SOME PROPERTIES OF INTUITIONISTIC FUZZY BI-IDEALS OF NEAR RINGS

K. DHILIP KUMAR*1  
Assistant Professor, Department of Mathematics,  
SSM College of Arts & Science, Komarapalayam -638 183, Tamil Nadu, India.

M. RAMACHANDRAN2  
Assistant Professor, Department of Mathematics,  
Government Arts & Science College, Sathyamangalam -638 401, Tamil Nadu, India.

E-mail: dhilipkumarmaths@gmail.com1, dr.ramachandran64@gmail.com2

ABSTRACT

In this present paper, we introduce the concept of intuitionistic fuzzy bi-ideals of near-rings. Also we investigate some algebraic nature of intuitionistic fuzzy bi-ideals of near-rings and some related properties of these fuzzy substructures.

Keywords: Near-rings, Bi-ideals, Fuzzy bi-ideals, Intuitionistic fuzzy set, Intuitionistic fuzzy subring, Intuitionistic fuzzy ideal, Intuitionistic fuzzy bi-ideal.

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1. INTRODUCTION

The notion of intuitionistic fuzzy set (IFS) was introduced by Atanassov [2] as a generalization of notion of fuzzy sets. The concept of near-rings was introduced by Pilz [9] and that of quasi-ideal in near ring was introduced by Yakabe [12]. The notion of bi-ideals was introduced by Chelvam and Ganesan [4].

In this paper we study the intuitionistic fuzzification of the notion of bi-ideals in near-rings. We show that every intuitionistic fuzzy bi-ideal of a near-ring is an intuitionistic fuzzy subnear-ring. We give characterizations of intuitionistic fuzzy bi-ideals in near-rings.

A near-ring is a non empty set N with two binary operations “+” and “.” such that

(i) (N,+) is a group not necessarily abelian
(ii) (N, .) is a semi group
(iii) (x + y).z = x.z + y.z , for all x, y, z∈N.

Precisely speaking it is a right near-ring because it satisfies the right distributive law. If the condition (iii) is replace by z(x + y) = z.x + z.y for all x, y, z∈N, then it is called left near-ring. We denote xy instead of x.y. A near-ring N is called zerosymmetric if x.0 = 0 for all x ∈N.

Given two subsets A and B of N, the product AB is defined as

AB = \{ab|a∈A, b∈B\}

A subgroup S of (N, +) is called left (right) N-subgroup of N if NS ≤S(SN ≤S). A subgroup M of (N, +) is called subnear-ring of N if MM ≤M. A subnear-ring M is called invariant in N if MN ≤ NM ≤ M.

2. PRELIMINARIES

Throughout this paper N stands for a right zero symmetric near-ring.

Definition 2.1: An ideal of a near-ring N is a subset I of N such that

(i) (I, +) is normal subgroup of (N, +)
(ii) 1 N ⊆I
(iii) y (x + i) – yx ∈ I for all x, y ∈ N and i ∈ I
Note that I is right ideal of N if I satisfies (i) and (ii), and I is left ideal of N if I satisfies (i) and (iii).

**Definition 2.2:** A subgroup Q of N is called a quasi-ideal of N if QN ∩ NQ = Q.

**Proposition 2.3:** Let Q be a quasi-ideal and M be a sub near-ring of a near-ring N then Q ∩ M is a quasi-ideal of N.

**Proposition 2.4:** Let N be a zero symmetric near-ring and Q be the subgroup of N. Then Q is a quasi-ideal of N if and only if QN ∩ NQ ⊆ Q.

**Proof:** Let n ∈ N, q ∈ Q be any element. As N is zero symmetric near-ring. Therefore nq = n(0 + q) = n0 ∈ N* Q implies NQ ⊆ N*Q so that NQ ∩ N*Q = NQ. Hence we have QN ∩ NQ ⊆ Q.

**Definition 2.5:** A subgroup B of N is called a bi-ideal of N if BNB ∩ (BN)*B ⊆ B.

**Proposition 2.6:** Let N be a zero symmetric near-ring and B be the subgroup of N. Then B is bi-ideal if and only if BNB ∩ (BN)*B ⊆ B.

**Proof:** Since N is a zero symmetric, therefore N ∩ N* B. Hence BNB = BNB ∩ BNB ⊆ BNB ∩ (BN)*B ⊆ B.

**Proposition 2.7:** Intersection of a bi-ideal B and a sub near-ring S of a near-ring is a bi-ideal of S.

**Lemma 2.8:** Let N be a zero symmetric near-ring and Q be a quasi-ideal in N. Then every Q is bi-ideal.

**Proof:** Let Q be a quasi ideal in a zero symmetric near-ring N. Then (Q, +) is a subgroup of N and QN ∩ NQ ⊆ Q. Now, QNQ ⊆ QN and QNQ ⊆ NQ. Thus, QNQ ⊆ QN ∩ NQ ⊆ Q. Hence Q is bi-ideal of N.

