International Journal of Mathematical Archive-9(1), 2018, 50-55
IJMA Available online through www.ijma.info ISSN 2229 – 5046

MINIMIZATION OF TIME AND COST FOR TRANSPORTATION AND CONSTRUCTION MANAGEMENT OF ROADS IN AFAR REGIONAL STATE BY TRAVELLING SALESMAN PROBLEM

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(Received On: 10-12-17; Revised & Accepted On: 03-01-18)

ABSTRACT

At first glance, this problem is extremely similar to the Chinese postman problem. However, in contrast with the Chinese postman problem, no efficient algorithm for solving the travelling salesman problem is known as far. We will here describe a new algorithm to solving the travelling salesman problem in both cases of complete graphs and incomplete connected graphs, following a comprehensive treatment of Hungarian Method. Transport management, Construction management, Tourists, Sales representatives, Postman and visitors are wide application in order to save time and cost. However, the TSP is closely related to several of the problem areas, like 2-matching, spanning tree, and cutting planes, which areas actually were stimulated by questions prompted by the TSP, and often provide subroutines in solving the TSP. Being NP-complete, the TSP has served as prototype for the development and improvement of advanced computational methods, to a large extent utilizing polyhedral techniques. This paper gives an introduction to the Traveling Salesman Problem that includes current research. Additionally, the algorithms are used to find the shortest route traveling through zones of Afar Regional State in Ethiopia.

Key words: *Travelling salesman problem, complete graph, Incomplete connected graph and afar regional state.*

1. INTRODUCTION

The Traveling Salesman Problem (TSP) is a problem in combinatorial optimization studied in operations research and theoretical computer science. Given a list of cities and their pair wise distances, the task is to find a shortest possible tour that visits each city exactly once.

The problem was first formulated as a mathematical problem by Menger(1930) and is one of the most intensively studied problems in optimization. However it was unnoticed till Menger (1994) published a book where he narrated the foundation of mathematical problem for the travelling salesman problem. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities can be solved. The TSP has several applications even in its purest formulation, such as planning, logistics, and the manufacture of microchips. If a slight modification is made in the problem, it appears as a sub-problem in many areas, such as genome sequencing. In these applications, the concept city represents, for example, customers, soldering points, or DNA fragments, and the concept distance represents traveling times or cost, or a similarity measure between DNA fragments. In many applications, additional constraints such as limited resources or time windows make the problem considerably harder. In the theory of computational complexity, the decision version of TSP belongs to the class of NP-complete problems. Thus, it is assumed that there is no efficient algorithm for solving TSP problems. In other words, it is likely that the worst case running time for any algorithm for TSP increases exponentially with the number of cities, so 99 even some instances with only hundreds of cities will take many CPU years to solve exactly. The origins of the traveling salesman problem are unclear. A handbook for traveling salesmen from 1832 mentions the problem and includes example tours through Germany and Switzerland, but contains no mathematical treatment. Hamilton (1800) and Kirkman (1800) expressed the concept of Mathematical problems related to the travelling salesman problem. The general form of the TSP appears to have been first studied by mathematicians notably by Menger(1930). Further Menger (1930) also defines the problem related with salesman ship based on brute-force algorithm, and observes the non-optimality of the nearest neighbor heuristic. However Whitney (1930) introduced the name travelling salesman problem. During the period 1950 to 1960, the travelling salesman problem started getting popularity in scientific circle is especially in Europe and the USA.

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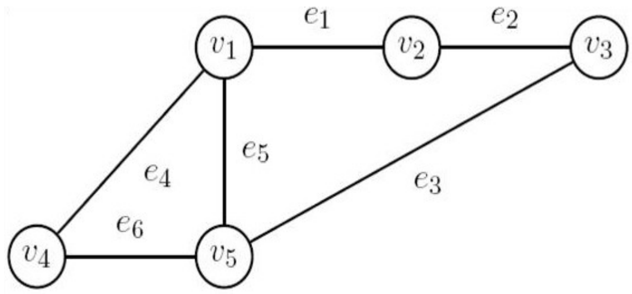
Many researchers like Dantzig, Fulkerson and Johnson (1954) at the RAND Corporation in Santa Monica expressed the problem as an integer linear program and developed the cutting plane method for its solution. With these new methods they solved an instance with 49 cities to optimality by constructing a tour and proving that no other tour could be shorter. In the following decades, the problem was studied by many researchers from mathematics, science, chemistry, physics, and other sciences. Karp (1972) showed that the Hamiltonian cycle problem was NP-complete, which implies the NP-hardness of TSP. This supplied a scientific explanation for the apparent computational difficulty of finding optimal tours.

