Volume 9, No. 1, Jan. - 2018 (Special Issue)
International Journal of Mathematical Archive-9(1), 2018, 238-244
Available online through www.ijma.info ISSN 2229 – 5046

INTERSECTING INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS

K. K. MYITHILI

Assistant Professor, PG Department of Mathematics,
Vellalar College for Women, Erode-638012, Tamil Nadu, India.

E-mail: mathsmyth@gmail.com

ABSTRACT

***If** the edges of the Intuitionistic Fuzzy Directed Hypergraphs (IFDHGs) $H = (V, E)$, are pairwise not disjoint, then H is said to be an intersecting IFDHG. The definitions like essentially intersecting, $\lozenge -$ intersecting and sequentially simple intersecting IFDHGs has been defined. Some of its properties have also been analyzed. Also it has been proved that the IFDHG H is strongly intersecting IFDHG if and only if $H^{r_i,s_i} \subseteq Tr(H^{r_i,s_i})$ for every $H^{r_i,s_i} \epsilon C(H)$.*

Keywords: *Essentially intersecting, $\lozenge -$ intersecting, sequentially simple intersecting IFDHGs.*

AMS Classification: *03E72.*

1. INTRODUCTION

Lotif. A. Zadeh introduced Fuzzy sets (FSs) in 1985[15], which are generalization of crisp sets. K.T. Atanassov introduced the concept of Intuitionistic Fuzzy Sets (IFSs) as an extension of FSs in 1999[1]. These sets include not only the membership of the set but also the non-membership of the set along with the degree of uncertainty. In order to expand the concept in application base, the notion of graph theory was generalized to that of a hypergraph. Claude Berge [2] introduced the concept of graph and hypergraph in 1976. In this paper, a few extensions of concepts in fuzzy hypergraphs by John N. Mordeson and Premchand S. Nair [3] have been carried out.

The paper has been organized as follows:

Section 2 deals with the definitions of fuzzy hypergraph, intuitionistic fuzzy hypergraph, IFDHG and the notations used in this paper. In section 3, a study is made on essentially intersecting, $\lozenge -$ intersecting and sequentially simple intersecting IFDHGs. Some properties of newly proposed hypergraph concepts are also discussed and it has been proved that H is strongly intersecting if and only if H^{r_i, s_i} is intersecting IFDHG, $\forall \langle r_i,\ s_i\rangle \,\epsilon\, F(H)$. Section 4, concludes the paper.

2. PRELIMINARIES

The notations used in this work are listed below:

$H = (V, E)$	- IFDHG with vertex set V and edge set E
$\langle \mu_i, \nu_i\rangle$	- degrees of membership and non-membership of the vertex
$\langle \mu_{ij}, \nu_{ij}\rangle$	- degrees of membership and non-membership of the edges
$\langle \mu_{ij}(v_i), \nu_{ij}(v_i)\rangle$	- degrees of membership and non-membership of the edges containing v_i
$h(H)$	- Height of a hypergraph H
$F(H)$	- Fundamental sequence of H
$C(H)$	- Core set of H
$H^{(r_i,s_i)}$	- $\langle r_i, s_i\rangle$ - level of H
$IF_p(v)$	- IF power set of V
$Tr(H)$	- Intuitionistic fuzzy transversals (IFT) of H

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

$Tr(H^{r_i,s_i})$ - $\langle r_i, s_i \rangle$ - level of $Tr(H)$
$H^{\square}$ - Skeleton of H

In this section, definitions of intuitionistic fuzzy set, intuitionistic fuzzy graph, IFDHG has been dealt with.

Definition 2.1: [1] Let a set E be fixed. An *intuitionistic fuzzy set (IFS) V* in *E* is an object of the form $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, where the function $\mu_i : E \to [0,1]$ and $\nu_i : E \to [0,1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E, 0 \le \mu_i(v_i) + \nu_i(v_i) \le 1$.

