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ON SOME NEW OPERATIONS OF FUZZY SOFT SETS

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ABSTRACT

The purpose of this paper is to study the notion of disjunctive sum, difference and symmetric difference of fuzzy soft sets and their basic properties. Finally we have put forward a decision making problem using the notion of cardinality of fuzzy soft sets.

Keywords: *Fuzzy Set, Soft Set, Fuzzy Soft Set, Disjunctive Sum, Difference, Symmetric Difference, Cardinality.*

1. INTRODUCTION

Fuzzy sets have been proved to be a very useful tool to handle uncertainty. Zadeh [9] introduced fuzzy set theory in 1965 and it was specifically designed to represent uncertainty and vagueness with formalized logical tools in dealing with the imprecision inherent in many real world problems. In 1999, Molodtsov [5] pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Rough Set Theory etc. which are considered as mathematical tools in dealing with uncertainties, cannot be successfully used to solve complicated problems in the fields of engineering, social science, economics, medical science etc. He further pointed out that the reason of these difficulties is the inadequacy of the parameterization tool of the theory. Accordingly, he put forward a noble concept known as soft set as a new mathematical tool for dealing with uncertainties. The soft set theory introduced by Molodstov is free of the difficulties present in these theories. The absence of any restriction on the approximate description in soft set theory makes this theory very convenient and easily applicable. Zadeh’s fuzzy sets are considered as a special case of the soft sets. Since its introduction, the concept of soft set has gained considerable amount of attention including some successful applications in information processing, decision, demand analysis, clustering and forecasting.

In 2003, Maji *et al.* [3] made a theoretical study on the soft set theory in more details. Especially, they introduced the concepts of subset, intersection, union, and complement of soft sets and discussed their properties. These operations make it possible to construct new soft sets from given soft sets.

In recent times, researchers have contributed a lot towards fuzzification of soft set theory. Maji *et al.* [4] combined fuzzy sets with soft sets and introduced the concept of fuzzy soft sets along with some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. In present times, works on the fuzzification of soft set theory is going on and these works have produced several new mathematical models such as intuitionistic fuzzy soft set, generalized fuzzy soft set, possibility fuzzy soft set etc.

Neog and Sut have introduced some new operations of fuzzy soft sets in [8]. In our work, we have put forward some more notions related to fuzzy soft sets. A decision problem solved with the help of cardinality of fuzzy soft sets has been proposed in our work.

2. PRELIMINARIES

In this section, we first recall the basic definitions related to fuzzy soft sets which would be used in the sequel.

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Definition 2.1 [5]: A pair (*F*, *E*) is called a soft set (over *U*) if and only if *F* is a mapping of *E* into the set of all subsets of the set *U*. In other words, the soft set is a parameterized family of subsets of the set *U*. Every set $F(\varepsilon),\ \varepsilon \in E$, from this family may be considered as the set of ε - elements of the soft set (*F*, *E*), or as the set of ε - approximate elements of the soft set.

Definition 2.2 [4]: A pair (*F*, *A*) is called a fuzzy soft set over *U* where $F : A \to \tilde{P}(U)$ is a mapping from *A* into $\tilde{P}(U)$. Here $\tilde{P}(U)$ represents the fuzzy subsets of *U*.

Definition 2.3 [1]: Let *U* be a universe and *E* a set of attributes. Then the pair (*U*, *E*) denotes the collection of all fuzzy soft sets on *U* with attributes from *E* and is called a fuzzy soft class.

Definition 2.4 [4]: A soft set (*F*, *A*) over *U* is said to be null fuzzy soft set denoted by ϕ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the null fuzzy set $\overline{0}$ of *U*, where $\overline{0}(x) = 0\ \ \forall x \in U$.

We would use the notation (ϕ, A) to represent the fuzzy soft null set with respect to the set of parameters *A*.

Definition 2.5 [4]: A soft set (*F*, *A*) over *U* is said to be absolute fuzzy soft set denoted by $\tilde{A}$ if $\forall \varepsilon \in A,\ F(\varepsilon)$ is the absolute fuzzy set $\overline{1}$ of *U* where $\overline{1}(x) = 1\ \ \forall x \in U$.

