International Journal of Mathematical Archive-9(2), 2018, 72-78
Available online through www.ijma.info ISSN 2229 – 5046

ADJOINT OF TRIANGULAR FUZZY NUMBER MATRICES

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(Received On: 19-12-17; Revised & Accepted On: 18-01-18)

ABSTRACT

Fuzzy matrix play an important role in many branches such as deals with Engineering, automata theory, Control theory etc. In this paper the notion called adjoint of matrix have been defined with the domain of triangular fuzzy number. We state a formula for the adjoint matrix of a square triangular fuzzy matrix and this formula shall be used anywhere in this paper. We investigate the relation between the notion of two triangular fuzzy matrices and corresponding some properties are verified. Also we establish the relevant numerical example under the notion of TFM.

Keywords: *Fuzzy Number, Triangular Fuzzy Number (TFN), Triangular Fuzzy Matrix (TFM), Adjoint of Triangular Fuzzy Matrix (ATFM).*

1. INTRODUCTION

Fuzzy sets have been introduced by Lofti.A.Zadeh[11] Fuzzy set theory permits the gradual assessments of the membership of elements in a set which is described in the interval [0,1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951, by means of Zadeh's extension principle [10, 11]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single – valued numbers.

The concept of Rank of a matrix with fuzzy numbers as its elements, which may be used to modern uncertain imprecise aspects of real-word problems. We studied main ideas based on rank of fuzzy matrix and arithmetic operations. We give some necessary and sufficient conditions for algorithm to find rank of fuzzy matrices based on Triangular fuzzy number. In Dubosis and Prade [1] arithmetic operations will be employed for the same purpose but with respect to the inherent difficulties which are derived from the positively restriction on Triangular fuzzy number. The concept of Rank and Nullity of a triangular fuzzy matrix and some of the relevant theorems will be revalued. Finally fuzzifing the defuzzified version of the original problem for introducing fuzzy rank.

The fuzzy matrices introduced first time by Thomson [9] and discussed about the convergence of powers of fuzzy matrix. Kim [5] presented some important results on determinant of square fuzzy matrices. A.K.Shyamal and M.Pal [7, 8] first time introduced triangular fuzzy matrices. In C.Jaisankar and S.Arunvasan [3, 4] introduced the concept on Hessenberg of Triangular Fuzzy Matrices. In Latha *et.al* [6] proposed Rank of a Type-2 Triangular Fuzzy Matrix.

The paper organized as follows, Firstly in section 2, we recall the definitions of Triangular fuzzy number and some operations on triangular fuzzy numbers (TFNs). In section 3, we have reviewed the definition of triangular fuzzy matrix (TFM) and some operations on Triangular fuzzy matrices (TFMs). In section 4, we defined the notion of adjoint triangular fuzzy matrix. In section 5, some of the relevant theorems and properties are justified. In section 6, we have been presented the numerical example with the aid of notion. Finally in section 7, conclusion is included.

2. PRELIMINARIES

In this section, we recapitulate some underlying definitions and basic results of fuzzy numbers.

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Definition 2.1: Fuzzy Set. A fuzzy set is characterized by a membership function mapping the element of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs

$$A = \{(x, \mu_A(x));\ x \in X\}$$

Here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

Definition 2.2: Normal Fuzzy Set. A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exists at least one $x \in X$ such that $\mu_A(x) = 1$.

Definition 2.3: Convex Fuzzy Set. A fuzzy set $A=\{(x,\mu_A(x))\} \subseteq X$ is called Convex fuzzy set if all $A_\propto$ are Convex set (i.e.,) for every element $x_1 \in A_\propto$ and $x_2 \in A_\propto$ for every $\alpha \in [0,1]$, $\lambda x_1 + (1-\lambda) x_2 \in A_\alpha$ for all $\lambda \in [0,1]$ otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.4: Fuzzy Number. A fuzzy set $\tilde{A}$ defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics

i. $\tilde{A}$ is normal
ii. $\tilde{A}$ is convex
iii. The support of $\tilde{A}$ is closed and bounded then $\tilde{A}$ is called fuzzy number.