Definition 2.2: [14] Let E be fixed set and $V = \{\langle v_i, \mu_i(v_i), \nu_i(v_i)\rangle / v_i \in E\}$, be an IFS. Six types of Cartesian products of *n* subsets[1] $V_1, V_2 \ldots.. V_n$ of *V* over *E* are defined as

$$V_1 \times_1 V_2 \times_1 \ldots \times_1 V_n = \{\langle (v_1, v_2 \ldots v_n), \prod_{i=1}^n \mu_i, \prod_{i=1}^n \nu_i \rangle | \, v_1 \in V_1, v_2 \in V_2 \ldots v_n \in V_n\},$$

$$V_{i_1} \times_2 V_{i_2} \times_2 \ldots \times_2 V_{i_n} = \{\langle (v_1, v_2 \ldots v_n), \sum_{i=1}^n \mu_i - \sum_{i=1}^n \mu_i \mu_j - \sum_{i\ne j\ne k} \mu_i \mu_j \mu_k - \ldots + (-1)^{n-2} \sum_{i\ne j\ne k\ne\cdots\ne n} \mu_i \mu_j \mu_k \ldots \mu_n + (-1)^{n-1} \prod_{i=1}^n \mu_i, \prod_{i=1}^n \nu_i \Big| \, v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

$$V_{i_1} \times_3 V_{i_2} \times_3 \ldots \times_3 V_{i_n} = \left\{ \langle (v_1, v_2 \ldots v_n), \prod_{i=1}^n \mu_i, \sum_{i=1}^n \nu_i - \sum_{i\ne j}^n \nu_i \nu_j + \sum_{i\ne j\ne k}^n \nu_i \nu_j \nu_k - \cdots + (-1)^{n-2} \sum_{i\ne j\ne k\ne\cdots\ne n} \nu_i \nu_j \nu_k \ldots \nu_n + (-1)^{n-1} \prod_{i=1}^n \nu_i \rangle \right| v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

$$V_{i_1} \times_4 V_{i_2} \times_4 \ldots \times_4 V_{i_n} = \{\langle (v_1, v_2 \ldots v_n), \min(\mu_1, \mu_2, \ldots \mu_n), max(\nu_1, \nu_2, \ldots \nu_n)\rangle | \, v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

$$V_{i_1} \times_5 V_{i_2} \times_5 \ldots \times_5 V_{i_n} = \{\langle (v_1, v_2 \ldots v_n), \max(\mu_1, \mu_2, \ldots \mu_n), min(\nu_1, \nu_2, \ldots \nu_n)\rangle | \, v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

$$V_{i_1} \times_6 V_{i_2} \times_6 \ldots \times_6 V_{i_n} = \{\langle (v_1, v_2 \ldots v_n), \frac{\sum_{i=1}^n \mu_i}{2}, \frac{\sum_{i=1}^n \nu_i}{2} \rangle | \, v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

It must be noted that $v_i \times_s v_j$ is an IFS, where *s* = 1, 2, 3, 4, 5, 6.

Definition 2.3: [4] An *intuitionistic fuzzy graph (IFG)* is of the form $G = (V, E)$ where (i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu : E \to [0,1]$ and $\nu : E \to [0,1]$ denote the degrees of membership and non-membership of the vertex $v_i \in V$ respectively and

$$0 \le \mu_i(v_i) + \nu_i(v_i) \le 1 \quad (1)$$

for every $v_i \in V, i = 1,2,3 \ldots n$.

(ii) $E \subseteq V \times V$ where $\mu_{ij}: V \times V \to [0,1]$ and $\nu_{ij}: V \times V \to [0,1]$ are such that

$$\mu_{ij} \le \mu_i \emptyset \mu_j \quad (2)$$

$$\nu_{ij} \le \nu_i \emptyset \nu_j \quad (3)$$

and $0 \le \mu_{ij} + \nu_{ij} \le 1$ (4)

[1]<u>subsets -</u> crisp sense

where μ_{ij} and ν_{ij} are the degrees of membership and non-membership of the edge (v_i, v_j); the values of $\mu_i \emptyset \mu_j$ and $\nu_i \emptyset \nu_j$ can be determined by one of the cartesian products $\times_s, s = 1,2,3, \ldots 6$ for all *i* and *j* given in Definition 2.2.

Note: Throughout this paper, it is assumed that the fifth Cartesian product in Definition 2.2

$$V_{i_1} \times_5 V_{i_2} \times_5 \ldots \times_5 V_{i_n} = \{\langle (v_1, v_2 \ldots v_n), \max(\mu_1, \mu_2, \ldots \mu_n), min(\nu_1, \nu_2, \ldots \nu_n)\rangle | \, v_1 \in V_1, v_2 \in V_2, \ldots v_n \in V_n\}$$

is used to determine the degrees of membership μ_{ij} and non-membership ν_{ij} of the edge e_{ij}.

