

## SORET AND DUFOUR EFFECTS ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OF MICROPOLAR FLUID IN A CIRCULAR ANNULUS

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### ABSTRACT

*In this paper, we make an attempt to investigate combined influence of magnetic field and dissipation on convective heat and mass transfer flow of a viscous chemically reacting fluid through a porous medium in the concentric cylindrical annulus with inner cylinder maintained at constant temperature and concentration on the other cylinder maintained constant heat flux. The equations governing the flow, heat, Mass and micro rotation are solved by employing Galerkin finite element analysis with quadratic approximation functions. The temperature, concentration and micro concentration distributions are analyzed for different values of  $Sr/Du$ ,  $\gamma$ ,  $Ec$ ,  $\Delta$ ,  $\lambda$  and  $A$ . The rate of heat and mass transfer and couple stress are numerically evaluated for different variations of the governing parameters.*

**Keywords:** Soret and Dufour effects, heat and mass transfer, Circular annulus, micropolar fluid.

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### 1. INTRODUCTION

An enclosed cylindrical annular cavity formed by three vertical, concentric cylinders, containing a fluid through which heat is transferred by natural convection, is a simplified representation of a number of practical and experimental situations. Also, the annulus represents a common geometry employed in a variety of heat transfer systems ranging from simple heat exchangers to the most complicated nuclear reactors. Since, the flow and heat transfer in a cylindrical annular configuration contains all the essential physics that are common to all confined natural convective flows, a complete understanding of the flow in such geometry is very essential. In addition, from a computational stand point, the annular configuration allows investigation of a wide range of geometrical effects. Convection is an important phenomenon in the crystal growth techniques as it can account for heat transfer in the liquid phase and can change the material properties.

The theory of micropolar fluids initiated by Eringen [5] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stress and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid randomly oriented (or sphenical) particles suspended in a viscous medium where the deformation of the particles is ignored. The fluid containing certain additives, some polymeric fluids and animal blood are examples of micropolar fluids. The mathematical theory of equations of micropolar fluids and application of these fluids in the theory of lubrication and porous media is presented by Lukaszewics [12]. Agarwal and Dhanpal [1] obtained numerical solution of micropolar fluid flow and heat transfer between two co-axial porous circular cylinders. Verma and Singh [27] have analyzed the behaviour of parametric fluid flow in a porous annulus in the presence of external magnetic field acting parallel to the common axis of the long co axial porous cylindrical tubes. Panja *et.al* [19] studied the flow of electrically conducting Reiner – Rivlin fluid between two non-conducting co axial circular cylinders with porous walls in the presence of uniform magnetic field. Shivashankar *et.al* [23] have obtained numerical solution to the MHD flow of micropolar fluid between two concentric porous cylinders; Murthy *et.al* [14] have considered study flow of micropolar fluid through a circular pipe under a transverse magnetic field with constant suction and injection.

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Gebhart [10], Gebhart and Mollendorf [11] have shown that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size at extremely low temperature or in high gravitational field. On the other hand Barletta [3] have pointed out that relevant effects of viscous dissipation on the temperature profiles and on Nusselt number may occur in the fully developed forced convection in tubes. In view of this several authors notably Soundalgekar and Pop [26]. Soundalgekar *et.al* [24], Barletta and Zandhien [4], Sreevani [25], El-Hakein [6] and Barletta [3] have studied the effect of viscous dissipation on the convective flows past an infinite vertical plate and through vertical channels and ducts. The effect of viscous dissipation on natural convection has been studied for some different cases including the natural convection from horizontal cylinder embedded in a porous media by Fand and Brucker [7], Giampietrao Fabbri [9] and Sreevani [25]. They reported that the viscous dissipation may not be neglected in all cases of natural convection from horizontal cylinders and further that the inclusion of a viscous dissipation term in a porous medium may lead to more accurate correlation equations, the effect of viscous dissipation has been studied by Nakayama and Pop [15] for steady free convection boundary layer over a non-isothermal body of arbitrary shape embedded in porous media. They used the integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. This observation has been pointed also by Murthy and Singh [13] for the natural connection flow along an isothermal wall embedded in a porous medium. They concluded that that the effect of viscous dissipation increases as we move from Non-Darcy regime to Darcy regime. Recently, Prasuna *et.al* (28) have analysed the effect of dissipation and Soret effect on convective heat and mass transfer flow through a porous medium in a concentric annulus.

