International Journal of Mathematical Archive-9(2), 2018, 8-13
Available online through www.ijma.info ISSN 2229 – 5046

ANALYTIC MEAN PRIME LABELING OF SOME CYCLE RELATED GRAPHS

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(Received On: 23-12-17; Revised & Accepted On: 15-01-18)

ABSTRACT

*Analytic mean prime labeling of a graph is the labeling of the vertices with {0, 1, 2,… …,p-1} and the edges with mean of the absolute difference of the squares of the labels of the incident vertices if square difference is even or mean of the absolute difference of the squares of the labels of the incident vertices and one if square difference is odd. The greatest common incidence number of a vertex (**gcin**) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of degree greater than one is one, then the graph admits analytic mean prime labeling. Here we identify some cycle related graphs for analytic mean prime labeling.*

Keywords: *Graph labeling, square difference, greatest common incidence number, prime labeling, analytic mean, cycles.*

AMS Mathematics Subject Classification (2010): *05C78.*

1. INTRODUCTION

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p, q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2], [3] and [4]. Some basic concepts are taken from [1] and [2]. The analytic mean labeling was defined by T Tharmaraj and P Sarasia in [5]. In this paper we introduced the concept of analytic mean prime labeling and proved that some cycle related graphs admit analytic mean prime labeling.

Definition 1.1: Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2. MAIN RESULTS

Definition 2.1: Let G = (V, E) be a graph with p vertices and q edges. Define a bijection f : V(G) → {0,1,2,3,---,p-1} by $f(v_i) = i-1$, $1\leq i\leq p$. Define a 1-1 mapping f^*_{ampl} : E(G) → set of natural numbers N by

$$f^*_{ampl}(uv) = \frac{|f(u)^2-f(v)^2|}{2}, \text{ if } f(u)^2 - f(v)^2 \text{ is even.}$$

$$f^*_{ampl}(uv) = \frac{|f(u)^2-f(v)^2|+1}{2}, \text{ if } f(u)^2 - f(v)^2 \text{ is odd.}$$

The induced function f^*_{ampl} is said to be analytic mean prime labeling, if the ***gcin*** of each vertex of degree at least 2, is 1.

Definition 2.2: A graph which admits analytic mean prime labeling is called analytic mean prime graph.

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Theorem 2.1: Cycle C_n admits analytic mean prime labeling, if n is even.

Proof: Let G = C_n and let v_1, v_2,---,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = n.

Define a function f : V → {0,1,2,3,---,n-1 } by
$f(v_i) = i-1$, i = 1, 2, ---,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_i\ v_{i+1})$ = i, i = 1, 2, ---,n-1

$f^*_{ampl}(v_1\ v_n)$ = $\frac{n(n-2)}{2}$ +1

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}(v_1\ v_2), f^*_{ampl}(v_1\ v_n)\}$
= gcd of {1, $\frac{n(n-2)}{2}$ +1}
= 1

gcin of (v_{i+1}) = gcd of $\{f^*_{ampl}(v_i\ v_{i+1}), f^*_{ampl}(v_{i+1}\ v_{i+2})\}$
= gcd of {i, i+1}
= 1, i = 1, 2, ---,n-2

gcin of (v_n) = gcd of $\{f^*_{ampl}(v_{n-1}\ v_n), f^*_{ampl}(v_1\ v_n)\}$
= gcd of {n-1, $\frac{n(n-2)}{2}$ +1}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence C_n , admits analytic mean prime labeling.

Theorem 2.2: Let G be the graph obtained by joining an edge to each vertex of cycle C_n. G is called crown graph. G admits analytic mean prime labeling.

Proof: Let G be the graph and let v_1, v_2, ---,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n.

Define a function f : V → {0,1,2,3,---,2n-1 } by
$f(v_i) = i-1$, i = 1,2,---,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_{2i-1}\ v_{2i})$ = 2i-1, i = 1,2---,n.

$f^*_{ampl}(v_{2i}\ v_{2i+2})$ = 4i, i = 1,2,---,n-1.

$f^*_{ampl}(v_2\ v_{2n})$ = $2n^2$-2n.

Clearly f^*_{ampl} is an injection.

gcin of (v_2) = gcd of $\{f^*_{ampl}(v_1\ v_2), f^*_{ampl}(v_2\ v_4)\}$
= gcd of {1, 4}= 1.

gcin of (v_{2i+2}) = gcd of $\{f^*_{ampl}(v_{2i}\ v_{2i+2}), f^*_{ampl}(v_{2i+1}\ v_{2i+2})\}$
= gcd of {4i, 2i+1}
= gcd of {2i, 2i+1}= 1, i = 1,2,---,n-1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

Theorem 2.3: Two copies of cycle C_n (n > 3) sharing a common edge admits analytic mean prime labeling.

Proof: Let G be the graph and let v_1, v_2,---,v_{2n-2} are the vertices of G

Here |V(G)| = 2n-2 and |E(G)| = 2n-1.