Definition 2.4: [5] An *intuitionistic fuzzy hypergraph* (IFHG) is an ordered pair $H = (V, E)$ where
(i) $V = \{v_1, v_2, \ldots, v_n\}$, is a finite set of intuitionistic fuzzy vertices,
(ii) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of crisp subsets of V
(iii) $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)): \mu_j(v_i), \nu_j(v_i) \ge 0$ and $\mu_j(v_i), \nu_j(v_i) \le 1\}, j = 1,2, \ldots, m$,
(iv) $E_j \ne \phi, j = 1,2,3, \ldots m$.

Here, the hyperedges E_j are crisp sets of intuitionistic fuzzy vertices $\mu_j(v_i)$ and $\nu_j(v_i)$ denote the degrees of membership and non-membership of vertex v_i to edge E_j. Thus, the elements of the incidence matrix of IFHG are of the form $(v_{ij}, \mu_j(v_i), \nu_j(v_i))$. The sets *(V,E)* are crisp sets.

Note: The support of an IFS *V* in *E* is denoted by $supp(E_j) = \{v_i / \mu_{ij}(v_i) > 0$ and $\nu_{ij}(v_i) > 0\}$.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Definition 2.5: [6] An IFDHG H is a pair *(V,E)*, where V is a non - empty set of vertices and E is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $E_i \in E$ is defined as a pair $t(E_i), h(E_i))$, where $(E_i) \subset V$, with $t(Ei) \neq \phi$, is its tail, and $h(E_i) \in V - t(E_i)$ is its head. A vertex s is said to be a source vertex in H if $h(E_i) \neq s$, for every $E_i \in E$. A vertex d is said to be a destination vertex in H if $d \neq t(E_i)$, for every $E_i \in E$.

Definition 2.6: [7] Let H be an IFDHG, let $H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})$ be the $\langle r_i, s_i\rangle$-level IFDHG of H. The sequence of real numbers $\{r_1, r_2, \ldots, r_n; s_1, s_2, \ldots, s_n\}$, such that $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$, satisfying the properties:

(i) If $r_1 < \alpha \leq 1$ and $0 \leq \beta < s_1$ then $E^{\alpha,\beta} = \varphi$,

(ii) If $r_i + 1 \leq \alpha \leq r_i$; $s_i \leq \beta \leq s_i + 1$ then $E^{\alpha,\beta} = E^{r_i,s_i}$

(iii) $E^{r_i,s_i} \sqsubset E^{r_{i+1},s_{i+1}}$

is called the fundamental sequence of H, and is denoted by *F(H)*. The core set of H is denoted by *C(H)* and is defined by $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}, \ldots, H^{r_n,s_n}\}$. The corresponding set of $\langle r_i, s_i\rangle$- level hypergraphs $H^{r_1,s_1} \subset H^{r_2,s_2} \subset \ldots. \subset H^{r_n,s_n}$ is called the H induced fundamental sequence and is denoted by *I(H)*. The $\langle r_n, s_n\rangle$- level is called the support level of H and the H^{r_n,s_n} is called the support of H.

Definition 2.7: [7] Let H be an IFDHG and $C(H) = \{H^{r_1,s_1}, H^{r_2,s_2}, \ldots, H^{r_n,s_n}\}$. H is said to be ordered if *C(H)* is ordered. That is $H^{r_1,s_1} \subset H^{r_2,s_2} \subset \ldots. \subset H^{r_n,s_n}$. The IFDHG is said to be simply ordered if the sequence $\{H^{r_i,s_i}/i = 1,2,3\ldots, n\}$ is simply ordered, that is if it is ordered and if whenever $E \in H^{r_{i+1},s_{i+1}} - H^{r_i,s_i}$ then $E \not\subseteq H^{r_i,s_i}$.

Definition 2.8: [9] Let H be an IFDHG with core set $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i}) | i = 1, 2, \ldots n\}$, where $E(H^{r_i,s_i}) = E_i$ is the crisp edge set of the core hypergraph H^{r_i,s_i}. Let *E (H)* denote the crisp edge set of H defined by $E(H) = \cup \{E_i/E_i = E(H^{r_i,s_i}); H^{r_i,s_i} \in C(H)\}$. *E (H)*, a crisp hypergraph on *V*, is called *core aggregate hypergraph* of H and is denoted by $\mathcal{H}(H) = (V, E(H))$.

Definition 2.9: [9] An IFDHG H is said to be an *intersecting intuitionistic fuzzy directed hypergraph*, if for each pair of intuitionistic fuzzy hyperedge $\{E_i, E_j\} \subseteq E$, $E_i \cap E_j \neq \phi$.