## 2. FORMULATION OF THE PROBLEM

We consider the steady flow of an incompressible, viscous, electrically conducting micropolar fluid through a porous medium in an annulus region between the concentric porous cylinders  $r = a$  and  $r = b$  ( $b > a$ ) under the influence of a radial magnetic field  $\frac{H_0}{r^2}$ .

The fluid is injected through the inner cylinder with radial velocity  $u_a$  and flows outward through the outer cylinder with a radial velocity  $u_b$ . We also take the viscous, Darcy and Ohmic dissipation into account

The velocity and micro rotation are taken in the form

$$\left. \begin{aligned} v_r = u(r), v_\theta = v = 0, & \quad v_z = w(r) \\ \omega_r = 0, \quad \omega_\theta = \omega(r), & \quad \omega_z = 0 \end{aligned} \right\} \quad (1)$$

The equations governing the flow and heat and mass transfer eqn.(1)

$$\frac{\partial u}{\partial r} + \frac{u}{r} = 0$$

$$\rho \mu \frac{\partial u}{\partial r} = -\frac{\partial p}{\partial r} + (\mu + k) \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \left( \frac{\mu}{k_1} \right) u \quad (2)$$

$$\rho \mu \frac{\partial w}{\partial r} = -\frac{\partial p}{\partial z} + (\mu + k) \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \rho \bar{g} - \left( \frac{\mu}{k_1} \right) w - k \frac{\partial w}{\partial r} (r\omega) \quad (3)$$

$$\rho j \mu \frac{\partial \omega}{\partial r} = r \left( \frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} - \frac{N}{r^2} \right) - k \frac{\partial \omega}{\partial r} - 2k\omega \quad (4)$$

$$\begin{aligned} \rho_0 C_p w \frac{\partial T}{\partial z} = k_f \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + (2\mu + k) \left[ \left( \frac{\partial u}{\partial r} \right)^2 + \frac{\mu^2}{r^2} + \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2 \right] \\ + 2k \left( \frac{1}{2} \frac{\partial w}{\partial r} + \omega \right)^2 - 2 \frac{\beta}{r} \omega \frac{\partial \omega}{\partial r} + r \left[ \left( \frac{\partial \omega}{\partial r} \right)^2 + \frac{\omega^2}{r^2} \right] - Q_H (T - T_e) + \frac{D_m K_T}{C_s} \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) \end{aligned} \quad (5)$$

$$w \frac{\partial C}{\partial z} = D_m \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - \gamma' C + \frac{D_m K_T}{T_m} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) \quad (6)$$

$$\rho - \rho_o = -\beta \rho_o (T - T_o) - \beta^* \rho_o (C - C_o) \quad (7)$$

where  $u$ ,  $w$  are the velocity components along  $0(r, z)$  directions,  $T$  is the temperature,  $\omega$  is the micro rotation,  $p$  is the pressure,  $\rho$  is the density,  $\mu$  is the dynamic viscosity,  $C_p$  is the specific heat at constant pressure,  $k_f$  is the thermal conductivity,  $k_1$  is the permeability of the porous permeability,  $\sigma$  is the electrical conductivity  $\mu$  is the magnetic permeability and  $k$ ,  $r$ ,  $\beta$  are the material constants.

