INTERSECTING INTUITIONISTIC FUZZY DIRECTED HYPERGRAPHS

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ABSTRACT

If the edges of the Intuitionistic Fuzzy Directed Hypergraphs (IFDHGs) \( H = (V, E) \), are pairwise not disjoint, then \( H \) is said to be an intersecting IFDHG. The definitions like essentially intersecting, \( \diamond \) – intersecting and sequentially simple intersecting IFDHGs have been defined. Some of its properties have also been analyzed. Also it has been proved that the IFDHG \( H \) is strongly intersecting IFDHG if and only if \( H^{r, s_i} \subseteq Tr(H^{r, s_i}) \) for every \( H^{r, s_i} \in \mathcal{C}(H) \).

Keywords: Essentially intersecting, \( \diamond \) – intersecting, sequentially simple intersecting IFDHGs.

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1. INTRODUCTION

Lotif. A. Zadeh introduced Fuzzy sets (FSs) in 1985[15], which are generalization of crisp sets. K.T. Atanassov introduced the concept of Intuitionistic Fuzzy Sets (IFSs) as an extension of FSs in 1999[1]. These sets include not only the membership of the set but also the non-membership of the set along with the degree of uncertainty. In order to expand the concept in application base, the notion of graph theory was generalized to that of a hypergraph. Claude Berge [2] introduced the concept of graph and hypergraph in 1976. In this paper, a few extensions of concepts in fuzzy hypergraphs by John N. Mordeson and Premchand S. Nair [3] have been carried out.

The paper has been organized as follows:

Section 2 deals with the definitions of fuzzy hypergraph, intuitionistic fuzzy hypergraph, IFDHG and the notations used in this paper. In section 3, a study is made on essentially intersecting, \( \diamond \) – intersecting and sequentially simple intersecting IFDHGs. Some properties of newly proposed hypergraph concepts are also discussed and it has been proved that \( H \) is strongly intersecting if and only if \( H^{r, s_i} \) is intersecting IFDHG, \( \forall \ (r_i, s_i) \in F(H) \). Section 4, concludes the paper.

2. PRELIMINARIES

The notations used in this work are listed below:

- \( H = (V, E) \) - IFDHG with vertex set \( V \) and edge set \( E \)
- \( \langle \mu_i, \nu_i \rangle \) - degrees of membership and non-membership of the vertex
- \( \langle \mu_{ij}, \nu_{ij} \rangle \) - degrees of membership and non-membership of the edges
- \( \langle \mu_{ij}(v_i), \nu_{ij}(v_i) \rangle \) - degrees of membership and non-membership of the edges containing \( v_i \)
- \( h(H) \) - Height of a hypergraph \( H \)
- \( F(H) \) - Fundamental sequence of \( H \)
- \( \mathcal{C}(H) \) - Core set of \( H \)
- \( H^{(r_i, s_i)} \) - (\( r_i, s_i \)) - level of \( H \)
- \( IF_p(v) \) - IF power set of \( V \)
- \( Tr(H) \) - Intuitionistic fuzzy transversals (IFT) of \( H \)
In this section, definitions of intuitionistic fuzzy set, intuitionistic fuzzy graph, IFDHG has been dealt with.

**Definition 2.1:** [1] Let a set $E$ be fixed. An *intuitionistic fuzzy set (IFS)* $V$ in $E$ is an object of the form $V = \{(v_i, \mu_i(v_i), \nu_i(v_i)) / v_i \in E\}$, where the function $\mu_i : E \to [0, 1]$ and $\nu_i : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $v_i \in E$, respectively and for every $v_i \in E, 0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$. $V$ is a fuzzy set in $E$.