Definition 2.10: [9] Let H be an IFDHG and $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1, 2, \ldots n\}$, if H^{r_i,s_i} is an intersecting IFDHG for each $i = 1, 2, \ldots, n$ then H is *K-intersecting IFDHG*.

Definition 2.11: [9] An IFDHG H is said to be *strongly intersecting*, if for any two edges E_i and E_j contain a common spike of height, $h = h(E_i) \wedge h(E_j)$.

Definition 2.12: [8] Let H be an IFDHG. A *primitive k-coloring A* of H is a partition $\{A_1, A_2, A_3, \ldots. A_k\}$ of V into k-subsets (colors) such that the support of each intuitionistic fuzzy hyperedge of H intersects atleast two colors of *A*, except spike edges.

Definition 2.13: [8] The *k-chromatic number* of an IFDHG H is the minimal number $\chi_k(H)$, of colors needed to produce a primitive coloring of *H*. The *chromatic number* of H is the minimal number, $\chi(H)$, of colors needed to produce a *K*-coloring of *H.*

Theorem 2.1: [8] If H is an ordered IFDHG and A is a primitive coloring of *H*, then A is a K - coloring of *H.*

Theorem 2.2: [9] Let H be an IFDHG and suppose $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1, 2, \ldots n\}$. Then H is intersecting if and only if $H^{r_n,s_n} = (V^{r_n,s_n}, E^{r_n,s_n})$ is intersecting.

Theorem 2.3: [9] Let H be an ordered IFDHG and let $C(H) = \{H^{r_i,s_i} = (V^{r_i,s_i}, E^{r_i,s_i})/i = 1, 2, \ldots n\}$, then H is intersecting if and only if H is *K*-intersecting.

Theorem 2.4: [9] If $H^{\square}$ is intersecting, then H is strongly intersecting.

Definition 2.14: [8] A *spike reduction* of $E_i \epsilon F_{\wp}(V)$, denoted by $\breve{E}$ is defined as

$$\breve{E}(v_i) = max_i\{\langle r_i,\ s_i\rangle / \left|E_i^{r_i,s_i}\right| \geq 2, (0 \leq r_i \leq E_\mu(v_i), 0 \leq s_i \leq E_\nu(v_i))\}.$$

Note:

i) If $A = \emptyset$ then $\breve{E}(v_i) = 0$

ii) If E_i is spike, then $\breve{E} = \chi_0$.

Definition 2.15: [8] Let H be an IFDHG and let $\breve{H} = (\breve{V}, \breve{E})$, where $\breve{E} = \{\breve{E}_i | E_i \epsilon E\}$ and $\breve{V} = \cup_{\breve{E}_i \epsilon \breve{E}}\, supp(\breve{E})$.

Theorem 2.5: [3] H is intersecting if and only if H is K-intersecting.

Theorem 2.6: [3] If H is a crisp intersecting hypergraph, then $\chi(H) \leq 3$.

Theorem 2.7: [3] A crisp hypergraph H is intersecting if and only if $H \subseteq Tr(H)$.

Theorem 2.8: [8] Let H be an IFDHG. Then H is strongly intersecting if and only if H is K-intersecting.

3. INTERSECTING IFDHG

Definition 3.1: Let H be an IFDHG. Then H is said to be *essentially intersecting* if $\breve{H}$ is intersecting. And H is said to be *essentially strongly intersecting* if $\breve{H}$ is strongly intersecting.

Example 3.1: Consider an IFDHG, H with the incidence matrix as given below:

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \\ \end{array}$$

$$H = \begin{array}{c|cccc} & E_1 & E_2 & E_3 & E_4 \\ \hline v_1 & \langle 0.7,0.1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.5,0.3\rangle \\ v_2 & \langle 0.7,0.1\rangle & \langle 0.5,0.2\rangle & \langle 0.5,0.2\rangle & \langle 0,1\rangle \\ v_3 & \langle 0.7,0.1\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0,1\rangle \\ v_4 & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0.5,0.3\rangle \end{array}$$

The corresponding graph of IFDHG H is displayed in Figure 3.1.