The boundary conditions are

$$\begin{aligned} u = u_a, \quad w = 0, \quad \omega = 0, \quad T = T_0 + A_0 z, \quad C = C_0 + B_0 z \quad \text{on} \quad r = a \\ u = u_b, \quad w = 0, \quad \omega = 0, \quad T = T_1 + A_0 z, \quad C = C_1 + B_0 z \quad \text{on} \quad r = b \end{aligned}$$

From the equation of continuity we obtain

$$ru = c, \text{ constant} \Rightarrow ru = au_a = bu_b \Rightarrow u = \frac{au_a}{r}$$

In view of the boundary condition on temperature and concentration, we may write

$$T = T_0 + A_0(z) + \theta(r), \quad C = C_0 + B_0(z) + \phi(r)$$

On introducing the non-dimensional variables  $r'$ ,  $w'$ ,  $\theta'$ ,  $p'$  and  $N'$  as

$$r' = \frac{r}{a}, \quad w' = \frac{w}{v/a}, \quad \theta = \frac{T - T_0}{T_i - T_0}, \quad \omega' = \frac{(\mu + k)\omega}{\rho\left(\frac{\mu^2}{a^2}\right)}, \quad p' = \frac{p}{\rho\left(\frac{\mu^2}{a^2}\right)}, \quad \phi = \frac{C - C_0}{C_i - C_0}$$

The equations (2) – (4) reduces to (on dropping the dashes)

$$w_{rr} + \left(1 - \frac{S}{1 + \Delta}\right) \frac{1}{r} w_r = -\pi_1 + \frac{M^2}{r^2} w + D_1^{-1} w - G_1(\theta + N\phi) - \frac{\Delta_1}{r} \frac{\partial}{\partial r}(r\omega)$$

$$\omega_{rr} + (1 - SA) \frac{1}{r} \omega_r - \left(\frac{1}{r^2} - \frac{2\Delta}{A}\right) \omega = \frac{\Delta_1}{Ar} \frac{dw}{dr} \quad (8)$$

$$P_r N_T w = \theta_{rr} + \frac{1}{r} \theta_r - \alpha \theta + Du \Pr \left( \phi_{rr} + \frac{1}{r} \phi_r \right) + Ec P_r \left\{ (2 + \Delta) \left( \frac{S^2}{r^4} + w_r^2 \right) + 2\Delta \left( \frac{1}{2} w_r + \omega \right)^2 \right. \\ \left. - 2\Delta \omega \frac{\partial \omega}{\partial r} + A_1 \left( w_r^2 + \frac{\omega^2}{r^4} \right) \right\} \quad (9)$$

$$Sc N_c w = \phi_{rr} + \frac{1}{r} \phi_r - \gamma \phi + Sc So \left( \theta_{rr} + \frac{1}{r} \theta_r \right)$$

where

$$G = \frac{\beta g a^3 (T_i - T_0)}{v^2} \text{ (Grashof number)}, \quad D^{-1} = \frac{a^2}{k_1} \quad \text{(Darcy parameter)}$$

$$P_r = \frac{\mu C_p}{k_f} \text{ (Prandtl number)}, \quad Ec = \frac{a^2}{C_p (T_i - T_0) v^2} \quad \text{(Eckert number)}$$

$$\alpha = \frac{Q_H a^2}{k_f C_p} \text{ (Heat source parameter)}, \quad S = \frac{au_a}{v} \quad \text{(Suction parameter)}$$

$$So = \frac{D_m K_T (C_i - C_0)}{T_m (T_i - T_0)} \text{ (Soret parameter)}, \quad Du = \frac{D_m K_T (T_i - T_0)}{C_s (C_i - C_0)} \quad \text{(Dufour parameter)}$$

$$\gamma = \frac{\gamma' a^2}{D_m} \text{ (Chemical Reaction parameter)}, \quad A = \frac{r}{\mu a^2}, \quad \Delta = \frac{\mu}{k} \quad \text{(Micropolar parameters)}$$

$$\Delta_1 = \frac{\Delta}{1 + \Delta}, \quad D_1^{-1} = \frac{D^{-1}}{1 + \Delta}, \quad G_1 = \frac{G}{1 + \Delta}, \quad r = \frac{b}{a}$$