**Definition 2.9:** Let X be a non-empty set. A mapping μ : X → [0, 1] is a fuzzy set in X. The complement of μ, denoted by μ′, is the fuzzy set in X given by μ′(x) = 1 − μ(x) for all x ∈ X. For any I ⊆ X, μI denotes the characteristic function of I.

**Definition 2.10:** For any fuzzy set μ in X and r ∈ [0, 1], we define two sets, U(μ, r) = {x ∈ X | μ(x) ≥ r} and L(μ, r) = {x ∈ X | μ(x) ≤ r}, which are called an upper and lower r-level cut of μ respectively and can be used to characterize μ.

**Definition 2.11:** A fuzzy set μ in N is a fuzzy subnear-ring of N if for all x, y ∈ N,
(i) μ(x − y) ≥ min{μ(x), μ(y)}
(ii) μ(x+y) ≥ μ(x), μ(y).

**Definition 2.12:** A fuzzy set μ in N is a fuzzy bi-ideal of N if for all x, y ∈ N,
(i) μ(x − y) ≥ min{μ(x), μ(y)}
(ii) μ(x+y) ≥ min{μ(x), μ(y)} for all x, y, z ∈ N.

### 3. INTUITIONISTIC FUZZY SETS AND BI-IDEALS

**Definition 3.1:** An intuitionistic fuzzy set A in a non-empty set X is an object having the form A = \{(x, μ(x), ν(x)) | x ∈ X\}, where the functions μ, ν: X → [0, 1] denote the degree of membership and the degree of non-membership of each element x ∈ X in the set A, respectively, and 0 ≤ μ(x) + ν(x) ≤ 1 for all x ∈ X.

For the sake of simplicity, we shall use the symbol A = (μ, ν) for the intuitionistic fuzzy set A = \{(x, μ(x), ν(x)) | x ∈ X\}.

**Definition 3.2:** An intuitionistic fuzzy set A = (μ, ν) of a group (G, +) is said to be an intuitionistic fuzzy subgroup of G if for all x, y ∈ G
(i) μ(x+y) ≥ min{μ(x), μ(y)}
(ii) μ(x−y) = μ(x)
(iii) ν(x+y) ≤ max{ν(x), ν(y)}
(iv) ν(x−y) = ν(x)

Equivalently, μ(x-y) ≥ min{μ(x), μ(y)} and ν(x-y) ≤ max{ν(x), ν(y)} for all x, y ∈ G.
Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic fuzzy subset of a near-ring $N$. We define the product of $A$ and $B$ as $AB = (\mu_{AB}, \nu_{AB})$. If $S \subseteq N$, then, we define the characteristic function $\chi_S$ on $N$ is defined as

$$
\chi_S(x) = \begin{cases}
(1,0) & \text{if } x \in S \\
(0,1) & \text{if } x \in N \setminus S
\end{cases}
$$

The characteristic function on $N$ is $\chi_N$ and $\chi_N(x) = (1, 0)$ for all $x \in N$.

**Definition 3.3:** An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in $N$ is an intuitionistic fuzzy subnear-ring of $N$ if for all $x, y \in N$,

1. $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$
2. $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$
3. $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\}$
4. $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$.

**Definition 3.4:** An intuitionistic fuzzy set $A = (\mu_A, \nu_A)$ in $N$ is an intuitionistic fuzzy bi-ideal of $N$ if for all $x, y, z \in N$,

1. $\mu_A(x - y) \geq \min\{\mu_A(x), \mu_A(y)\}$
2. $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$
3. $\nu_A(x - y) \leq \max\{\nu_A(x), \nu_A(y)\}$
4. $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$.

**Lemma 3.5:** Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy set in $N$. Then $A$ is an intuitionistic fuzzy bi-ideal of $N$ if and only if the fuzzy sets $\mu_A$ and $\nu_A$ are fuzzy bi-ideals of $N$.

**Proof:** If $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy bi-ideal of $N$, then clearly $\mu_A$ is a fuzzy bi-ideal of $N$.

For all $x, y \in N$,

$$
\nu^\prime_A(x - y) = 1 - \nu_A(x - y)
\geq 1 - \max\{\nu_A(x), \nu_A(y)\}
\geq \min/1 - \nu_A(x), 1 - \nu_A(y)
\geq \min/\nu^\prime_A(x), \nu^\prime_A(y).
$$

For all $x, y, z \in N$,

$$
\nu^\prime_A(xyz) = 1 - \nu_A(xyz)
\geq 1 - \max\{\nu_A(x), \nu_A(y)\}
\geq \min/1 - \nu_A(x), 1 - \nu_A(z)
\geq \min/\nu^\prime_A(x), \nu^\prime_A(z).
$$

Thus $\nu^\prime_A$ is a fuzzy bi-ideal of $N$.