**Definition 2.2:** [14] Let $E$ be fixed set and $V = \{(v_i, \mu_i(v_i), \nu_i(v_i)) / v_i \in E\}$ be an IFS. Six types of Cartesian products of $n$ subsets $V_1, V_2, \ldots, V_n$ of $V$ over $E$ are defined as

- $V_1 \times V_2 \times \ldots \times V_n = \{(v_1, v_2, \ldots, v_n) / v_i \in V_i, v_i \in V_i, \ldots, v_n \in V_n\}$
- $V_1 \times V_2 \times \ldots \times V_n = \{((v_1, v_2, \ldots, v_n)) \}$
- $V_1 \times V_2 \times \ldots \times V_n = \{((v_1, v_2, \ldots, v_n)) \}$
- $V_1 \times V_2 \times \ldots \times V_n = \{((v_1, v_2, \ldots, v_n)) \}$
- $V_1 \times V_2 \times \ldots \times V_n = \{((v_1, v_2, \ldots, v_n)) \}$

It must be noted that $v_i \in V_i$ is an IFS, where $s = 1, 2, 3, 4, 5, 6$.

**Definition 2.3:** [4] An *intuitionistic fuzzy graph (IFG)* is of the form $G = (V, E)$ where (i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu : E \to [0, 1]$ and $\nu : E \to [0, 1]$ denote the degrees of membership and non-membership of the vertex $v_i \in V$ respectively and

- $0 \leq \mu_i(v_i) + \nu_i(v_i) \leq 1$ for every $v_i \in V, i = 1, 2, 3, \ldots, n$.
- $\mu_i \leq \mu_i \forall v_i \in V$ where $\mu_i : V \times V \to [0, 1]$ and $v_i : V \times V \to [0, 1]$ are such that

$$\mu_i = \mu_i(v_i)$$

and $0 \leq \mu_i + \nu_i \leq 1$.

**Definition 2.4:** [5] An *intuitionistic fuzzy hypergraph (IFHG)* is an ordered pair $H = (V, E)$ where

- $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set of intuitionistic fuzzy vertices,
- $E = (E_1, E_2, \ldots, E_m)$ is a family of crisp subsets of $V$.
- $E_j = \{(v_i, \mu_j(v_i), \nu_j(v_i)) : \mu_j(v_i), \nu_j(v_i) \geq 0 \text{ and } \mu_j(v_i), \nu_j(v_i) \leq 1, j = 1, 2, \ldots, m\}$,
- $E_j \neq \phi, j = 1, 2, 3, \ldots, m$.

Here, the hyperedges $E_j$ are crisp sets of intuitionistic fuzzy vertices $\mu_i(v_i)$ and $\nu_i(v_i)$ denote the degrees of membership and non-membership of vertex $v_i$ to edge $E_j$. Thus, the elements of the incidence matrix of IFHG are of the form $\{(v_i, \mu_j(v_i), \nu_j(v_i))\}$.

**Note:** The support of an IFS $V$ in $E$ is denoted by $\text{supp}(E_i) = \{v_i / \mu_j(v_i) > 0 \text{ and } \nu_j(v_i) > 0\}$.
Definition 2.5: [6] An IFDHG $H$ is a pair $(V,E)$, where $V$ is a non-empty set of vertices and $E$ is a set of intuitionistic fuzzy hyperarcs; an intuitionistic fuzzy hyperarc $E_i \in E$ is defined as a pair $(t(E_i), h(E_i))$, where $(E_i) \subset V$, with $t(E_i) \neq \emptyset$, is its tail, and $h(E_i) \supset V - t(E_i)$ is its head. A vertex $s$ is said to be a source vertex in $H$ if $h(E_i) \neq s$, for every $E_i \in E$. A vertex $d$ is said to be a destination vertex in $H$ if $d \neq t(E_i)$, for every $E_i \in E$.

Definition 2.6: [7] Let $H$ be an IFDHG, let $H^{r_i}_{\mathcal{C}} = (V^{r_i}_{\mathcal{C}}, E^{r_i}_{\mathcal{C}})$ be the $(r_i, s_i)$-level IFDHG of $H$. The sequence of real numbers $\{r_1, r_2, \ldots, r_n; s_1, s_2, \ldots, s_n\}$, such that $0 \leq r_i \leq h(H) \text{ and } 0 \leq s_i \leq h_v(H)$, satisfying the properties:

(i) If $r_i < s_i \leq 1$ and $0 \leq \beta \leq s_i$ then $E^{\beta}_{\mathcal{C}} = \emptyset$.

(ii) If $r_i + 1 \leq s_i$ then $E^{s_i}_{\mathcal{C}} = E^{r_i}_{\mathcal{C}}$.