We would use the notation (U, A) to represent the fuzzy soft absolute set with respect to the set of parameters *A*.

Definition 2.6 [4]: For two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a fuzzy soft class (*U*, *E*), we say that (*F*, *A*) is a fuzzy soft subset of (*G*, *B*), if

(*i*) $A \subseteq B$

(*ii*) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as (*F*, *A*) $\tilde{\subseteq}$ (*G*, *B*).

Definition 2.7 [4]: Union of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.8 [1]: Let (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets in a soft class (*U*, *E*) with $A \cap B \neq \phi$. Then Intersection of two fuzzy soft sets (*F*, *A*) and (*G*, *B*) in a soft class (*U*, *E*) is a fuzzy soft set (*H*, *C*) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A) \tilde{\cap} (G, B) = (H, C)$, where $\cap$ is the operation intersection of two fuzzy sets.

Definition 2.9 [6]: The complement of a fuzzy soft set (*F*, *A*) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to \tilde{P}(U)$ is a mapping given by $F^c(\alpha) = [F(\alpha)]^c,\ \forall \alpha \in A$.

Definition 2.10[4]: If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) AND (*G*, *B*)" is a fuzzy soft set denoted by $(F,\ A) \wedge (G,\ B)$ and is defined by $(F,\ A) \wedge (G,\ B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$, where $\cap$ is the operation intersection of two fuzzy sets.

Definition 2.11[4]: If (*F*, *A*) and (*G*, *B*) be two fuzzy soft sets, then "(*F*, *A*) OR (*G*, *B*)" is a fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta),\ \forall \alpha \in A$ and $\forall \beta \in B$, where $\cup$ is the operation union of two fuzzy sets.

3. ON SOME NEW OPERATIONS ON FUZZY SOFT SETS

In this section, we have put forward some more notions related to fuzzy soft sets. A decision problem solved with the help of cardinality of fuzzy soft sets has been proposed in our work.

Definition 3.1 [8]: Let (F, A) and (G, B) be two fuzzy soft sets over (U, E). We define the disjunctive sum of (F, A) and (G, B) as the fuzzy soft set (H, C) over (U, E), written as $(F, A)\tilde{\oplus}(G, B) = (H, C)$, where $C = A \cap B \neq \phi$ and $\forall \varepsilon \in C$, $x \in U$, $\mu_{H(\varepsilon)}(x) = \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\right), \min\left(1-\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right)$.

Example 3.1: Let $U = \{c_1, c_2, c_3, c_4\}$ be the set of four cars under consideration and $E = \{e_1(\text{Costly}), e_2(\text{Beautiful}), e_3(\text{Fuel Efficient}), e_4(\text{ModernTechnology}), e_5(\text{Luxurious})\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$. We consider

$$(F, A) = \{F(e_1) = \{(c_1, 0.7), (c_2, 0.1), (c_3, 0.2), (c_4, 0.6)\},$$
$$F(e_2) = \{(c_1, 0.3), (c_2, 0.8), (c_3, 0.4), (c_4, 0.5)\},$$
$$F(e_3) = \{(c_1, 0.1), (c_2, 0.2), (c_3, 0.7), (c_4, 0.3)\}$$

as the fuzzy soft set representing the 'attractiveness of a car' according to Mr. *X*.

Let $B = \{e_3, e_4\} \subseteq E$ and

$$(G, B) = \{G(e_3) = \{(c_1, 0.2), (c_2, 0.5), (c_3, 0.1), (c_4, 0.7)\},$$
$$G(e_4) = \{(c_1, 0.8), (c_2, 0.4), (c_3, 0.1), (c_4, 0.6)\}\}$$

be the fuzzy soft set representing a 'good car' according to the same person Mr. *X*.
Then $(F, A)\tilde{\oplus}(G, B) = (H, C)$ where $C = A \cap B = \{e_3\}$ and