Define a function f : V → {0,1,2,3,---,2n-3 } by
$f(v_i)$ = i-1, i = 1,2,---,2n-2.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ampl}^* is defined as follows

$f_{ampl}^*(v_i\ v_{i+1}) = i$, i = 1, 2,---,2n-3

$f_{ampl}^*(v_1\ v_{2n-2}) = 2n^2-6n+5$.

Case (i): n is even

$f_{ampl}^*(v_{\frac{n}{2}}\ v_{\frac{3n-2}{2}}) = \frac{2n^2-5n+4}{2}$.

Case (ii): n is odd

$f_{ampl}^*(v_{\frac{n+1}{2}}\ v_{\frac{3n-1}{2}}) = n^2-2n+1$.

Clearly f_{ampl}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ampl}^*(v_1\ v_2), f_{ampl}^*(v_1\ v_{2n-2})\}$
= gcd of {1, $2n^2$-6n+5}
= 1

gcin of (v_{i+1}) = gcd of $\{f_{ampl}^*(v_i\ v_{i+1}), f_{ampl}^*(v_{i+1}\ v_{i+2})\}$
=1, i = 1, 2,---,2n-4

gcin of (v_{2n-2}) = gcd of $\{f_{ampl}^*(v_{2n-3}\ v_{2n-2}), f_{ampl}^*(v_1\ v_{2n-2})\}$
= gcd of {2n-3, $2n^2$-6n+5}
= gcd of {n-1, 2n-3}
= gcd of {n-2, n-1}= 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G , admits analytic mean prime labeling.

Theorem 2.4: Let G be the graph obtained by joining path P_m to any one vertex of a cycle C_n. G is called tadpole Graph. G admits analytic mean prime labeling, if n > 4 and m < n.

Proof: Let G be the graph and let v_1, v_2,---,v_{n+m-1} are the vertices of G

Here |V(G)| = n+m-1 and |E(G)| = n+m-1.

Define a function f : V → {0,1,2,3,---,n+m-2 } by
$f(v_i)$ = i-1 , i = 1,2, ---,n+m-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ampl}^* is defined as follows

$f_{ampl}^*(v_i\ v_{i+1}) = i$, i = 1, 2,---,n+m-2

$f_{msqsp}^*(v_1\ v_n) = \frac{(n-1)^2}{2}$, if n is odd
$= \frac{(n-1)^2+1}{2}$, if n is even

Clearly f_{ampl}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ampl}^*(v_1\ v_2), f_{ampl}^*(v_1\ v_n)\}$
= 1

gcin of (v_{i+1}) = gcd of $\{f_{ampl}^*(v_i\ v_{i+1}), f_{ampl}^*(v_{i+1}\ v_{i+2})\}$
= 1, i = 1, 2,---,n+m-3

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence G, admits analytic mean prime labeling.

Theorem 2.5: Let G be the graph obtained by joining two copies of path P_n to two consecutive vertices of cycle C_n. G admits analytic mean prime labeling, if n > 3.

Proof: Let G and let $v_1, v_2, ---, v_{3n-2}$ are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 3n-2.

Define a function $f : V \to \{0,1,2,3,---,3n-3\}$ by

$f(v_i) = i-1$, i = 1,2,---,3n-2.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_i\ v_{i+1}) = i$, i = 1, 2,---,3n-3

$f^*_{ampl}(v_n\ v_{2n-1}) = \frac{3n^2-6n+3}{2}$, if n is odd.

$= \frac{3n^2-6n+4}{2}$, if n is even.

Clearly f^*_{ampl} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{ampl}(v_i\ v_{i+1}), f^*_{ampl}(v_{i+1}\ v_{i+2})\}$

= 1, i = 1, 2,---,3n-4

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

Theorem 2.6: Let G be the graph obtained by joining two copies of cycle C_n by an edge. G is called dum bell graph. G admits analytic mean prime labeling, when n is odd and n > 5.

Proof: Let G be the graph and let $v_1, v_2, ----------------, v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n+1.

Define a function $f : V \to \{0,1,2,3,----------------,2n-1\}$ by

$f(v_i) = i-1$, i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ampl} is defined as follows

$f^*_{ampl}(v_i\ v_{i+1}) = i$, i = 1, 2,------------,2n-1

$f^*_{ampl}(v_1\ v_n) = \frac{(n-1)^2}{2}$.

$f^*_{ampl}(v_{n+1}\ v_{2n}) = \frac{3n^2-4n+1}{2}$.

Clearly f^*_{ampl} is an injection.

gcin of (v_1) = gcd of $\{f^*_{ampl}(v_1\ v_2), f^*_{ampl}(v_1\ v_n)\}$

= 1

gcin of (v_{i+1}) = 1, i = 1, 2,------------,2n-2

gcin of (v_{2n}) = gcd of $\{f^*_{ampl}(v_{n+1}\ v_{2n}), f^*_{ampl}(v_{2n-1}\ v_{2n})\}$

= gcd of $\{2n-1, \frac{3n^2-4n+1}{2}\}$

= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

Theorem 2.7: Dum bell graph admits analytic mean prime labeling, when n is even > 5 and $(n-2) \not\equiv 0 (\mathrm{mod}\ 6)$.