(iii) $E^{s_i}_{\mathcal{C}} = E^{r_i}_{\mathcal{C}}$.

is called the fundamental sequence of $H$, and is denoted by $F(H)$. The core set of $H$ is denoted by $C(H)$ and is defined by $C(H) = \{H^{r_1}_{\mathcal{C}}, H^{r_2}_{\mathcal{C}}, \ldots, H^{r_n}_{\mathcal{C}}\}$. The corresponding set of $(r_i, s_i)$-level hypergraphs $H^{r_1}_{\mathcal{C}} \subset H^{r_2}_{\mathcal{C}} \subset \ldots \subset H^{r_n}_{\mathcal{C}}$ is called the $H$-induced fundamental sequence and is denoted by $I(H)$. The $(r_n, s_n)$-level is called the support level of $H$ and the $H^{r_n}_{\mathcal{C}}$ is called the support of $H$.

Definition 2.7: [7] Let $H$ be an IFDHG and $C(H) = \{H^{r_1}_{\mathcal{C}}, H^{r_2}_{\mathcal{C}}, \ldots, H^{r_n}_{\mathcal{C}}\}$. $H$ is said to be ordered if $C(H)$ is ordered. That is $H^{r_1}_{\mathcal{C}} \subset H^{r_2}_{\mathcal{C}} \subset \ldots \subset H^{r_n}_{\mathcal{C}}$. The IFDHG is said to be simply ordered if the sequence $\{H^{r_i}_{\mathcal{C}}/i = 1, 2, 3, \ldots, n\}$ is simply ordered, that is it is ordered and if whenever $E \not\in H^{r_1+1}_{\mathcal{C}} - H^{r_1}_{\mathcal{C}}$ then $E \not\in H^{r_2}_{\mathcal{C}}$.

Definition 2.8: [9] Let $H$ be an IFDHG with core set $C(H) = \{H^{r_1}_{\mathcal{C}}, (V^{r_1}_{\mathcal{C}}, E^{r_1}_{\mathcal{C}})|i = 1, 2, \ldots, n\}$, where $E(H^{r_1}_{\mathcal{C}}) = E_i$ is the crisp edge set of the core hypergraph $H^{r_1}_{\mathcal{C}}$. Let $E(H)$ denote the crisp edge set of $H$ defined by $E(H) = \bigcup E_i$. $H^{r_1}_{\mathcal{C}} \subset C(H) \subset E(H)$, a crisp hypergraph on $V$, is called core aggregate hypergraph of $H$ and is denoted by $\mathfrak{H}(H) = (V, E(H))$.

Definition 2.9: [9] An IFDHG $H$ is said to be an intersecting intuitionistic fuzzy directed hypergraph, if for each pair of intuitionistic fuzzy hyperedge $\{E_i, E_j\} \subseteq E$, $E_i \cap E_j \neq \emptyset$.

Definition 2.10: [9] Let $H$ be an IFDHG and $C(H) = \{H^{r_1}_{\mathcal{C}}, (V^{r_1}_{\mathcal{C}}, E^{r_1}_{\mathcal{C}})|i = 1, 2, \ldots, n\}$, if $H^{r_1}_{\mathcal{C}}$ is an intersecting IFDHG for each $i = 1, 2, \ldots, n$ then $H$ is a $k$-intersecting IFDHG.

Definition 2.11: [9] An IFDHG $H$ is said to be strongly intersecting, if for any two edges $E_i$ and $E_j$ contain a common spike of height, $h = h(E_i) \wedge h(E_j)$.

Definition 2.12: [8] Let $H$ be an IFDHG. A primitive $k$-coloring $A$ of $H$ is a partition $\{A_1, A_2, A_3, \ldots, A_k\}$ of $V$ into $k$-subsets (colors) such that the support of each intuitionistic fuzzy hyperedge of $H$ intersects at least two colors of $A$, except spike edges.

Definition 2.13: [8] The $k$-chromatic number of an IFDHG $H$ is the minimal number $\chi_k(H)$, of colors needed to produce a primitive coloring of $H$. The chromatic number of $H$ is the minimal number, $\chi(H)$, of colors needed to produce a $K$-coloring of $H$.