Application of New Alternate Method of TSP

we discuss how the new alternate method for solving for problem can be applied for Travelling Salesman Problem. For this we have considered an example related with travelling salesman problem and explain in detail how to find optimal solution using new alternate method. In example we are taken the zones of Afar Regional State as vertices and we find the shortest Hamiltonian Cycle in order to minimize time and cost for Travelers, Sales representatives, Tourists, Transportation, Road construction authorities.

Definition 1: A graph G = (V; E) is a pair of vertices (or nodes) V and a set of edges E, assumed finite i.e. |V| = n and |E| = m. Here V(G) = {v_1; v_2; …; v_5 } and E(G) = {e_1; e_2; …. e_6}. An edge $e_k = (v_i ; v_j)$ is incident with the vertices vi and vj .



Definition 2: A graph in which every pair of distinct vertices is adjacent is called a complete graph. A complete graph of order n is denoted Kn.

Definition 3: For two, not necessarily distinct, vertices u and v in a graph G, a {u→v} walk W in G is a sequence of vertices in G, beginning with u and ending at v such that consecutive vertices in W are adjacent in G.

Definition 4: A walk in a graph G in which no vertex is repeated is called a path.

Definition 5: A walk whose initial and terminal vertices are the same and every other vertex is distinct is called a cycle.

Definition 6: A cycle in a given graph G that contains every vertex of G is called a Hamiltonian Cycle of G.

Definition 7: A graph is called Hamiltonian if it contains a Hamiltonian cycle.

Definition 8: A graph G is connected if G contains a {u→v} walk for every two vertices u and v of G, otherwise G is disconnected.

Definition 9: Let G = (V; E) be a graph. G is called a weighted graph if each edge e is assigned a non-negative number w (e), called the weight of e.

Chinese Postman Problem 10: In 1962, a Chinese mathematician called Kuan Mei-Ko was Interested in a postman delivering mail to a number of streets such that the total distance walked by the postman was as short as possible. How could the postman ensure that the distance walked was a minimum?

The Hungarian Method 11:

A considerable number of methods has been so far presented for assignment problem in which the Hungarian method is more convenient method among them. This iterative method is based on add or subtract a constant to every element of a row or column of the cost matrix, in a minimization model and create some zeros in the given cost matrix and then try to find a omplete assignment in terms of zeros. By a complete assignment for a cost $n \times n$ matrix, we mean an assignment plan containing exactly n assigned independent zeros, one in each row and one in each column.

The Hungarian algorithm to a given n × n cost matrix to find an optimal assignment that is the shortest Hamiltonian cycle.

Step-1: Subtract the smallest entry in each row from all the entries of its row.

Step-2: Subtract the smallest entry in each column from all the entries of its column.

Step-3: Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

Step-4: Test for Optimality:
- (i) If the minimum number of covering lines is n, an optimal assignment of zeros is possible and we are finished.
- (ii) If the minimum number of covering lines is less than n, an optimal assignment of zeros is not yet possible. In that case, proceed to Step 5.

Step-5: Determine the smallest entry not covered by any line. Subtract this entry from each uncovered row, and then add it to each covered column. Return to Step 3.

2. OUR APPROACH FOR SOLVING TRAVELING SALESMAN PROBLEM

This section presents a method to solve the traveling salesman problem which is different from the preceding method. We call it “Abduselam method", because of making a reduced distance or cost matrix. The new method is based on minimum element in the distance or cost matrix and then try to find a complete solution in terms of minima. By a complete solution we mean selecting exactly n minimum elements one in each row of n×n matrix.

Historically the TSP deals with finding the shortest tour in an n-city situation where each city is visited exactly once. In this problem, traveling salesman wants to minimize the total distance traveled (or time or money) during his visit of n cities. $D_{ij,}$ i =1, 2,...,n is the distance of i^{th} node to j^{th} node. Note that $dij = 0$ when $i = j$, and no edge between i^{th} node to j^{th} node i.e. we do not produce the item i again after i . The individual set up costs can be arranged in the form of a square matrix. A traveling salesman problem is to determine a set of n elements of this matrix. Traveling salesman problem is similar to the assignment problem, but here two restrictions are imposed. The first restriction is that we can not select the element in the leading diagonal. The second restriction is that we do not produce an item again until all the items are produced once. The second restriction means no city is visited twice until the tour of all the cities is completed. This paper attempts to propose a method, namely Abduselam method, for solving traveling salesman problem, which is different from the preceding methods.