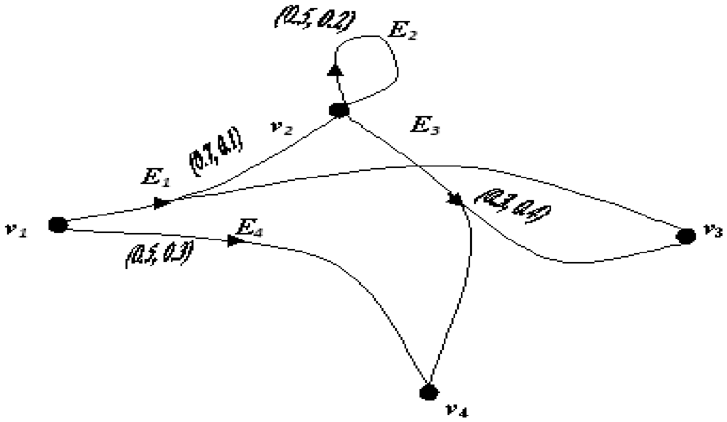

Figure-3.1: Intersecting IFDHG

Figure 3.2 depicts essentially intersecting IFDHG

$$H = \begin{array}{c|ccc} & E_1 & E_2 & E_3 \\ \hline v_1 & \langle 0.7,0.1\rangle & \langle 0,1\rangle & \langle 0.5,0.3\rangle \\ v_2 & \langle 0.7,0.1\rangle & \langle 0.5,0.2\rangle & \langle 0,1\rangle \\ v_3 & \langle 0.7,0.1\rangle & \langle 0.3,0.4\rangle & \langle 0,1\rangle \\ v_4 & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0.5,0.3\rangle \end{array}$$

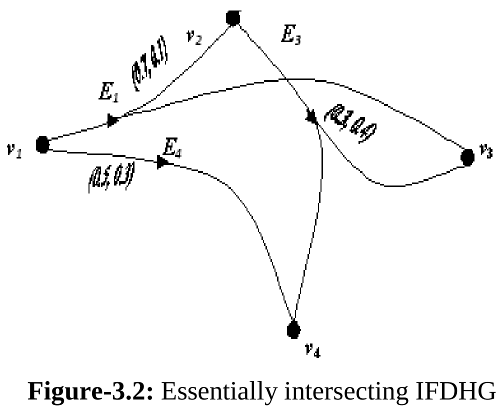

Figure-3.2: Essentially intersecting IFDHG

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

Definition 3.2: Let H be an IFDHG and $H^{\lozenge} = (\breve{H})^{\square}$, then H is called $\lozenge -$ *intersecting* if $H^{\lozenge}$ is intersecting.

Note:
(i) For our convenience, assume $F(H^{\lozenge}) = \{r_1^s, r_2^s, \dots, r_k^s; s_1^s, s_2^s, \dots, s_k^s\}$, where $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$ and
(ii) $F(\breve{H}) = \{r_1, r_2, \dots, r_m; s_1, s_2, \dots, s_m\}$, where $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$.

Theorem 3.1: If H is $\lozenge -$ *intersecting IFDHG*, then H is *essentially strongly intersecting IFDHG*.

Note: In general, the converse need not be true.

Theorem 3.2: If H is ordered and *essentially intersecting IFDHG*, then $\chi(H) \leq 3$.

Proof: Assume $\breve{H}$ exists, then $\chi(H) = 1$. Let $(\breve{H})^{r_m, s_m} \epsilon C(\breve{H})$, where $\langle r_m, s_m \rangle \epsilon F(H)$ will be the smallest value. Since $\breve{H}$ is intersecting IFDHG, it follows from theorem 2.2 that $(\breve{H})^{r_m, s_m}$ is also intersecting. Hence by theorem 2.6 $\chi((\breve{H})^{r_m, s_m}) \leq 3$. Also since H is ordered, $\breve{H}$ is also ordered. By definition 2.13 and theorem 2.1 it follows that, $\chi((\breve{H})^{r_m, s_m}) \leq 3$ and by definition 2.14, $\chi(\breve{H}) \leq 3$. Hence, $\chi(H) = \chi(\breve{H})$.

Theorem 3.3: If H is elementary and essentially intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: Since H is ordered, the result is obvious from theorem 3.2.

Theorem 3.4: If H is of the form $\mu \otimes H$ and essentially intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: The result is obvious, since H is elementary.

Theorem 3.5: If H is $\lozenge$ $-$ intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: Given $H^{\lozenge}$ is intersecting. Also $H^{\lozenge}$ is elementary. Hence by theorem 3.3 $\chi(H) \leq 3 \Longrightarrow \chi(H^{\lozenge}) \leq 3$. Since $\chi(H^{\lozenge}) = \chi(\breve{H}) = \chi(H)$.The result follows obviously.