$$(H,C) = \Big\{ H(e_3) = \Big\{\Big(c_1, \max\left(\min\left(\mu_{F(e_3)}(c_1), 1-\mu_{G(e_3)}(c_1)\right), \min\left(1-\mu_{F(e_3)}(c_1), \mu_{G(e_3)}(c_1)\right)\right)\Big),$$
$$\Big(c_2, \max\left(\min\left(\mu_{F(e_3)}(c_2), 1-\mu_{G(e_3)}(c_2)\right), \min\left(1-\mu_{F(e_3)}(c_2), \mu_{G(e_3)}(c_2)\right)\right)\Big),$$
$$\Big(c_3, \max\left(\min\left(\mu_{F(e_3)}(c_3), 1-\mu_{G(e_3)}(c_3)\right), \min\left(1-\mu_{F(e_3)}(c_3), \mu_{G(e_3)}(c_3)\right)\right)\Big),$$
$$\Big(c_4, \max\left(\min\left(\mu_{F(e_3)}(c_4), 1-\mu_{G(e_3)}(c_4)\right), \min\left(1-\mu_{F(e_3)}(c_4), \mu_{G(e_3)}(c_4)\right)\right)\Big)\Big\}$$
$$= \Big\{ H(e_3) = \Big\{\left(c_1, \max\left(\min(0.1, 0.8), \min(0.9, 0.2)\right)\right), \left(c_2, \max\left(\min(0.2, 0.5), \min(0.8, 0.5)\right)\right),$$
$$\left(c_3, \max\left(\min(0.7, 0.9), \min(0.3, 0.1)\right)\right), \left(c_4, \max\left(\min(0.3, 0.3), \min(0.7, 0.7)\right)\right)\Big\}$$
$$= \Big\{ H(e_3) = \Big\{\left(c_1, \max(0.1, 0.2)\right), \left(c_2, \max(0.2, 0.5)\right), \left(c_3, \max(0.7, 0.1)\right), \left(c_4, \max(0.3, 0.7)\right)\Big\}$$
$$= \Big\{ H(e_3) = \Big\{(c_1, 0.2), (c_2, 0.5), (c_3, 0.7), (c_4, 0.7)\Big\}$$

Proposition 3.1: Let (F, A) and (G, B) be two fuzzy soft sets over (U, E). Then the following results hold.

(i) (U, E) is closed with respect to the operation disjunctive sum of fuzzy soft sets.
(ii) $(F, A)\tilde{\oplus}(G, B) = (G, B)\tilde{\oplus}(F, A)$, (Commutative law)
(iii) $(F, A)\tilde{\oplus}\left((G, B)\tilde{\oplus}(H, C)\right) = \left((F, A)\tilde{\oplus}(G, B)\right)\tilde{\oplus}(H, C)$, (Associative law)
(iv) $(F, A)\tilde{\oplus}(\phi, A) = (F, A)$, (Law of identity element, (φ, A) is the identity of $\tilde{\oplus}$) $(F, A)\tilde{\oplus}(F, A) = (F, A)$, (Idempotent law)
(v) $(F, A)\tilde{\oplus}(U, A) = (F, A)^c$, (Law of (U, A))

Proof:

(i) It is obvious that the disjunctive sum of two fuzzy soft sets over the fuzzy soft class (U, E) is again a fuzzy soft set over (U, E).
(ii) Let $(F, A)\tilde{\oplus}(G, B) = (H, C)$, where $C = A \cap B \neq \phi$ and $\forall \varepsilon \in C, x \in U$, we have
$\mu_{H(\varepsilon)}(x) = \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\right), \min\left(1-\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right)$.
Now consider $(G, B)\tilde{\oplus}(F, A) = (I, C)$, where $C = A \cap B \neq \phi$ and $\forall \varepsilon \in C, x \in U$. Then
$\mu_{I(\varepsilon)}(x) = \max\left(\min\left(\mu_{G(\varepsilon)}(x), 1-\mu_{F(\varepsilon)}(x)\right), \min\left(1-\mu_{G(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right)\right)$.
It follows that $(F, A)\tilde{\oplus}(G, B) = (G, B)\tilde{\oplus}(F, A)$
(iii) Associativity of disjunctive sum of fuzzy soft sets is straight forward.
(iv) Let $(F, A)\tilde{\oplus}(\varphi, A) = (H, C)$, where $C = A \cap A = A$ and $\forall \varepsilon \in C = A, x \in U$, we have