Theorem 2.1: [8] If $H$ is an ordered IFDHG and $A$ is a primitive coloring of $H$, then $A$ is a $K$-coloring of $H$.

Theorem 2.2: [9] Let $H$ be an IFDHG and suppose $C(H) = \{H^{r_1}_{\mathcal{C}}, (V^{r_1}_{\mathcal{C}}, E^{r_1}_{\mathcal{C}})|i = 1, 2, \ldots, n\}$. Then $H$ is intersecting if and only if $H^{r_1}_{\mathcal{C}} = (V^{r_1}_{\mathcal{C}}, E^{r_1}_{\mathcal{C}})$ is intersecting.

Theorem 2.3: [9] Let $H$ be an ordered IFDHG and let $C(H) = \{H^{r_1}_{\mathcal{C}}, (V^{r_1}_{\mathcal{C}}, E^{r_1}_{\mathcal{C}})|i = 1, 2, \ldots, n\}$, then $H$ is intersecting if and only if $H$ is $K$-intersecting.

Theorem 2.4: [9] If $H^0$ is intersecting, then $H$ is strongly intersecting.

Definition 2.14: [8] A spike reduction of $E \in F(H)$, denoted by $\overline{E}$, is defined as $\overline{E}(v_i) = \max\{|r_i, s_i|/E^{r_i}_{\mathcal{C}}| \geq 2, 0 \leq r_i \leq E^{r_i}_{\mathcal{C}}(v_i), 0 \leq s_i \leq E^{r_i}_{\mathcal{C}}(v_i)|\}$. Note:

i) If $A = \emptyset$ then $\overline{E}(v_i) = 0$

ii) If $E_i$ is spike, then $\overline{E} = \chi_0$

Definition 2.15: [8] Let $H$ be an IFDHG and let $\bar{H} = (\bar{V}, \bar{E})$, where $\bar{E} = (\bar{E}_i|E\in E)$ and $\bar{V} = \cup E \in E \supp(\bar{E})$.\n
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Theorem 2.5: [3] $H$ is intersecting if and only if $H$ is $K$-intersecting.

Theorem 2.6: [3] If $H$ is a crisp intersecting hypergraph, then $\chi(H) \leq 3$.

Theorem 2.7: [3] A crisp hypergraph $H$ is intersecting if and only if $H \subseteq Tr(H)$.

Theorem 2.8: [8] Let $H$ be an IFDHG. Then $H$ is strongly intersecting if and only if $H$ is $K$-intersecting.

3. INTERSECTING IFDHG

Definition 3.1: Let $H$ be an IFDHG. Then $H$ is said to be essentially intersecting if $\tilde{H}$ is intersecting. And $H$ is said to be essentially strongly intersecting if $\tilde{H}$ is strongly intersecting.

Example 3.1: Consider an IFDHG, $H$ with the incidence matrix as given below:

$$H = \begin{pmatrix}
    (0.7, 0.1) & (0.1) & (0.1) & (0.5, 0.3) \\
    (0.7, 0.1) & (0.5, 0.2) & (0.5, 0.2) & (0.1) \\
    (0.7, 0.1) & (0.1) & (0.3, 0.4) & (0.1) \\
    (0.1) & (0.1) & (0.3, 0.4) & (0.5, 0.3)
\end{pmatrix}$$

The corresponding graph of IFDHG $H$ is displayed in Figure 3.1.

Figure 3.2 depicts essentially intersecting IFDHG

$$H = \begin{pmatrix}
    (0.7, 0.1) & (0.1) & (0.5, 0.3) \\
    (0.7, 0.1) & (0.5, 0.2) & (0.1) \\
    (0.7, 0.1) & (0.3, 0.4) & (0.1) \\
    (0.1) & (0.3, 0.4) & (0.5, 0.3)
\end{pmatrix}$$

Figure 3.2: Essentially intersecting IFDHG
Definition 3.2: Let $H$ be an IFDHG and $H^3 = (\tilde{H})^3$, then $H$ is called $\emptyset - \text{intersecting}$ if $H^3$ is intersecting.