The non-dimensional boundary conditions are

$$\begin{aligned} w = 0, \quad \theta = 1, \quad \phi = 1, \quad N = 0 \quad \text{on} \quad r = 1 \\ w = 0, \quad \theta = 0, \quad \phi = 0, \quad N = 0 \quad \text{on} \quad r = s \end{aligned} \quad (10)$$

### 3. METHOD OF SOLUTION

The finite element analysis with quadratic polynomial approximation functions is carried out along the radial distance across the circular cylindrical annulus. The behavior of the velocity, temperature and concentration profiles has been discussed computationally for different variations in governing parameters. The Gelarkin method has been adopted in the variation formulation in each element to obtain the global coupled matrices for the velocity, temperature and concentration in course of the finite element analysis. Choose an arbitrary element  $e_k$  and let  $w^k$ ,  $\theta^k$ ,  $\phi^k$  and  $N^k$  be the values of  $w$ ,  $\theta$ ,  $\phi$  and  $N$  in the element  $e_k$ .

We define the error residuals as

$$E_w^k = \frac{d}{dr} \left( r \frac{dw^k}{dr} \right) - \frac{\lambda}{1+\Delta} \frac{dw^k}{dr} + \Delta_1(r\omega^k) + \pi_1 r - D^{-1}rw^k - Gr(\theta^k + N\phi^k) \quad (11)$$

$$E_\theta^k = \frac{d}{dr} \left( r \frac{d\theta^k}{dr} \right) - P_r N_t w_r - \alpha\theta + EcP_r \left\{ \begin{aligned} & \left( (2+\Delta) \frac{\lambda^2}{r^3} + ((2+\Delta)r + A) \left( \frac{dw}{dr} \right)^2 + \right. \\ & \left. 2\Delta r \left( \frac{1}{2} \frac{dw}{dr} + \right)^2 \omega - 2A\omega \frac{d\omega}{dr} + A_1 \frac{\omega^2}{r} \right) \end{aligned} \right\} \quad (12)$$

$$+ Du Pr \frac{d}{dr} \left( r \frac{d\phi^k}{dr} \right) \quad (13)$$

$$E_c^k = \frac{d}{dr} \left( r \frac{d\phi^k}{dr} \right) - ScN_c w_r - \gamma\phi^k + ScSo \frac{d}{dr} \left( r \frac{d\theta^k}{dr} \right) \quad (14)$$

$$E_N^k = \frac{d}{dr} \left( r \frac{d\omega^k}{dr} \right) - \omega Ar \frac{d\omega^k}{dr} - \left( \frac{1}{r} - \frac{2\Delta r}{A} \right) \omega^k - \frac{\Delta}{A} \frac{dw^k}{dr} \quad (15)$$

Where  $w^k$ ,  $\theta^k$ ,  $\phi^k$  and  $\omega^k$  are values of  $w$ ,  $\theta$ ,  $\phi$  and  $\omega$  in the arbitrary element  $e_k$ . These are expressed as linear combinations in terms of respective local nodal values.

$$\begin{aligned} w^k &= w_1^k \psi_1^k + w_2^k \psi_2^k + w_3^k \psi_3^k = \sum_{i=1}^3 w_i^k \psi_i^k \\ \theta^k &= \theta_1^k \psi_1^k + \theta_2^k \psi_2^k + \theta_3^k \psi_3^k = \sum_{i=1}^3 \theta_i^k \psi_i^k \\ \phi^k &= \phi_1^k \psi_1^k + \phi_2^k \psi_2^k + \phi_3^k \psi_3^k = \sum_{i=1}^3 \phi_i^k \psi_i^k \\ \omega^k &= \omega_1^k \psi_1^k + \omega_2^k \psi_2^k + \omega_3^k \psi_3^k = \sum_{i=1}^3 \omega_i^k \psi_i^k \end{aligned} \quad (15)$$

Where  $\psi_1^k, \psi_2^k, \dots$  etc are Lagrange's quadratic polynomials.

