{440}{49}} & \frac{\frac{-20}{7}}{s^2-\frac{440}{49}} & 0 & 0 \\ \frac{\frac{-22}{7}}{s^2-\frac{440}{49}} & \frac{s}{s^2-\frac{440}{49}} & 0 & 0 \\ 0 & 0 & \frac{s}{s^2-\frac{440}{49}} & \frac{\frac{-20}{7}}{s^2-\frac{440}{49}} \\ 0 & 0 & \frac{\frac{-22}{7}}{s^2-\frac{440}{49}} & \frac{s}{s^2-\frac{440}{49}} \end{pmatrix} \tag{31}$$

$$e^{Ax} = L^{-1}((sI_4 - A)^{-1}) \tag{32}$$

$$= \begin{pmatrix} Cos\sqrt{\frac{440}{49}}t & -\frac{20}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t & 0 & 0 \\ -\frac{22}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t & Cos\sqrt{\frac{440}{49}}t & 0 & 0 \\ 0 & 0 & Cos\sqrt{\frac{440}{49}}t & -\frac{20}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t \\ 0 & 0 & -\frac{22}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t & Cos\sqrt{\frac{440}{49}}t \end{pmatrix} \tag{33}$$

The solution of equation (17) is given by

$$\begin{pmatrix} x_l(t,\alpha) \\ x_r(t,\alpha) \\ x_l(t,\beta) \\ x_r(t,\beta) \end{pmatrix} = \begin{pmatrix} C_l(\alpha)Cos\sqrt{\frac{440}{49}}t - C_r(\alpha)\frac{20}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t \\ C_r(\alpha)Cos\sqrt{\frac{440}{49}}t - C_l(\alpha)\frac{22}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t \\ C_l(\beta)Cos\sqrt{\frac{440}{49}}t - C_r(\beta)\frac{20}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t \\ C_r(\beta)Cos\sqrt{\frac{440}{49}}t - C_l(\beta)\frac{22}{\sqrt{440}}Sin\sqrt{\frac{440}{49}}t \end{pmatrix} \tag{34}$$

Where from the initial condition (18) we have $C_l(\alpha) = 1 + 4\alpha, C_r(\alpha) = 5 - 4\alpha, C_l(\beta) = 1 + \frac{20(1-\beta)}{7}$ and $C_r(\beta) = 5 - \frac{20(1-\beta)}{7}$.

5. APPLICATIONS

5.1 Population Growth

A typical application is population growth in which the rate of change of population of a certain species $\frac{dx}{dt}$ at Any time t is proportional to the value of P at that instant. The model can be written as $\frac{dp}{dt} = kp$, Where k is known as the growth constant, it is a positive constant. Here if we consider k as TIFN $\tilde{k}^i$ then it becomes a IFDE $\frac{d\tilde{P}^i}{dt} = \tilde{k}^i\tilde{P}^i$ $that$ can be solved by our proposed method. Let $\tilde{k}^i = \langle 5,2,2; 0.6,0.3\rangle$ then the intuitionistic fuzzy population growth model can be in matrix form as

$$\frac{d}{dt}\begin{pmatrix} P_l(\alpha) \\ P_r(\alpha) \\ P_l(\beta) \\ P_r(\beta) \end{pmatrix} = \begin{pmatrix} C_l(\alpha) & 0 & 0 & 0 \\ 0 & C_r(\alpha) & 0 & 0 \\ 0 & 0 & C_l(\beta) & 0 \\ 0 & 0 & 0 & C_r(\beta) \end{pmatrix}\begin{pmatrix} P_l(\alpha) \\ P_r(\alpha) \\ P_l(\beta) \\ P_r(\beta) \end{pmatrix} \tag{35}$$

$$a_l(\alpha) = [3 + \frac{5\alpha}{3}] \tag{36}$$

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$$a_r(\alpha) = [7 - \frac{5\alpha}{3}] \tag{37}$$
$$a_l(\beta) = [3 + \frac{20(1-\beta)}{3}] \tag{38}$$
$$a_l(\beta) = [7 - \frac{20(1-\beta)}{3}] \tag{39}$$