Note:
(i) For our convenience, assume $F(H^3) = \{r_1^i, r_2^i, \ldots, r_m^i, s_1^i, s_2^i, \ldots, s_k^i\}$, where $0 \leq r_i \leq h_u(H)$ and $0 \leq s_i \leq h_v(H)$ and
(ii) $F(\tilde{H}) = \{r_1, r_2, \ldots, r_m, s_1, s_2, \ldots, s_m\}$, where $0 \leq r_1 \leq h_u(H)$ and $0 \leq s_1 \leq h_v(H)$.

Theorem 3.1: If $H$ is $\emptyset - \text{intersecting IFDHG}$, then $H$ is essentially strongly intersecting IFDHG.

Note: In general, the converse need not be true.

Theorem 3.2: If $H$ is ordered and essentially intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: Assume $\tilde{H}$ exists, then $\chi(H) = 1$. Let $(\tilde{H})^{r_m \cdot s_m} \in C(\tilde{H})$, where $(r_m, s_m) \in F(H)$ will be the smallest value. Since $\tilde{H}$ is intersecting IFDHG, it follows from theorem 2.2 that $(\tilde{H})^{r_m \cdot s_m}$ is also intersecting. Hence by theorem 2.6 $\chi((\tilde{H})^{r_m \cdot s_m}) \leq 3$. Also since $H$ is ordered, $\tilde{H}$ is also ordered. By definition 2.13 and theorem 2.1 it follows that, $\chi((\tilde{H})^{r_m \cdot s_m}) \leq 3$ and by definition 2.14, $\chi(\tilde{H}) \leq 3$. Hence, $\chi(H) = \chi(\tilde{H})$.

Theorem 3.3: If $H$ is elementary and essentially intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: Since $H$ is ordered, the result is obvious from theorem 3.2.

Theorem 3.4: If $H$ is of the form $\mu \otimes H$ and essentially intersecting IFDHG, then $\chi(H) \leq 3$.

Proof: The result is obvious, since $H$ is elementary.

Theorem 3.5: If $H$ is $\emptyset - \text{intersecting IFDHG}$, then $\chi(H) \leq 3$.

Proof: Given $H^3$ is intersecting. Also $H^3$ is elementary. Hence by theorem 3.3 $\chi(H) \leq 3 \implies \chi(H^3) \leq 3$. Since $\chi(H^3) = \chi(\tilde{H}) = \chi(\tilde{H})$. The result follows obviously.

Note: Since $H = \tilde{H}$, $K$ – coloring of skeleton $F^3$, of $H$ may not be extendible to $K$ – coloring of $H$, or if extendible, then it may not use the new colors. Therefore, if $H = \tilde{H}$ then $\chi(H^3) < \chi(H)$.

Definition 3.3: Let $H = \{v_i \in IP(V) | i = 1, 2, \ldots, n\}$ is a finite collection of intuitionistic fuzzy subsets of $V$ and let $0 \leq r_i \leq h_u(H)$ and $0 \leq s_i \leq h_v(H)$. Then $H_{(r,s)} = \{v \in IP(V) | h(v) = (r_i, s_i)\}$ denotes the set of edges in $K$ of height $(r_i, s_i)$. In general, $H^{r \cdot s}$ denotes the partial IFDHG of $H = (V, E)$ with the edge set $E^{r \cdot s}$ provided $E^{r \cdot s} \neq \emptyset$.

Definition 3.4: Let $H_i = (V_i, E_i)$, $i = 1, 2$ be an IFDHG. Then $H_1 \subseteq H_2$ if every edge of $H_1$ contains an edge $H_2$.

Theorem 3.6: $H$ is strongly intersecting IFDHG if and only if $H^{r \cdot s} \subseteq Tr(H^{r \cdot s})$ for every $H^{r \cdot s} \in C(H)$.

Proof: By theorem 2.8, definition 2.11 and theorem 2.7 it is obvious that $H$ is strongly intersecting IFDHG $\iff H$ is $K$ – intersecting IFDHG $\iff H^{r \cdot s}$ is intersecting for all $H^{r \cdot s} \in C(H)$ $\iff H^{r \cdot s} \subseteq Tr(H^{r \cdot s})$ for every $H^{r \cdot s} \in C(H)$.