Algorithm: The following are the steps involved in the algorithm

Step-1: Form the n×n distance or cost square matrix for graph.

Step-2: Select minimum weight edge other than zeros in the distance or cost square matrix, labeled it is as 1 and identify the corresponding two nodes of minimum weight edge. (In case of equal select anyone)

Step-3: Observe the Step 2 corresponding nodes rows in distance or cost matrix and then select next minimum weight edge, labeled it as 2 and identify the corresponding two nodes of minimum weight edge.

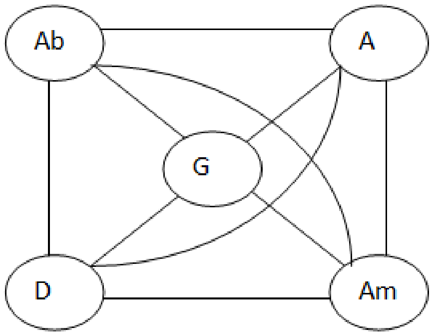
Step-4: From step2 and step3 we obtained one node is twice so ignore that node and again select the minimum weight edge from the remaining two nodes rows in the matrix without repeating ignore node, labeled it is as 3 and identify the corresponding two nodes of minimum weight edge. If the both nodes repeat at a time then select next minimum weight edge otherwise path will be closed without covering remaining nodes.

Step-5: Ignore the twice repeated node and again select the minimum weight edge from the remaining two nodes rows in the matrix without repeating ignore node, labeled it is as 4 and identify the corresponding two nodes of minimum weight edge.

Step-6: Repeat the step 4 until selecting n minimum weight edges in n×n distance or cost square matrix without repeatation.

Step-7: Draw the graph with selected edges which gives the shortest route or the shortest Hamiltonian cycle.

Example 1: You work as a sales manager for a mobile manufacturer, and you have currently five salespeople in capital towns of five zones Asaita(A), Abala(Ab), Amibara(Am), Golina(G) and Delifaga(D) in Afar Regional State. You want to visit five towns on business work. Design of visit five towns to the sales manager such that minimize the total distance. Distance between cities is shown in the following graph.



Step-1: Formation of 5x5 distance square matrix

$$\begin{array}{cc} & \begin{array}{ccccc} Ab & A & Am & D & G \end{array} \\ \begin{array}{c} Ab \\ A \\ Am \\ D \\ G \end{array} & \begin{bmatrix} 0 & 90 & 10^{1} & 20^{2} & 80 \\ 90 & 0 & 100 & 40^{3} & 70^{5} \\ 10 & 100 & 0 & 30 & 60^{4} \\ 20 & 40 & 30 & 0 & 50 \\ 80 & 70^{5} & 60 & 50 & 0 \end{bmatrix} \end{array}$$

Step-2: Select least weight edge in the matrix that is 10 (other than 0), labeled it as 1 and nodes of edge are Ab and Am.

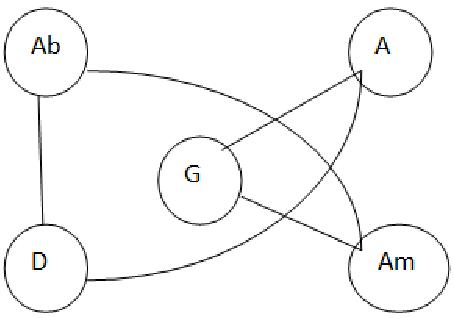
Step-3: Select least weight edge in Ab and Am rows that is 20, labeled it as 2 and nodes of edge are Ab and D.

Step-4: Ab node was repeated twice from the step2 and step3 so ignore Ab node and select least weight edge from Am and D node rows that is 40, labeled it as 3 and nodes of edge are D and A.

Step-5: D node was repeated twice so ignore D node select least edge from Am and A nodes that is 60, labeled it as 4 and nodes of edge are Am and G.

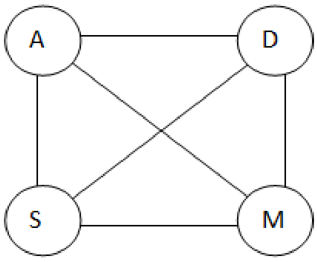
Similarly, Am node was repeated twice so ignore Am node and select least weight edge from A and G node rows that is 70, labeled it as 5 and nodes of edge are G and A or A and G.