Definition 2.5: Triangular Fuzzy Number. A fuzzy number $\tilde{A} = (a_{1,} a_{2,} a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ;\ x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & ;\ a_1 \le x \le a_2 \\ 1 & ;\ x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & ;\ a_2 \le x \le a_3 \\ 0 & ;\ x > a_3 \end{cases}$$

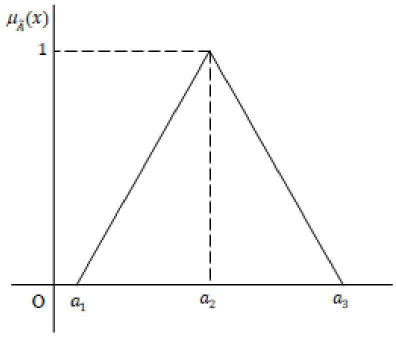

Figure-1: Triangular Fuzzy Number

Definition 2.6: Ranking Function. We defined a ranking function $\Re$: F(R)$\to$R which maps each fuzzy numbers to real line F(R) represent the set of all triangular fuzzy number. If R be any linear ranking function

$$\Re(\tilde{A}) = \left(\frac{a_1 + a_2 + a_3}{3}\right)$$

Also we defined orders on F(R) by

$\Re(\tilde{A}) \ge \Re(\tilde{B})$ if and only if $\tilde{A} \ge \Re(\tilde{B})$
$\Re(\tilde{A}) \le \Re(\tilde{B})$ if and only if $\tilde{A} \ge \Re(\tilde{B})$
$\Re(\tilde{A}) = \Re(\tilde{B})$ if and only if $\tilde{A} = \Re(\tilde{B})$

Definition 2.7: Arithmetic Operations on Triangular Fuzzy Numbers (TFNs). Let $\tilde{A} = (a_{1,} a_{2,} a_3)$ and $\tilde{B} = (b_1, b_{2,} b_3)$ be triangular fuzzy numbers (TFNs) then we defined,

Addition

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

Subtraction

$\tilde{A}$-$\tilde{B}$ = $(a_1 - b_3, a_2 - b_2, a_3 - b_1)$

Multiplication

$\tilde{A}$×$\tilde{B}$ = ($a_1\Re$(B), $a_2\Re$(B), $a_3\Re$(B))

where $\Re(\tilde{B})$= $\left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b})$= $\left(\frac{b_1+b_2+b_3}{3}\right)$

Division

$\tilde{A}$÷$\tilde{B}$ = $\left(\frac{a_1}{\Re\,(\tilde{B})}, \frac{C}{\Re\,(\tilde{B})}, \frac{a_3}{\Re\,(\tilde{B})}\right)$

where $\Re(\tilde{B})$= $\left(\frac{b_1+b_2+b_3}{3}\right)$ or $\Re(\tilde{b})$= $\left(\frac{b_1+b_2+b_3}{3}\right)$

Scalar Multiplication

$$k\tilde{A}=\begin{cases}(ka_{1,}ka_{2,}ka_3)\; if\; K\; \geq 0\\ (ka_{3,}ka_2, ka_1) if k < 0\end{cases}$$

Definition 2.8: Zero Triangular Fuzzy Number. If $\tilde{A}$ = (0, 0, 0) then $\tilde{A}$ is said to be zero triangular fuzzy number. It is defined by 0.

Definition 2.9: Zero Equivalent Triangular Fuzzy Number. A triangular fuzzy number $\tilde{A}$ is said to be a zero equivalent triangular fuzzy number if $\Re$ ($\tilde{A}$)=0. It is defined by $\tilde{0}$.

Definition 2.10: Unit Triangular Fuzzy Number. If $\tilde{A}$ = (1, 1, 1) then $\tilde{A}$ is said to be a unit triangular fuzzy number. It is denoted by 1.

Definition 2.11: Unit Equivalent Triangular Fuzzy Number. A triangular fuzzy number $\tilde{A}$ is said to be unit equivalent triangular fuzzy number. If $\Re$ ($\tilde{A}$) = 1. It is denoted by $\tilde{1}$.

3. TRIANGULAR FUZZY MATRICES (TFMs)

In this section, we introduced the triangular fuzzy matrix and the operations of the matrices some examples provided using the operations.