Theorem 3.7: $H$ is strongly intersecting IFDHG if and only if for every $(r_i, s_i) \in F(H), H^{r \cdot s} \mid (r_i, s_i) \subseteq Tr(H^{r \cdot s})$.

Proof: Let for every $(r_i, s_i) \in F(H), H^{r \cdot s} \mid (r_i, s_i) \subseteq Tr(H^{r \cdot s})$. For each $H^{r \cdot s} \in C(H)$, the edge set $E(H^{r \cdot s}) = \{E^{r \cdot s} \mid E^{r \cdot s} \subseteq Tr(H^{r \cdot s})\} \subseteq \{E^{r \cdot s} \mid E^{r \cdot s} \subseteq Tr(H^{r \cdot s})\}$. Hence, $H^{r \cdot s} \subseteq Tr(H^{r \cdot s})$ for all $H^{r \cdot s} \in C(H)$ and by theorem 3.6, $H$ is strongly intersecting IFDHG.

Conversely, let $H$ is strongly intersecting IFDHG. And suppose $E^{r \cdot s} \mid (r_i, s_i)$ where $(r_i, s_i)$ the largest member of $F(H)$ be. Let $H^{r \cdot s} \in C(H)$.

To Prove: $E^{r \cdot s}$ is the transversal of $H^{r \cdot s}$.
Let $E \in H^r_{\gamma \beta}$. Then there exists an edge $E_i$ of $H$ such that $E_i^r = E$. Since, $H$ is strongly intersecting IFDHG, there is a spike $s_i$ with height $h(s_i) = h(E) \wedge h(E_i) = h(E_i)$. \(\geq (r_i, s_i).\) And support of \(v\), which is contained in both $E$ and $E_i$. Hence, $\forall E \in E \cap E^r_{\gamma \beta}$. Thus $E$ is a transversal of $H$ and therefore it contains a member of $Tr(H)$. Therefore, $H^r_{\gamma \beta} |_{(r_i, s_i)} \subseteq Tr(H^r_{\gamma \beta})$.

By theorem 2.8, it is true that $H$ is $K$ – intersecting IFDHG. Again by the same theorem, it follows that $H^r_{\beta \gamma}$ is strongly intersecting. Hence, $H^r_{\beta \gamma} |_{(r_i, s_i)} \subseteq Tr(H^r_{\beta \gamma})$ for every $(r_i, s_i) e F(H)$.

**Theorem 3.8:** Let $H$ be an IFDHG with $C(H) = \{H^r_{\gamma \beta}((r_i, s_i)) e F(H)\}$. Then $H^r_{\beta \gamma} \subseteq Tr(H^r_{\beta \gamma})$ for every $H^r_{\beta \gamma} e C(H)$ if and only if $H^r_{\beta \gamma} |_{(r_i, s_i)} \subseteq Tr(H^r_{\beta \gamma})$ for every $(r_i, s_i) e F(H)$.

**Proof:** By theorem 3.6 and 3.7, the proof is obvious.

**Theorem 3.9:** $H$ is strongly intersecting IFDHG if and only if $H^r_{\beta \gamma}$ is intersecting for every $(r_i, s_i) e F(H)$.

**Proof:** By theorem 2.2 and 2.8, the following equivalencies holds good.

$H^r_{\beta \gamma}$ is intersecting for every $(r_i, s_i) e F(H) \iff E(H^r_{\beta \gamma})$ is intersecting for each $H^r_{\gamma \beta} e C(H)$ \[ \iff H$ is K – intersecting IFDHG \[ \iff H$ is strongly intersecting IFDHG.

**Definition 3.5:** An IFDHG is said to be non-trivial if it has at least one edge $E$ such that $|support(E)| \geq 2$.

**Definition 3.6:** An IFDHG is said to be sequentially simple if $C(H) = \{H^r_{\gamma \beta}((r_i, s_i)) e F(H)\}$ satisfies the property that if $E \in (E_{r_{i+1} \gamma_{i+1}} \setminus E_{r_i \gamma_i})$, then $E \nsubseteq V^r_{\gamma \beta}$ where $0 \leq r_i \leq h_{\mu}(H)$ and $0 \leq s_i \leq h_{\nu}(H)$. $H$ is said to be essentially sequentially simple if $H$ is sequentially simple.