Note: Since $H = \breve{H}, K -$ coloring of skeleton $H^{\square}$, of H may not be extendible to $K -$ coloring of H, or if extendible, then it may not use the new colors. Therefore, if $H = \breve{H}$ then $\chi(H^{\square}) < \chi(H)$.

Definition 3.3: Let $H = \{v_i \epsilon IF_{\wp}(V) | i = 1,2, \dots, n\}$ is a finite collection of intuitionistic fuzzy subsets of V and let $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$. Then $H|_{\langle r_i, s_i \rangle} = \{v \epsilon F_{\wp}(V) | h(v) = \langle r_i, s_i \rangle\}$ denotes the set of edges in K of height $\langle r_i, s_i \rangle$. In general, H^{r_i, s_i} denotes the partial IFDHG of $H = (V, E)$ with the edge set E^{r_i, s_i} provided $E^{r_i, s_i} \neq \emptyset$.

Definition 3.4: Let $H_i = (V_i, E_i), i = 1,2$ be an IFDHG. Then $H_1 \subseteq H_2$ if every edge of H_1contains an edge H_2.

Theorem 3.6: H is strongly intersecting IFDHG if and only if $H^{r_i, s_i} \subseteq Tr(H^{r_i, s_i})$ for every $H^{r_i, s_i} \epsilon C(H)$.

Proof: By theorem 2.8, definition 2.11 and theorem 2.7 it is obvious that

H is strongly intersecting IFDHG $\Leftrightarrow$ H is $K -$ intersecting IFDHG

$\Leftrightarrow H^{r_i, s_i}$ is intersecting for all $H^{r_i, s_i} \epsilon C(H)$

$\Leftrightarrow H^{r_i, s_i} \subseteq Tr(H^{r_i, s_i})$ for every $\langle r_i, s_i \rangle \epsilon F(H)$, $H^{r_i, s_i} \epsilon C(H)$.

Theorem 3.7: H is strongly intersecting IFDHG if and only if for every $\langle r_i, s_i \rangle \epsilon F(H), H^{r_i, s_i}|_{\langle r_i, s_i \rangle} \subseteq Tr(H^{r_i, s_i})$.

Proof: Let for every $\langle r_i, s_i \rangle \epsilon F(H), H^{r_i, s_i}|_{\langle r_i, s_i \rangle} \subseteq Tr(H^{r_i, s_i}$. For each $H^{r_i, s_i} \epsilon C(H)$, the edge set $E(H^{r_i, s_i}) = \{E^{r_i, s_i} | E \epsilon H^{r_i, s_i}|_{\langle r_i, s_i \rangle}\} \subseteq \{\{\tau^{r_i, s_i} | \tau \epsilon Tr(H^{r_i, s_i})\} = Tr(E(H^{r_i, s_i}))$. Hence, $H^{r_i, s_i} \subseteq Tr(H^{r_i, s_i})$ for all $H^{r_i, s_i} \epsilon C(H)$ and by theorem 3.6, H is strongly intersecting IFDHG.

Conversely, let H is strongly intersecting IFDHG. And suppose $E \epsilon H|_{\langle r_i, s_i \rangle}$ where $\langle r_i, s_i \rangle$ the largest member of F (H) be. Let $H^{r_i, s_i} \epsilon C(H)$.

To Prove: E^{r_i, s_i} is the transversal of H^{r_i, s_i}.

Let $E \epsilon H^{r_i, s_i}$. Then there exists an edge E_1 of H such that $E_1^{r_i, s_i} = E$. Since, H is strongly intersecting IFDHG, there is a spike σ_v with height $h(\sigma_v) = h(E) \wedge h(E_1) = h(E_1) \geq \langle r_i, s_i \rangle$. And support of $\{v\}$, which is contained in both E and E_1. Hence, $v \epsilon E \cap E^{r_i, s_i}$. Thus E is a transversal of H and therefore it contains a member of $Tr(H)$. Therefore, $H^{r_i, s_i}|_{\langle r_i, s_i \rangle} \subseteq Tr(H^{r_i, s_i})$.

By theorem 2.8, it is true that H is $K-$ intersecting IFDHG. Again by the same theorem, it follows that H^{r_i, s_i} is strongly intersecting. Hence, $H^{r_i, s_i}|_{\langle r_i, s_i \rangle} \subseteq Tr(H^{r_i, s_i})$ for every $\langle r_i, s_i \rangle \epsilon F(H)$.