Definition 3.1: Triangular Fuzzy Matrix (TFM). A triangular fuzzy matrix of order m×n is defined as A=$(\tilde{a}_{ij})_{m\times n}$, where $\tilde{a}_{ij}$=(a_{ij}, b_{ij}, c_{ij}) is the ij^{th} element of A.

Definition 3.2: Operations on Triangular Fuzzy Matrices (TFMs). As for classical matrices. We define the following operations on triangular fuzzy matrices. Let A = $(\tilde{a}_{ij})$ and B = $(\tilde{b}_{ij})$ be two triangular fuzzy matrices (TFMs) of same order. Then, we have the following

i. A+B = $(\tilde{a}_{ij} + \tilde{b}_{ij})$
ii. A-B = $(\tilde{a}_{ij} - \tilde{b}_{ij})$
iii. For A = $(\tilde{a}_{ij})_{m\times n}$ amd B = $(\tilde{b}_{ij})_{n\times k}$ then AB = $(\tilde{c}_{ij})_{m\times k}$ where $\tilde{c}_{ij}$ =$\sum_{p=1}^{n} \tilde{a}_{ip}$ **.** $\tilde{b}_{pj}$, i=1,2,…m and j=1,2,…k
iv. A^T or A^1 = $(\tilde{a}_{ji})$
v. $KA = (K\tilde{a}_{ij})$ where K is scalar.

Definition 3.3: Diagonal TFM. A square TFM A = $(\tilde{a}_{ij})$ is said to be diagonal TFM is all the elements outside the principal diagonal are 0.

Definition 3.4: Diagonal-Equivalent TFM. A square TFM A = $(\tilde{a}_{ij})$ is said to be diagonal-equivalent TFM is all the elements outside the principal diagonal are $\tilde{0}$.

Definition 3.5: Equal Triangular Fuzzy Matrices. Two triangular fuzzy matrices A=$(\tilde{a}_{ij})$ and B=$(\tilde{b}_{ij})$ of the same order are said to be equal if the rank of their elements in the corresponding position are equal. Also it is denoted by A=B.

4. ADJOINT OF TRIANGULAR FUZZY MATRIX

Definition 4.1: Determinant of TFM. The determinant of a n× n TFM A=($\tilde{a}_{ij}$) is denoted by $|A|$or det(A) and is defined as follows:

$$|A|=\sum_{p\in s_n} sign\prod_{i=1}^{n} \tilde{a}_{ip(i)} = \sum_{p\in s_n} sign\prod_{i=1}^{n} \tilde{a}_{1p(1)}, \tilde{a}_{2p(2)} \dots\dots\dots\dots. \tilde{a}_{np(n).}$$

Where $\tilde{a}_{ip(i)}$ are TFN and $S_n$denote the symmetric group of all permutations of the indices {1, 2, 3,….n} and sign p=1 or -1 according as the permutation P = $\begin{pmatrix} 1 & 2\dots\dots\dots & n \\ p(1) & p(2)\dots\dots.. & p(n) \end{pmatrix}$ is even or odd respectively.

Definition 4.2: Minor. Let A = ($\tilde{a}_{ij}$) be a square TFM of order n. The minor of an element ($\tilde{a}_{ij}$) in A is a determinant of order (n-1)×(n-1) which is obtained by deleting i_{th}row and j_{th}column from A and is denoted by $\widetilde{M}_{ij}$.

Definition 4.3: Cofactor. Let A = ($\tilde{a}_{ij}$) be a square TFM of order n. The cofactor of an element $\tilde{a}_{ij}$ in A is denoted by $\tilde{A}_{ij}$ and is defined as $\tilde{A}_{ij} = (-1)^{i+j}\widetilde{M}_{ij}$.

Definition 4.4: Adjoint. Let A=($\tilde{a}_{ij}$) be a square TFM of order n. Find the cofactor $\tilde{A}_{ij}$of every element $\tilde{a}_{ij}$ in A and replace every $\tilde{a}_{ij}$ by its cofactor $\tilde{A}_{ij}$in A and Let it be B. ie., B=($\tilde{A}_{ij}$). Then the transpose of B is called the adjoint or adjugate of A and is denoted by adj A. ie., B'= ($\tilde{A}_{ji}$)=adj A.