**Theorem 3.10:** If $H$ is an ordered IFDHG. Then the following statements holds:

i) $H$ is intersecting if and only if $H^\delta$ is intersecting.

ii) $\overline{H}$ is intersecting if and only if $H^\delta$ is intersecting.

**Proof:** Since $H^\delta = (\overline{H})^\gamma$ and $\overline{H}$ is ordered whenever $H$ is non-trivial ordered IFDHG (ii) is true. Also since $H$ is ordered, $support(H) = \cup \{E^r_{(r_{i+1} \gamma_{i+1})}((r_i, s_i)) e F(H)\}$. Thus, $support(H^\delta) \subseteq support(H)$. Again by construction of $H^\delta$, every member of the edge set $E^r_{(r_{i+1} \gamma_{i+1})}$ is either a member or it contains a member of $support(H^\delta)$. Hence, for any two edges $E_1, E_2 \subseteq support(H)$ there exists corresponding edges $E_1^', E_2^' \subseteq support(H^\delta)$ such that $E_1^' \subseteq E_1$ and $E_2^' \subseteq E_2$. Therefore, $support(H^\delta)$ is intersecting $\iff support(H)$ is intersecting. Hence (i) is proved.

**Theorem 3.11:** Let $H$ be an IFDHG. Then the following conditions holds good.

i) If $H^\delta$ is intersecting, then $H$ is strongly intersecting.

ii) If $H^\delta$ is intersecting, then $\overline{H}$ is strongly intersecting.

**Proof:** It is obvious that the edge $E$ in the core hypergraph $H^r_{\gamma \beta} e C(H)$ contains a member of $support(H^\delta)$ by the construction process explained in [8] and also by $H^\delta$ is elementary. Hence, if $support(H^\delta)$ is intersecting, then every core hypergraph, $H^r_{\gamma \beta}$ of $H$ is also intersecting. Therefore, $H$ is $K$ – intersecting and by theorem 2.8, $H$ is strongly intersecting.

**Example 3.2:** Consider an IFDHG, $H$ with the incidence matrix as given below:

$$
\begin{array}{cccc}
E_1 & E_2 & E_3 & E_4 \\
\begin{array}{c}
\begin{pmatrix}
0.7,0.1 \\
0.7,0.1 \\
0.7,0.1 \\
0.1 
\end{pmatrix}
\end{array} & \begin{array}{c}
(0,1) \\
(0.5,0.2) \\
(0.2,0.1) \\
(0.1) 
\end{array} & \begin{array}{c}
(0,1) \\
(0.5,0.2) \\
(0.2,0.1) \\
(0.1) 
\end{array} & \begin{array}{c}
(0.5,0.4) \\
(0.3,0.4) \\
(0.5,0.2) \\
(0.1) 
\end{array}
\end{array}
$$

In example 3.2, $H$ is strongly intersecting.
The incidence matrix for $H^\Omega$ is

$$
\begin{bmatrix}
E_1 & E_2 & E_3 & E_4 \\
 v_1 & (0.7,0.1) & (0.1) & (0.1) & (0.5,0.4) \\
v_2 & (0.1) & (0.5,0.2) & (0.1) & (0.1) \\
v_3 & (0.7,0.1) & (0.1) & (0.3,0.4) & (0.1) \\
v_4 & (0.1) & (0.1) & (0.3,0.4) & (0.5,0.2)
\end{bmatrix}
$$

Here, $H^\Omega$ is not intersecting.

4. CONCLUSION

In this paper, an attempt has been made to study the intersecting intuitionistic fuzzy directed hypergraphs. Also, essentially intersecting, $\triangledown$ intersecting and sequentially simple intersecting IFDHGs have been defined. Some of its properties have also been analyzed. Also it has been proved that the IFDHG $H$ is strongly intersecting IFDHG if and only if $H^{\Omega,\triangledown} \subseteq Ty(H^{\Omega,\triangledown})$ for every $H^{\Omega,\triangledown} \in C(H)$.

5. REFERENCES