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-0\right), \min\left(1-\mu_{F(\varepsilon)}(x), 0\right)\right)\\ &= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1\right), \min\left(1-\mu_{F(\varepsilon)}(x), 0\right)\right)\\ &= \max\left(\mu_{F(\varepsilon)}(x), 0\right)\\ &= \mu_{F(\varepsilon)}(x).\end{aligned}$$

It follows that $(F, A)\tilde{\oplus}(\varphi, A) = (F, A)$.

Second Part:

Let $(F, A)\tilde{\oplus}(F, A) = (H, C)$, where $C = A \cap A = A$ and $\forall \varepsilon \in A, x \in U$, we have

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-\mu_{F(\varepsilon)}(x)\right), \min\left(1-\mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right)\right)\\ &= \mu_{F(\varepsilon)}(x).\end{aligned}$$

It follows that $(F, A)\tilde{\oplus}(F, A) = (F, A)$.

(v) Let $(F, A)\tilde{\oplus}(U, A) = (H, C)$, where $C = A \cap A = A$ and $\forall \varepsilon \in A, x \in U$, we have

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 1-1\right), \min\left(1-\mu_{F(\varepsilon)}(x), 1\right)\right)\\ &= \max\left(\min\left(\mu_{F(\varepsilon)}(x), 0\right), \min\left(1-\mu_{F(\varepsilon)}(x), 1\right)\right)\\ &= \max\left(0, 1-\mu_{F(\varepsilon)}(x)\right)\\ &= 1-\mu_{F(\varepsilon)}(x).\end{aligned}$$

It follows that $(F, A)\tilde{\oplus}(U, A) = (F, A)^c$.

Definition 3.2 [8]: Let (F, A) and (G, B) be two fuzzy soft sets over (U, E) . We define the difference of (F, A) and (G, B) as the fuzzy soft set (H, C) over (U, E) , written as $(F, A)\tilde{\Theta}(G, B) = (H, C)$, where $C = A \cap B \neq \phi$ and $\forall \varepsilon \in C$, $x \in U$, $\mu_{H(\varepsilon)}(x) = \min\left(\mu_{F(\varepsilon)}(x), 1-\mu_{G(\varepsilon)}(x)\right)$.

Example 3.2: We consider the fuzzy soft sets (F, A) and (G, B) given in Example 3.1. Then the fuzzy soft set $(F, A)\tilde{\Theta}(G, B)$ would represent the fuzzy soft set representing attractive but not good cars.

Let $(F, A)\tilde{\Theta}(G, B) = (H, C)$, where $C = A \cap B = \{e_3\}$ and

$$\begin{aligned}(H, C) = \Big\{ H(e_3) = \{&\left(c_1, \min\left(\mu_{F(e_3)}(c_1), 1-\mu_{G(e_3)}(c_1)\right)\right), \left(c_2, \min\left(\mu_{F(e_3)}(c_2), 1-\mu_{G(e_3)}(c_2)\right)\right),\\ &\left(c_3, \min\left(\mu_{F(e_3)}(c_3), 1-\mu_{G(e_3)}(c_3)\right)\right), \left(c_4, \min\left(\mu_{F(e_3)}(c_4), 1-\mu_{G(e_3)}(c_4)\right)\right)\Big\}\\ = \Big\{ H(e_3) = \{&\left(c_1, \min(0.1, 1-0.2)\right), \left(c_2, \min(0.2, 1-0.5)\right), \left(c_3, \min(0.7, 1-0.1)\right), \left(c_4, \min(0.3, 1-0.7)\right)\Big\}\\ = \Big\{ H(e_3) = \{&\left(c_1, \min(0.1, 0.8)\right), \left(c_2, \min(0.2, 0.5)\right), \left(c_3, \min(0.7, 0.9)\right), \left(c_4, \min(0.3, 0.3)\right)\Big\}\\ = \Big\{ H(e_3) = \{&(c_1, 0.1), (c_2, 0.2), (c_3, 0.7), (c_4, 0.3)\}\Big\}.\end{aligned}$$

Proposition 3.2: Let (F, A) and (G, B) be two fuzzy soft sets over (U, E) . Then the following results hold.