Definition 4.5: Aliter Definition for Adjoint. The adjoint matrix of an n× n fuzzy matrix A is denoted by Adj A and is defined as $\tilde{b}_{ij} = |A_{ji}|$ where $|A_{ji}|$ is the determinant of the (n-1)× $(n-1)$ fuzzy matrix formed by deleting row j and column i from A and $B = adjA$.

Definition 4.6: Aliter definition for determinant. Alternatively, the determinant of square TFM $A = (\tilde{a}_{ij})$ of order n may be expanded in the form

$$|A| = \sum_{j=1}^{n} \tilde{a}_{ij}\,\tilde{A}_{ij}\,, i \in \{1,2,\dots\dots\dots\dots n\} \quad \text{Where } \tilde{A}_{ij} \text{ is the cofactor of } \tilde{a}_{ij}.$$

Thus the determinant is the sum of the products of the elements of any row (or column) and the cofactors of the corresponding elements of the same row (or column).

Definition 4.7: Let A be a square TFM of order n then following hold:

(i) A is said to be reflexive TFM $A \geq I_n.\ ie.$, iff all diagonal elements in fuzzy matrix A are unity $ie., iff$ $a_{ii} = 1\ \forall i$.

(ii) A is said to be symmetric $iff\ A' = A\ ie., iff$ the square TFM A remains unaltered by interchanging its rows and columns $ie., a_{ij} = a_{ji}\ \forall\ i,j \in \{1,2,\dots\dots n\}$.

(iii) A is said to be transitive $iff\ \ A^2 \leq A\ ie., iff$ the square TFM A multiplied by itself gives the elements less than or equal to the corresponding elements of the square TFM A $ie., iff\ a_{ik}a_{kj} \leq a_{ij}$ for every $k = 1,2,\dots n$. A square TFM is similarity (equivalence relation) iff it is reflexive, symmetric and transitive.

5. THEOREM

Theorem 5.1: For $n \times n$ fuzzy matrices A and B we have the following

1. $A \leq B\ implies\ adj\ A \leq\ adj\ B$
2. $adj\ A\ +\ adj\ B \leq\ adj\ (A + B)$
3. $adj\ A^T = (adj\ A)^T$

Proof:

1. Let $c =\ adj\ A$ and $D =\ adj\ B$.
   That is, C_{ij}=$\sum_{\pi\epsilon s_{n_i n_j}} \prod_{t\epsilon n_i} a_t\,\pi(t)$ and d_{ij} = $\sum_{\pi\epsilon s_{n_i n_j}} \prod_{t\epsilon n_i} a_t\,\pi(t)$
   It is clear that c_{ij}≤ d_{ij} because $a_{t\pi(t)}$≤ $b_{t\pi(t)}$ for every tϵn_j.
2. Since A, B ≤ A+B, if is clear that $adj\ A, adj\ B\ \leq\ adj(A + B)$ and So $adj\ A +\ adj\ B\ \leq\ adj(A + B)$
3. Let B= adj A and C= adj A^T
   Then b_{ij}=$\sum_{\pi\epsilon s_{n_i n_j}} \prod_{t\epsilon n_i} a_t\,\pi(t)$ and C_{ij}=$\sum_{\pi\epsilon s_{n_i n_j}} \prod_{t\epsilon n_i} a_t\,\pi(t)$

Which is the element $b_{ij.}$ Hence $(adj\ A)^T$= $adj\ A^T$.

Theorem 5.2: For a fuzzy matrix A we have$|A|$=$|adj\ A|$.