Theorem 3.8: Let H be an IFDHG with$C(H) = \{H^{r_i, s_i} | \langle r_i, s_i \rangle \epsilon F(H)\}$. Then $H^{r_i, s_i} \subseteq Tr(H^{r_i, s_i})$ for every $H^{r_i, s_i} \epsilon C(H)$ if and only if $H^{r_i, s_i}|_{\langle r_i, s_i \rangle} \subseteq Tr(H^{r_i, s_i})$ for every $\langle r_i, s_i \rangle \epsilon F(H)$.

Proof: By theorem 3.6 and 3.7, the proof is obvious.

Theorem 3.9: H is strongly intersecting IFDHG if and only if H^{r_i, s_i} is intersecting for every $\langle r_i, s_i \rangle \epsilon F(H)$.

Proof: By theorem 2.2 and 2.8, the following equivalencies holds good.
H^{r_i, s_i} is intersecting for every $\langle r_i, s_i \rangle \epsilon F(H) \Leftrightarrow E(H^{r_i, s_i})$ is intersecting for each $H^{r_i, s_i} \epsilon C(H)$
$\Leftrightarrow H$ is $K-$ intersecting IFDHG
$\Leftrightarrow H$ is strongly intersecting IFDHG.

Definition 3.5: An IFDHG is said to be *non-trivial* if it has atleast one edge E such that $|supp(E)| \geq 2$.

Definition 3.6: An IFDHG is said to be *sequentially simple* if $C(H) = \{H^{r_i, s_i} = (V^{r_i, s_i}, E^{r_i, s_i}) | \langle r_i, s_i \rangle \epsilon F(H)$ satisfies the property that if $E \epsilon E^{r_{i+1}, s_{i+1}} \setminus E^{r_i, s_i}$, then $E \nsubseteq V^{r_i, s_i}$ where $0 \leq r_i \leq h_\mu(H)$ and $0 \leq s_i \leq h_\nu(H)$. H is said to be *essentially sequentially simple* if $\breve{H}$ is *sequentially simple.*

Theorem 3.10: If H is an ordered IFDHG. Then the following statements holds:

i) If H is intersecting if and only if $H^{\square}$ is intersecting.
ii) $\breve{H}$ is intersecting if and only if $H^{\lozenge}$ is intersecting.

Proof: Since $H^{\lozenge} = (\breve{H})^{\square}$ and $\breve{H}$ is ordered whenever H is non-trivial ordered IFDHG (ii) is true. Also since H is ordered, $supp(H) = \cup \{E(H^{r_i, s_i}) | H^{r_i, s_i} \epsilon C(H)\}$. Thus, $supp(H^{\square}) \subseteq supp(H)$. Again by construction of $H^{\square}$, every member of the edge set $E(H^{r_i, s_i})$ is either a member or it contains a member of $supp(H^{\square})$. Hence, for any two edges $E_1, E_2 \subseteq supp(H)$ there exists corresponding edges $E_1', E_2' \subseteq supp(H^{\square})$ such that $E_1' \subseteq E_1$ and $E_2' \subseteq E_2$. Therefore, $supp(H^{\square})$ is intersecting $\Leftrightarrow supp(H)$ is intersecting. Hence (i) is proved.

Theorem 3.11: Let H be an IFDHG. Then the following conditions holds good.

i) If $H^{\square}$ is intersecting, then H is strongly intersecting.
ii) If $H^{\lozenge}$ is intersecting, then $\breve{H}$ is strongly intersecting.

Proof: It is obvious that the edge E in the core hypergraph $H^{r_i, s_i} \epsilon C(H)$ contains a member of $supp(H^{\square})$ by the construction process explained in [8] and also by $H^{\square}$ is elementary. Hence, if $supp(H^{\square})$ is intersecting, then every core hypergraph, H^{r_i, s_i} of H is also intersecting. Therefore, H is $K-$ intersecting and by theorem 2.8, H is strongly intersecting.

Example 3.2: Consider an IFDHG, H with the incidence matrix as given below:

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{pmatrix} \langle 0.7,0.1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.5,0.4 \rangle \\ \langle 0.7,0.1 \rangle & \langle 0.5,0.2 \rangle & \langle 0.5,0.2 \rangle & \langle 0.2,0.1 \rangle \\ \langle 0.7,0.1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.4 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.4 \rangle & \langle 0.5,0.2 \rangle \end{pmatrix} \end{array}$$

In example 3.2, H is strongly intersecting.