Thus for α=0.3 and β=0.4 such that $0 \le 0.7 \le 1$ and using $\tilde{A}_{\alpha,\beta} = min\{\tilde{A}_{\alpha,}\tilde{A}_{\beta}\}$ we have $\tilde{A}_{\alpha,\beta} = [\frac{33}{7}, \frac{37}{7}]$

$$P_l(t,\alpha) = C_l(\alpha)e^{\frac{33t}{7}} \tag{40}$$
$$P_r(t,\alpha) = C_r(\alpha)e^{\frac{37t}{7}} \tag{41}$$
$$P_l(t,\beta) = C_l(\beta)e^{\frac{33t}{7}} \tag{42}$$
$$P_r(t,\beta) = C_r(\beta)e^{\frac{33t}{7}} \tag{43}$$

Where $C_l(\alpha), C_r(\alpha), C_l(\beta), C_r(\beta)$ are the initial conditions.

5.1 Exponential decay

The rate of decay of a radioactive substance at time t is proportional to the mass x(t) of the substance left at that time. Thus the IFDE is

$\frac{d\tilde{x}^i}{dt} = -\tilde{\mu}^i\tilde{x}^i$ Let $-\tilde{\mu}^i = \langle -6,2,1; 0.7,0.2\rangle$ then the intuitionistic fuzzy differential equations becomes

$$\frac{d}{dt}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} = \begin{pmatrix} 0 & a_r(\alpha) & 0 & 0 \\ a_l(\alpha) & 0 & 0 & 0 \\ 0 & 0 & 0 & a_r(\beta) \\ 0 & 0 & a_l(\beta) & 0 \end{pmatrix}\begin{pmatrix} x_l(\alpha) \\ x_r(\alpha) \\ x_l(\beta) \\ x_r(\beta) \end{pmatrix} \tag{44}$$

Where

$$a_l(\alpha) = [-8 + \frac{20\alpha}{7}] \tag{45}$$
$$a_r(\alpha) = [-5 + \frac{10\alpha}{7}] \tag{46}$$
$$a_l(\beta) = [-8 + \frac{20(1-\beta)}{7}] \tag{47}$$
$$a_l(\beta) = [-8 + \frac{20(1-\beta)}{7}] \tag{48}$$

Solving the above system of differential equations by our proposed method we have the solutions

$$x_l(t,\alpha) = c_l(\alpha)Cos\sqrt{a_l(\alpha)a_r(\alpha)}t + c_r(\alpha)\sqrt{\frac{a_l(\alpha)}{a_r(\alpha)}}Sin\sqrt{a_l(\alpha)a_r(\alpha)}\ t \tag{49}$$
$$x_r(t,\alpha) = c_r(\alpha)Cos\sqrt{a_l(\alpha)a_r(\alpha)}t + c_l(\alpha)\sqrt{\frac{a_l(\alpha)}{a_r(\alpha)}}Sin\sqrt{a_l(\alpha)a_r(\alpha)}\ t \tag{50}$$
$$x_l(t,\beta) = c_l(\beta)Cos\sqrt{a_l(\beta)a_r(\beta)}t + c_r(\beta)\sqrt{\frac{a_l(\beta)}{a_r(\beta)}}Sin\sqrt{a_l(\beta)a_r(\beta)}\ t \tag{51}$$
$$x_r(t,\beta) = c_r(\beta)Cos\sqrt{a_l(\beta)a_r(\beta)}t + c_l(\beta)\sqrt{\frac{a_l(\beta)}{a_r(\beta)}}Sin\sqrt{a_l(\beta)a_r(\beta)}\ t \tag{52}$$

Here $c_l(\alpha)$, $c_r(\alpha)$, $c_l(\beta)$ and $c_r(\beta)$ are the initial conditions.

6. CONCLUSIONS

In our paper we have discussed the solution procedure of intuitionistic fuzzy differential equations with constant coefficient and the initial condition as triangular intuitionistic fuzzy number. This process is based on finding the exponential of the coefficient matrix using inverse Laplace Transform. Both the cases when the constant intuitionistic fuzzy number coefficient is positive and negative have been discussed with applications in this article. This process is very helpful when the eigenvalues are repeated or complex. As both the membership and non-membership functions have been considered fuzzy intuitionistic differential equations provides more realistic results than fuzzy differential equations.

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