Proof: By definition,
For A ϵ there exist n, the determinant of A, denoted as det(A) is defined as det(A)=$\sum_{\pi\epsilon S_n} a_{1\pi(1)}\,a_{2\pi(2)}$……..$a_{n\pi(n)}$,
Where the summation is taken over all π of S_n.

$$adj\ A=\begin{bmatrix} |A_{11}| & |A_{21}| \dots & |A_{n1}| \\ |A_{12}| & |A_{22}| \cdots & |A_{n2}| \\ \vdots & \vdots & \vdots \\ |A_{1n}| & |A_{2n}| \dots & |A_{nn}| \end{bmatrix}$$

$$\begin{aligned} |adj\ A| &= \textstyle\sum_{\pi\epsilon S_n} |A_{1\pi(1)}|\,|A_{2\pi(2)}| \dots\dots |A_{n\pi(n)}| \\ &= \textstyle\sum_{\pi\epsilon S_n} \prod_{i=1}^{n} |A_{i\pi(i)}| \\ &= \textstyle\sum_{\pi\epsilon S_n} [\prod_{i=1}^{n} (\sum_{\theta\epsilon s_{n_i n\pi(i)}} \prod_{t\epsilon n_i}^{n} a_{t\theta(t)})] \\ &= \textstyle\sum_{\pi\epsilon S_n} [(\sum_{\theta\epsilon s_{n_1 n\pi(1)}} \prod_{t\epsilon n_1} a_{t\theta(t)})\,(\sum_{\theta\epsilon s_{n_2 n\pi(2)}} \prod_{t\epsilon n_2} a_{t\theta(t)}) \dots (\sum_{\theta\epsilon s_{n_n n\pi(n)}} \prod_{t\epsilon n_n} a_{t\theta(t)})] \\ &= \textstyle\sum_{\pi\epsilon S_n} [(\prod_{t\epsilon n_1}^{n} a_{t\theta_1(t)})\,(\prod_{t\epsilon n_2}^{n} a_{t\theta_2(t)}) \dots (\prod_{t\epsilon n_n}^{n} a_{t\theta_n(t)})] \end{aligned}$$

For some

$\theta_1 \epsilon s_{n(1)n\pi(1)}, \theta_2 \epsilon s_{n(2)n\pi(2)}, \dots\dots\dots, \theta_n \epsilon s_{n(n)n\pi(n)}$

$=\sum_{\pi\epsilon S_n}[(a_{2\theta_1(2)} a_{3\theta_1(3)} \dots a_{n\theta_1(n)})(a_{1\theta_2(1)} a_{3\theta_2(3)} \dots a_{n\theta_2(n)}) \dots (a_{1\theta_n(1)} a_{2\theta_n(2)} \dots a_{n-1\theta_n(n-1)})]$

$=\sum_{\pi\epsilon S_n}[(a_{1\theta_2(1)} a_{1\theta_3(1)} \dots a_{1\theta_n(1)})(a_{2\theta_1(2)} a_{2\theta_3(2)} \dots a_{2\theta_n(2)})(a_{3\theta_1(3)} a_{3\theta_2(3)} \dots a_{3\theta_n(3)}) \dots (a_{n\theta_1(n)} a_{n\theta_2(n)} \dots a_{n\theta_{n-1}(n)})]$

$=\sum_{\pi\epsilon S_n}[a_{1\theta_{f_1}(1)} a_{2\theta_{f_2}(2)} \dots a_{n\theta_{f_n}(n)}]$

For some $f_h \epsilon \{1, 2, \dots, n\}/\{h\}$, h=1,2,3…,n. However because $a_{h\theta_{f_h(h)}} \neq a_{h\pi(h)}$, we can see that $a_{n\theta_{f_n(h)}} = a_{h\pi(f_n)}$.

Therefore $|adj\ A| = \sum_{\pi\epsilon S_n} (a_{1\pi(1)}\, a_{2\pi(2)} \dots a_{n\pi(n)})$, which is the expansion of $|A|$. This completes the proof.

Theorem 5.3: Let A be an $n \times n$ constant fuzzy matrix. Then we have

1. $(adj\ A)^T$ is constant
2. $C = A(adj\ A)$ is constant and $c_{ii}=|A|$ which is the greatest element in A.

Proof:

1. Let B= $adj\ A$. Then $b_{ij}=\sum_{\pi\epsilon s_{n_j n_i}} \prod_{t\epsilon n_j} a_{t\pi(t)}$ and $b_{ik}=\sum_{\pi\epsilon s_{n_k n_i}} \prod_{t\epsilon n_k} a_{t\pi(t)}$

   We notice that $b_{ij} = b_{ik}$ because the number π(t) of columns cannot be changed in the two expansions of b_{ij} so $(adj\ A)^T$ is constant.