(i) (U, E) is closed with respect to the operation difference of fuzzy soft sets.

(ii) $(F, A)\tilde{\Theta}(G, B) \neq (G, B)\tilde{\Theta}(F, A)$.

(iii) $(F, A)\tilde{\Theta}(\varphi, A) = (F, A)$.

(iv) $(F, A)\tilde{\Theta}(U, A) = (\varphi, A)$.

(v) $(F, A)\ \tilde{\Theta}\ (G, B) = (F, A)\ \tilde{\cap}\ (G, B)^c$.

Proof:

(i) It is obvious that the difference of two fuzzy soft sets over the fuzzy soft class (U, E) is again a fuzzy soft set over (U, E).

(ii) Proof follows from the definition.

(iii) Let $(F, A)\tilde{\Theta}(\phi, A) = (H, C)$, where $C = A \cap A = A$ and $\forall \varepsilon \in C = A, x \in U$, we have

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \min\left(\mu_{F(\varepsilon)}(x), 1-0\right)\\ &= \min\left(\mu_{F(\varepsilon)}(x), 1\right)\\ &= \mu_{F(\varepsilon)}(x).\end{aligned}$$

It follows that $(F,A)\tilde{\Theta}(\phi,A)=(F,A)$.

(iv) Let $(F,A)\tilde{\Theta}(U,A)=(H,C)$, where $C=A\cap A=A$ and $\forall \varepsilon\in C=A,\ x\in U$, we have

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \min\left(\mu_{F(\varepsilon)}(x),1-1\right)\\ &= \min\left(\mu_{F(\varepsilon)}(x),0\right)\\ &= 0.\end{aligned}$$

It follows that $(F,A)\tilde{\Theta}(U,A)=(\phi,A)$.

(v) Let $(F,A)\tilde{\Theta}(G,B)=(H,C)$, where $C=A\cap B\neq\phi$ and $\forall\varepsilon\in C,\ x\in U$,

$$\begin{aligned}\mu_{H(\varepsilon)}(x) &= \min\left(\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)\right)\\ &= \min\left(\mu_{F(\varepsilon)}(x),\mu_{G^c(\varepsilon)}(x)\right).\end{aligned}$$

Thus the proof follows.

Definition 3.3: Let (F,A) and (G,B) be two fuzzy soft sets over (U,E). We define the symmetric difference of (F,A) and (G,B) as the fuzzy soft set (H,C) over (U,E), written as $(F,A)\tilde{\Delta}(G,B)=(H,C)$, where $C=A\cap B\neq\phi$ and $\forall\varepsilon\in C,\ x\in U,\ \mu_{H(\varepsilon)}(x)=\max\left(\min\left(\mu_{F(\varepsilon)}(x),1-\mu_{G(\varepsilon)}(x)\right),\min\left(\mu_{G(\varepsilon)}(x),1-\mu_{F(\varepsilon)}(x)\right)\right)$.

We may write $\left((F,A)\tilde{\Theta}(G,B)\right)\tilde{\cup}\left((G,B)\tilde{\Theta}(F,A)\right)=(F,A)\tilde{\Delta}(G,B)$

Example 3.3: We consider the fuzzy soft sets (F, A) and (G, B) given in Example 3.1.