Proof: Let G be the graph and let $v_1, v_2, ---, v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n+1.

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Define a function f : V → {0,1,2,3,---,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,---,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ampl}^* is defined as follows

$f_{ampl}^*(v_i\ v_{i+1})$ = i, i = 1,2,---,2n-1

$f_{ampl}^*(v_1\ v_n)$ $= \frac{n^2-2n+2}{2}$.

$f_{ampl}^*(v_{n+1}\ v_{2n})$ $= \frac{3n^2-4n+2}{2}$.

Clearly f_{ampl}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ampl}^*(v_1\ v_2), f_{ampl}^*(v_1\ v_n)\}$
= 1

gcin of (v_{i+1}) = 1, i = 1,2,---,2n-2

gcin of (v_{2n}) = gcd of $\{f_{ampl}^*(v_{n+1}\ v_{2n}), f_{ampl}^*(v_{2n-1}\ v_{2n})\}$
= gcd of $\{2n-1, \frac{3n^2-4n+2}{2}\}$
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

Theorem 2.8: Let G be the graph obtained by duplicating a vertex in cycle C_n. G admits analytic mean prime labeling, if n is odd.

Proof: Let G be the graph and let v_1, v_2,---,v_{n+1} are the vertices of G

Here |V(G)| = n+1 and |E(G)| = n+2.

Define a function f : V → {0,1,2,3,---,n } by
$f(v_i)$ = i-1, i = 1,2,---,n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ampl}^* is defined as follows

$f_{ampl}^*(v_i\ v_{i+1})$ = i, i = 1,2, ---,n

$f_{ampl}^*(v_1\ v_n)$ $= \frac{n^2-2n+1}{2}$.

$f_{ampl}^*(v_{n+1}\ v_2)$ = n^2-1.

Clearly f_{ampl}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ampl}^*(v_1\ v_2), f_{ampl}^*(v_1\ v_n)\}$
= 1

gcin of (v_{i+1}) = 1, i = 1,2, ---,n-1

gcin of (v_{n+1}) = gcd of $\{f_{ampl}^*(v_{n+1}\ v_n), f_{ampl}^*(v_{n+1}\ v_2)\}$
= gcd of {n, n^2-1}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence G, admits analytic mean prime labeling.

Theorem 2.9: 2- tuple graph of cycle C_n admits analytic mean prime labeling.

Proof: Let G = $T^2(C_n)$ and let v_1,v_2,---,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n.

Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n

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Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ampl}^* is defined as follows

$f_{ampl}^*(v_{2i-1}\ v_{2i})$ = 2i-1, i = 1, 2, ---,n.
$f_{ampl}^*(v_{2i-1}\ v_{2i+1})$ = 4i-2, i = 1, 2, ---,n-1.
$f_{ampl}^*(v_{2i}\ v_{2i+2})$ = 4i, i = 1, 2, ---,n-1.
$f_{ampl}^*(v_1\ v_{2n-1})$ = $2n^2$-4n+2.
$f_{ampl}^*(v_2\ v_{2n})$ = $2n^2$-2n.

Clearly f_{ampl}^* is an injection.

gcin of (v_1) = gcd of $\{f_{ampl}^*(v_1\ v_2)\ \ f_{ampl}^*(v_1\ v_3)\}$
= gcd of {1, 2}= 1.
gcin of (v_2) = gcd of $\{f_{ampl}^*(v_1\ v_2), f_{ampl}^*(v_2\ v_4)\}$
= gcd of {1, 4}= 1.
gcin of (v_{2i+1}) = gcd of $\{f_{ampl}^*(v_{2i-1}\ v_{2i+1}), f_{ampl}^*(v_{2i+1}\ v_{2i+2})\}$
= gcd of {4i-2, 2i+1}
= gcd of {2i-1, 2i+1}
= gcd of {2i-1, 2}= 1, i = 1,2,---,n-1.
gcin of (v_{2i+2}) = gcd of $\{f_{ampl}^*(v_{2i}\ v_{2i+2}), f_{ampl}^*(v_{2i+1}\ v_{2i+2})\}$
= gcd of {4i, 2i+1}
= gcd of {2i, 2i+1}= 1, i = 1,2,---,n-1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $T^2(P_n)$, admits analytic mean prime labeling.

REFERENCES

1. Apostol. Tom M, Introduction to Analytic Number Theory, Narosa, (1998).
2. F Harary, Graph Theory, Addison-Wesley, Reading, Mass, (1972).
3. Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics (2016), #DS6, 1 – 408.
4. T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala 2000.
5. T Thamaraj, P B Sarasija, Analytic Mean Graphs, International Journal of Mathematical Analysis, Vol. 8, No.12, 2014, 595-609.

Source of support: Nil, Conflict of interest: None Declared.
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