Theorem 5.4: For an $n \times n$ fuzzy matrix A we have the following

a. If A is symmetric, then $adj\ A$ is symmetric

b. If A is transitive, then $adj\ A$ is transitive

Proof:

a. Let $B = adj\ A$. Then $b_{ij} = \sum_{\pi\epsilon s_{n_j n_i}} \prod_{t\epsilon n_j} a_{t\pi(t)} = \sum_{\pi\epsilon s_{n_i n_j}} \prod_{t\epsilon n_j} a_{t\pi(t)} = b_{ij}$(because A is symmetric)

b. Let D=A_{ij}. We can determine the elements of D in terms of the elements of A as

$$d_{hk}=\begin{Bmatrix} a_{hk,}\ if\ h < i, k < j, \\ a_{(h+1)k}, if\ h \geq i, k < j, \\ a_{h(k+1)}, if\ h < i, k \geq j, \\ a_{(h+1)(k+1)}, if\ h \geq i, k \geq j \end{Bmatrix}$$

Where A_{ij} denotes the (n-1)×(n-1) fuzzy matrix obtained from A by deleting the i^{th} row and column j. Now we show that $A_{st}A_{tu} \leq A_{su}$ for every $t\epsilon\{1,2, \dots n\}. Let$ R=A_{st}, C=A_{tu}, F=A_{su}, W=$A_{st}A_{tu}$ and note that A is transitive. Then

$W_{ij} = \sum_{k=1}^{n-1} r_{ij} c_{kj}$

$= \sum_{k=1}^{n-1} a_{ij} a_{kj} \leq a_{ij} = f_{ij}$ (if i<s, k<t, j<u)

$= \sum_{k=1}^{n-1} a_{ik} a_{k(j+1)}\, a_{i(j+1)} = f_{ij}$ (if i<s, k<t, j≥u)

$= \sum_{k=1}^{n-1} a_{i(k+1)} a_{(k+1)j}\, a_{i(j+1)} \leq a_{ij} = f_{ij}$ (if i<s, k≥t, j<u)

$= \sum_{k=1}^{n-1} a_{i(k+1)} a_{(k+1)(j+1)} \leq a_{i(j+1)} = f_{ij}$ (if i<s, k≥t, j≤u)

$= \sum_{k=1}^{n-1} a_{(i+1)k} a_{kj} \leq a_{(i+1)j} = f_{ij}$ (if i≥s, k≥t, j<u)

$= \sum_{k=1}^{n-1} a_{(i+1)(k+1)} a_{(k+1)j} \leq a_{(i+1)j} = f_{ij}$ (if i≥s, k≥t, j<u)

$= \sum_{k=1}^{n-1} a_{(i+1)(k+1)} a_{(k+1)(j+1)} \leq a_{(i+1)(j+1)} = f_{ij}$

(if i≥s, k≥t, j<u)

$= \sum_{k=1}^{n-1} a_{(i+1)k} a_{k(j+1)} \leq a_{(i+1)(j+1)} = f_{ij}$ (if i≥s, k≥t, j<u)

Thus $W_{ij} \leq f_{ij}$ in every case and therefore $a_{st}\ A_{tu} \leq A_{su}$ for every tϵ {1,2,…,n}.

By theorem. we get $|A_{st}||A_{tu}| \leq |A_{st}A_{tu}| \leq |A_{su}|$.

This means $b_{ts}\, b_{ut} \leq b_{us}$, $ie,.\, b_{ut} b_{ts} \leq b_{us}$ for every tϵ {1,2,…,n}.

Hence $B = adj\ A$ is transitive.

Theorem 5.5: To construct a transitive fuzzy matrix from a given TFM through adjoint matrix. For any $n \times n$ TFM A, the TFM $A(adjA)$ is transitive.