Let $(F,A)\tilde{\Theta}(G,B)=(H,C)$, where $C=A\cap B=\{e_3\}$ and

$$\begin{aligned}(H,C) &= \{\,H(e_3)=\{(c_1,\min(0.1,0.8)),\ (c_2,\min(0.2,0.5)),\ (c_3,\min(0.7,0.9)),\ (c_4,\min(0.3,0.3))\}\,\}\\ &= \{\,H(e_3)=\{(c_1,0.1),\ (c_2,0.2),\ (c_3,0.7),\ (c_4,0.3)\}\,\}\end{aligned}$$

Let $(G,B)\tilde{\Theta}(F,A)=(I,C)$, where $C=A\cap B=\{e_3\}$ and

$$\begin{aligned}(I,C) &= \{\,I(e_3)=\{(c_1,\min(0.2,0.9)),\ (c_2,\min(0.5,0.8)),\ (c_3,\min(0.1,0.3)),\ (c_4,\min(0.7,0.7))\}\,\}\\ &= \{\,I(e_3)=\{(c_1,0.2),\ (c_2,0.5),\ (c_3,0.1),\ (c_4,0.7)\}\,\}\end{aligned}$$

It follows that

$(F,A)\tilde{\Delta}(G,B)=(H,C)\tilde{\cup}(I,C)=(J,C)$, where

$$(J,C)=\{\,J(e_3)=\{(c_1,0.2),(c_2,0.5),(c_3,0.7),(c_4,0.7)\}\,\}$$

3.5 Application of fuzzy soft sets in a decision making problem:

In this section, we take a hypothetical case study to apply the notion of fuzzy soft sets in a decision making problem. Our study is based on cardinality of fuzzy soft sets initiated by Cagman *et al.* in [2].

Let (F,A) be a fuzzy soft set over (U,E), where U is the universe and E is the set of attributes. Then the cardinal set of (F,A) is a fuzzy set over E, denoted by $CS(F,A)$ and is defined as $CS(F,A)=\{(x,\mu_{CS(F,A)}(x)),x\in E\}$. The membership function $\mu_{CS(F,A)}$ of $CS(F,A)$ is defined by $\mu_{CS(F,A)}:E\to[0,1],\ \mu_{CS(F,A)}(x)=\dfrac{|\gamma_A(x)|}{|U|}$, where $|U|$ is the cardinality of universe U and $|\gamma_A(x)|$ is the scalar cardinality of fuzzy set $\gamma_A(x)$.

Suppose the authority of an institution wants to give award to the performing group of students in a science project competition. Each group consists of three student members. Let $U=\{s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8,s_9\}$ be our universal set containing 9 students. Here the set of parameters E is the set of certain attributes determined by the authority. Let $E=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$, where e_1 = Originality of idea and concept, e_2 = Relevance of the project to the theme, e_3 = Understanding of the issue, e_4=Data collection and analysis, e_5 = Experimentation & validation, e_6 = Interpretation and problem solving attempt, e_7 = Team work, e_8 = Oral Presentation. We construct the fuzzy soft sets (F, E), (G, E) and (H, E) for the 9 students under consideration as given below. The membership of the students except s_1, s_2 and s_3 in each F (e) is zero indicating that this fuzzy soft set is corresponding to the group containing the students s_1, s_2 and s_3. In the same way, the fuzzy soft set (G, E) is corresponding to the group containing the students s_4, s_5 and s_6 and (H, E) is corresponding to the group containing the students s_7, s_8 and s_9.

$(F, E) = \{ F(e_1) = \{ (s_1, 0.7), (s_2,0.5), (s_3,0.2), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_2) = \{ (s_1, 0.4), (s_2,0.3), (s_3,0.5), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_3) = \{ (s_1, 0.2), (s_2,0.1), (s_3,0.6), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_4) = \{ (s_1, 0.5), (s_2,0.4), (s_3,0.6), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_5) = \{ (s_1, 0.4), (s_2,0.8), (s_3,0.7), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_6) = \{ (s_1, 0.6), (s_2,0.6), (s_3,0.8), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_7) = \{ (s_1, 0.2), (s_2,0.2), (s_3,0.2), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\},$
$F(e_8) = \{ (s_1, 0.1), (s_2,0.3), (s_3,0.8), (s_4,0), (s_5,0), (s_6,0), (s_7,0), (s_8,0), (s_9,0)\}\},$