Conversely, suppose that $\mu_A$ and $\nu^\prime_A$ are fuzzy bi-ideals of $N$, then clearly the conditions (i) and (ii) of Definition 3.4 are valid.

Now for all $x, y \in N$,

$$1 - \nu_A(x - y) = \nu^\prime_A(x - y)
\geq \min/\nu^\prime_A(x), \nu^\prime_A(y)
\geq 1 - \max/\nu_A(x), \nu_A(y).
$$

Therefore $\nu_A(x - y) \leq \max/\nu_A(x), \nu_A(y)$.

For all $x, y, z \in N$,

$$1 - \nu_A(xyz) = \nu^\prime_A(xyz)
\geq \min/\nu^\prime_A(x), \nu^\prime_A(z)
\geq 1 - \max/\nu_A(x), \nu_A(z).
$$

Thus $\nu_A(xyz) \leq \max/\nu_A(x), \nu_A(z)$.

Thus $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy bi-ideal of $N$. 
Theorem 3.6: Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy set in \( N \). Then \( A \) is an intuitionistic fuzzy bi-ideal of \( N \) if and only if \( A = (\mu_A, \mu'_A) \) and \( A = (\nu_A, \nu'_A) \) are intuitionistic fuzzy bi-ideals of \( N \).

Proof: If \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy bi-ideal of \( N \), then \( \mu_A = (\mu'_A)^* \) and \( \nu_A = (\nu'_A)^* \) are fuzzy bi-ideals of \( N \), from Lemma 3.5. Therefore \( A = (\mu_A, \mu'_A) \) and \( A = (\nu_A, \nu'_A) \) are intuitionistic fuzzy bi-ideals of \( N \).

Conversely, if \( A \) and \( A \) are intuitionistic fuzzy bi-ideals of \( N \), then the fuzzy sets \( \mu_A \) and \( \nu'_A \) are fuzzy bi-ideals of \( N \). Therefore \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy bi-ideal of \( N \).

Theorem 3.7: Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy subset of \( N \). Then \( A \) is a fuzzy bi-ideal of \( N \) if and only if all the non-empty sets \( U(\mu_A, r) \) and \( V(\nu_A, t) \) are bi-ideals of \( N \) for all \( r \in \text{Im}(\mu_A) \) and \( t \in \text{Im}(\nu_A) \) respectively.

Proof: Suppose that \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy bi-ideal of \( N \). For \( x, y \in U(\mu_A, r) \), we have \( \mu_A(x-y) \geq \min(\mu_A(x), \mu_A(y)) \). Hence, \( x-y \in U(\mu_A, r) \). Let \( x, z \in U(\mu_A, r) \) and \( y \in N \). Then \( \mu_A(xyz) \geq \min(\mu_A(x), \mu_A(z)) \) for all \( x, y, z \in U(\mu_A, r) \).

Hence \( U(\mu_A, r) \) is a bi-ideal of \( N \) for all \( r \in \text{Im}(\mu_A) \). Similarly, we can show that \( L(\nu_A, t) \) is also a bi-ideal of \( N \) for all \( t \in \text{Im}(\nu_A) \).

Conversely suppose that \( U(\mu_A, r) \) and \( L(\nu_A, t) \) are bi-ideals of \( N \) for all \( r \in \text{Im}(\mu_A) \) and \( t \in \text{Im}(\nu_A) \) respectively. Suppose that \( x, y \in N \) and \( \mu_A(x-y) < \min(\mu_A(x), \mu_A(y)) \). Choose \( r \) such that \( \mu_A(x-y) < r < \min(\mu_A(x), \mu_A(y)) \). Then we get \( x-y \in U(\mu_A, r) \) but \( x-y \notin U(\mu_A, r) \), a contradiction.

Hence \( \mu_A(x-y) \geq \min(\mu_A(x), \mu_A(y)) \). A similar argument shows that \( \nu_A(xyz) \geq \min(\nu_A(x), \nu_A(z)) \) for all \( x, y, z \in N \). Likewise we can show that \( \nu_A(x-y) \geq \max(\nu_A(x), \nu_A(y)) \) and \( \nu_A(xyz) \geq \max(\nu_A(x), \nu_A(z)) \). Hence \( A = (\mu_A, \nu_A) \) is an intuitionistic fuzzy bi-ideal of \( N \).

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