Proof:

Let $C = \left[C_{ij}\right] = A(adjA)$

Then $C_{ij} = \sum_{k=1}^{n} a_{ik} \left|A_{jk}\right| = a_{if} \left|A_{if}\right|$ for some $f \in \{1,2, \dots \dots n\}$

and $C^2{}_{ij} = \sum_{s=1}^{n} C_{is} C_{sj} = \sum_{s=1}^{n} .\left[\left(\sum_{i=1}^{n} a_{i1}\, |A_{sl}|\right)\left(\sum_{i=1}^{n} a_{st}\, \left|A_{jt}\right|\right)\right]$

$$\begin{aligned}
&= \sum_{s=1}^{n} (a_{ih} |A_{sh}|)\left(a_{su}\left|A_{ju}\right|\right)\ for\ some\ h, u \in 1,2, \dots .n \\
&= a_{ih}\left|A_{gh}\right|\left|a_{gu}\right|\left|A_{ju}\right| \\
&\leq a_{ih}\left|A_{ju}\right| \\
&\leq a_{if}\left|A_{jf}\right| = C_{ij}
\end{aligned}$$

Thus $C^2{}_{ij} \leq C_{ij}$ and so $(A\ adjA)^2 \leq A(adjA)$

Hence $A(adjA)$ is transitive.

6. NUMERICAL EXAMPLE

If $A = \begin{pmatrix} [(-2,-1,3),(-1,0,4),(3,4,8)] & [(-3,-2,2),(-2,-1,3),(2,3,7)] \\ [(0,1,5),(1,2,6),(5,6,10)] & [(1,2,6),(2,3,7),(6,7,11)] \end{pmatrix}$

Then $|A| = [(-10,-5,15),(-5,0,20),(15,20,40)] - [(0,1,5),(1,2,6),(5,6,10)]$
$= [(-20,-11,10),(-11,-2,19),(10,19,40)]$

Now, $\breve{R}(|A|) = 54/9$
$ie., \breve{R}(|A|) = 6.$
Also,

$$adjA = \begin{pmatrix} [(1,2,6),(2,3,7),(6,7,11)] & [(-7,-3,-2),(-3,1,2),(-2,2,3)] \\ [(-10,-6,-5),(-6,-2,-1),(-5,-1,0)] & [(-2,-1,3),(-1,0,4),(3,4,8)] \end{pmatrix}$$

$$A(adjA) = \begin{pmatrix} [(-2,-1,3),(-1,0,4),(3,4,8)] & [(-3,-2,2),(-2,-1,3),(2,3,7)] \\ [(0,1,5),(1,2,6),(5,6,10)] & [(1,2,6),(2,3,7),(6,7,11)] \end{pmatrix} \begin{pmatrix} [(1,2,6),(2,3,7),(6,7,11)] & [(-7,-3,-2),(-3,1,2),(-2,2,3)] \\ [(-10,-6,-5),(-6,-2,-1),(-5,-1,0)] & [(-2,-1,3),(-1,0,4),(3,4,8)] \end{pmatrix}$$

$$C = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$ie., A(adjA) = \breve{R}\ (|\mathrm{A}|)\mathrm{I}^2$

$$(adjA)A = \begin{pmatrix} [(1,2,6),(2,3,7),(6,7,11)] & [(-7,-3,-2),(-3,1,2),(-2,2,3)] \\ [(-10,-6,-5),(-6,-2,-1),(-5,-1,0)] & [(-2,-1,3),(-1,0,4),(3,4,8)] \end{pmatrix} \begin{pmatrix} [(-2,-1,3),(-1,0,4),(3,4,8)] & [(-3,-2,2),(-2,-1,3),(2,3,7)] \\ [(0,1,5),(1,2,6),(5,6,10)] & [(1,2,6),(2,3,7),(6,7,11)] \end{pmatrix}$$

$$C = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$ie., (adjA)A = \breve{R}\ (|\mathrm{A}|)\mathrm{I}^2$

7. CONCLUSION

In this article, adjoint of triangular fuzzy matrix are defined and also some special properties of adjoint of Triangular Fuzzy Matrices are proved. Using these results of Triangular Fuzzy Matrices, some important properties of Triangular Fuzzy Matrices involving the notion like inverse of matrix can be studied in future.

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Source of support: Nil, Conflict of interest: None Declared.