$(G, E) = \{G(e_1) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.2), (s_5,0.7), (s_6,0.6), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_2) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.3), (s_5,0.7), (s_6,0.6), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_3) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.5), (s_5,0.6), (s_6,0.2), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_4) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.4), (s_5,0.8), (s_6,0.5), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_5) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.7), (s_5,0.9), (s_6,0.8), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_6) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.1), (s_5,0.4), (s_6,0.6), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_7) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.2), (s_5,0.3), (s_6,0.7), (s_7,0), (s_8,0), (s_9,0)\},$
$G(e_8) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0.1), (s_5,0.2), (s_6,0.9), (s_7,0), (s_8,0), (s_9,0)\}\},$

$(H, E) = \{H(e_1) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.5), (s_8,0.2), (s_9,0.9)\},$
$H(e_2) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.6), (s_8,0.1), (s_9,0.8)\},$
$H(e_3) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.7), (s_8,0.7), (s_9,0.8)\},$
$H(e_4) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.8), (s_8,0.9), (s_9,0.7)\},$
$H(e_5) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.6), (s_8,0.8), (s_9,0.6)\},$
$H(e_6) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.5), (s_8,0.8), (s_9,0.7)\},$
$H(e_7) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.7), (s_8,0.9), (s_9,0.2)\},$
$H(e_8) = \{ (s_1, 0), (s_2,0), (s_3,0), (s_4,0), (s_5,0), (s_6,0), (s_7,0.6), (s_8,0.2), (s_9,0.5)\}\}.$

$CS\,(F, E) = \{(e_1, 1.4/9), (e_2, 1.2/9), (e_3, 0.9/9), (e_4, 1.5/9), (e_5, 1.9/9), (e_6, 2.0/9), (e_7, 0.6/9), (e_8, 1.2/9)\}$
$= \{(e_1, 0.16), (e_2, 0.13), (e_3, 0.1), (e_4, 0.17), (e_5, 0.21), (e_6, 0.22), (e_7, 0.07), (e_8, 0.13)\}$

It follows that $|\, CS\,(F, E)| = 0.16 + 0.13 + 0.1 + 0.17 + 0.21 + 0.22 + 0.07 + 0.13 = 1.19$
$CS\,(G, E) = \{(e_1, 1.5/9), (e_2, 1.6/9), (e_3, 1.3/9), (e_4, 1.7/9), (e_5, 2.4/9), (e_6, 1.1/9), (e_7, 1.2/9), (e_8, 1.2/9)\}$
$= \{(e_1, 0.17), (e_2, 0.18), (e_3, 0.14), (e_4, 0.19), (e_5, 0.27), (e_6, 0.12), (e_7, 0.13), (e_8, 0.13)\}$

In the same way $|\, CS\,(G, E)| = 0.17 + 0.18 + 0.14 + 0.19 + 0.27 + 0.12 + 0.13 + 0.13 = 1.33$
$CS\,(H, E) = \{(e_1, 1.6/9), (e_2, 1.5/9), (e_3, 2.2/9), (e_4, 2.4/9), (e_5, 2.0/9), (e_6, 2.0/9), (e_7, 1.8/9), (e_8, 1.3/9)\}$
$= \{(e_1, 0.18), (e_2, 0.17), (e_3, 0.24), (e_4, 0.27), (e_5, 0.22), (e_6, 0.22), (e_7, 0.20), (e_8, 0.14)\}$

And $|\, CS\,(H, E)| = 0.18 + 0.17 + 0.24 + 0.27 + 0.22 + 0.22 + 0.20 + 0.14 = 1.64$

It is seen that the scalar cardinality corresponding to the fuzzy soft set (H, E) is the highest. It follows that the group consisting of the students s_7, s_8 and s_9 is in the first rank.

4. CONCLUSION

In our work, we have put forward some new notions of fuzzy soft sets. Some related properties have been established with proof and examples. A decision problem has been considered to get the optimal solution with the help of cardinality of fuzzy soft sets. Future work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

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