Step-6: The graph obtained from the labeled edges as given below



The shortest route or Hamiltonian cycle which covers all the towns with minimum distance is
Ab→D→A→G→Am→Ab and the minimum distance is 200Kms

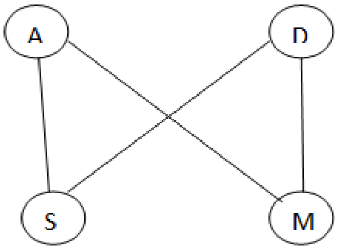
Example 2: Amount of Construct of roads among the four towns Asaita(A), Dupti(D), Semera(S) and Mille (M) as shown in the weighted graph. The determination of design of construction of road which covers four towns such that minimize the total cost.



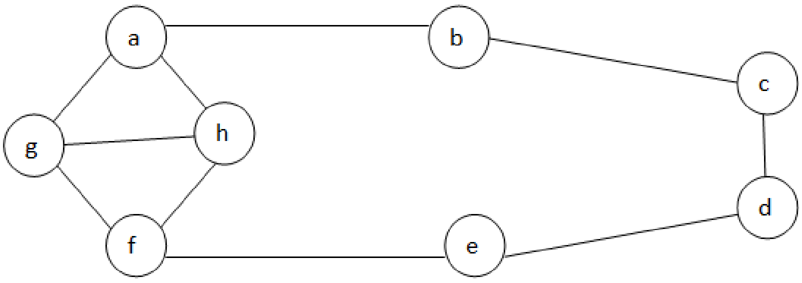
Formation 4x4 cost square matrix is
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 3 & 2 \\ 4 & 0 & 1 & 2 \\ 3 & 1 & 0 & 5 \\ 2 & 2 & 5 & 0 \end{bmatrix} \end{array}$$

Labeled 4x4 cost square matrix is
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 3^4 & 2 \\ 4 & 0 & 1^1 & 2^2 \\ 3^4 & 1 & 0 & 5 \\ 2^3 & 2 & 5 & 0 \end{bmatrix} \end{array}$$

The design of construction of road which covers four towns such that minimize the total cost is 8Trilians birr and the road is A→S→D→M→A.



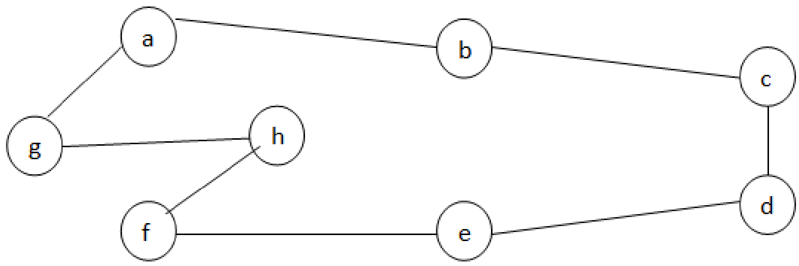
Example 3: Determination shortest route of trains which covers the all train stations in the metro city in order to save time and cost. The following graph indicates stations as nodes and time taken to travel as weight of edge in minutes.



Formation of 8x8 time square matrix is

$$\begin{array}{cc} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} & \begin{bmatrix} 0 & 20^{2} & 0 & 0 & 0 & 0 & 10^{1} & 30 \\ 20 & 0 & 10^{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 30^{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 30 & 0 & 20^{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 20^{6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 & 70 & 60^{7} \\ 10 & 0 & 0 & 0 & 0 & 70 & 0 & 50^{8} \\ 30 & 0 & 0 & 0 & 0 & 60 & 50^{8} & 0 \end{bmatrix} \end{array}$$

Shortest route of trains which covers the all train stations in the metro city in order to save time and cost is $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow h \rightarrow g \rightarrow a$ and time taken for cycle is 220 minutes.



3. CONCLUSION

In this Paper, we have applied new alternate method for solving Travelling salesman problem where it is shown that this method also gives optimal solution. Moreover the optimal solution obtained using this method is same as that of optimal solution obtained by Hungarian method. So we conclude that the Hungarian method and our method give same optimal solution. However the technique for solving Travelling Salesman problem using our method is more simple and easy as it takes few steps for the optimal solution.

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Source of support: Nil, Conflict of interest: None Declared.