CONFERENCE PAPER
International Conference dated 08-10 Jan. 2018, on Intuitionistic Fuzzy Sets and Systems (ICIFSS - 2018), Organized by Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.

The incidence matrix for $H^{\square}$ is

$$H = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} E_1 & E_2 & E_3 & E_4 \end{array} \\ \begin{pmatrix} \langle 0.7,0.1\rangle & \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.5,0.4\rangle \\ \langle 0,1\rangle & \langle 0.5,0.2\rangle & \langle 0,1\rangle & \langle 0,1\rangle \\ \langle 0.7,0.1\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0,1\rangle \\ \langle 0,1\rangle & \langle 0,1\rangle & \langle 0.3,0.4\rangle & \langle 0.5,0.2\rangle \end{pmatrix} \end{array}$$

Here, $H^{\square}$ is not intersecting.

4. CONCLUSION

In this paper, an attempt has been made to study the intersecting intuitionistic fuzzy directed hypergraphs. Also, essentially intersecting, $\lozenge -$ intersecting and sequentially simple intersecting IFDHGs have been defined. Some of its properties have also been analyzed. Also it has been proved that the IFDHG H is strongly intersecting IFDHG if and only if $H^{r_i,s_i} \subseteq Tr(H^{r_i,s_i})$ for every $H^{r_i,s_i} \epsilon C(H)$.

5. REFERENCES

1. Atanassov. K. T, *Intuitionistic fuzzy sets - Theory and Applications*, New York, Physica-verlag, Berlin (1999).
2. Berge .C, *Graphs and Hypergraphs*, North-Holland, NewYork, 1976.
3. Mordeson N. John, Nair S. Premchand, *Fuzzy graphs and fuzzy hypergraphs*, New York, Physica-verlag, (2000).
4. Parvathi. R and Karunambigai. M. G, *Intuitionistic fuzzy graphs*, Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence, Advances in soft computing: Computational Intelligence, Theory and Applications, Springer-Verlag, 20(2006), 139-150.
5. Parvathi. R, Thilagavathi. S and Karunambigai. M. G, *Intuitionistic fuzzy hypergraph*, Bulgarian Academy of Sciences, Cybernetics and Information Technologies, Vol.9 (2009) , No.2,46-53.
6. Parvathi. R, Thilagavathi. S, *Intuitionistic fuzzy directed hypergraphs*, Advances in Fuzzy Sets and Systems 14(1), 2013, 39-52.
7. Parvathi. R, Myithili. K. K and Akram. M, *Certain types of intuitionistic fuzzy directed hypergraphs*, International Journal of Cybernatics and Machine Learning, http://link.springer.com/article/10.1007/s13042-014-0253-1 (2014), 1-9.
8. Parvathi. R and Myithili. K. K, *Coloring of intuitionistic fuzzy directed hypergraphs*, International Journal of Computer Applications, 6(3), 2016, 159-166.
9. Parvathi. R and Myithili. K. K, *Chromatic values of intuitionistic fuzzy directed hypergraph colorings*, International Journal of Soft Computing and Engineering, 6(1), 2016, 32-37.
10. Rosenfeld. A, *Fuzzy graphs*, Fuzzy sets and their applications, L.A.Zadeh, K.S.Fu and M.Shimura Eds., Academic Press, NewYork, 1975, 77-95.
11. Roy H. Goetschel Jr., *Introduction to fuzzy hypergraphs and hebbian structures*, Fuzzy Sets and Systems, 76 (1995), 113-130.
12. Roy H. Goetschel Jr., William L.Craine and William Voxman, *Fuzzy transversals of fuzzy hypergraphs*, Fuzzy Sets and Systems, 84 (1996), 235-254.
13. Roy H. Goetschel Jr., *Fuzzy colorings of fuzzy hypergraphs*, Fuzzy Sets and Systems, 94 (1998), 185-204.
14. Tony Shannon. A, Atanassov. K, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceedings of the First Workshop on Fuzzy Based Expert Systems (D. Lakov, Ed.), Sofia, Sept. 28-30, 1994, 59-61.
15. L.A.Zadeh, *Fuzzy sets*, Information and Control, 8, (1965), 338-353.

Source of support: Proceedings of UGC Funded International Conference on Intuitionistic Fuzzy Sets and Systems (ICIFSS-2018), Organized by: Vellalar College for Women (Autonomous), Erode, Tamil Nadu, India.