Substituting eqn(15) in equations (11 – 14) and evaluating the resulting integral we get local stiffness matrices. These matrices are assembled into global matrix by using inter element continuity, equilibrium and boundary conditions. These global matrices are solved by iteration procedure. The iteration process is repeated until the convergence is obtained i.e.  $|u_{i+1} - u_i| < 10^{-6}$ .

### 4. SHEAR STRESS, NUSSELT NUMBER, SHERWOOD NUMBER

The shear stress ( $\tau$ ) is evaluated using the formula  $\tau = \left( \frac{du}{dr} \right)_{r=1,S}$

The rate of heat transfer (Nusselt number) is evaluated using the formula  $Nu = - \left( \frac{d\theta}{dr} \right)_{r,S}$

The rate of mass transfer (Sherwood number) is evaluated using the formula  $Sh = - \left( \frac{d\phi}{dr} \right)_{r,S}$

The couple stress at the inner and outer cylinder are evaluated by  $M^* = - \left( \frac{d\omega}{dr} \right)_{r,S}$

## COMPARISON

In the absence of Heat sources ( $\alpha=0$ ) and Dufour effect ( $Du=0$ ) the results are in good agreement with those of Prasuna *et.al* (28)

Table - 1

Parameters					Prasuna et al(28)				Present results( $\alpha=0$ )			
N	Ec	$\gamma$	$\Delta$	$\lambda$	Nu(1)	Nu(2)	Sh(1)	Sh(2)	Nu(1)	Nu(2)	Sh(1)	Sh(2)
1	0.01	0.5	1	0.1	2.8636	-5.1236	4.2065	-10.781	2.8632	-5.1237	4.2068	-10.780
2	0.01	0.5	1	0.1	6.4175	-11.548	6.7759	-15.409	6.4173	-11.546	6.7756	-15.408
-0.5	0.01	0.5	1	0.1	-0.8592	-1.5012	1.7038	-3.6098	-0.8591	-1.5011	1.7036	-3.6096
-1.5	0.01	0.5	1	0.1	0.1787	-0.01638	-0.9098	-1.5997	0.1779	-0.01633	-0.9099	-1.5994
1	0.03	0.5	1	0.1	1.8355	-3.3072	4.2058	-10.7794	1.8356	-3.3071	4.2055	-10.7796
1	0.05	0.5	1	0.1	2.8756	-5.1236	4.2065	-10.8802	2.8754	-5.1233	4.2066	-10.9801
1	0.01	1.5	1	0.1	2.8853	-5.1777	3.2232	-8.7041	2.8857	-5.1774	3.2229	-8.7042
1	0.01	0.5	3	0.1	1.778	-5.2273	1.5148	-5.5912	1.779	-5.2271	1.5149	-5.5911
1	0.01	0.5	5	0.1	1.035	-5.0836	4.0588	-8.4699	1.033	-5.0833	4.0589	-8.4696
1	0.01	0.5	1	0.3	2.8636	-2.8052	3.0811	-6.5156	2.8633	-2.8051	3.0810	2.8636
1	0.01	0.5	1	0.5	2.9613	-5.1236	3.4047	-9.5098	2.9611	-5.1233	3.4049	-9.5099

## 5. RESULTS AND DISCUSSION

In this analysis we investigate the effect of thermo-diffusion, diffusion-thermo, dissipation, heat sources and chemical reaction on mixed convective heat and mass transfer flow of a micro polar fluid through a porous medium in circular annulus between the cylinders  $r=a$  and  $r=b$  which are maintained at constant temperature and concentration. The non-linear coupled equations governing the flow heat and mass transfer are solved by Galerkin finite element analysis with quadratic approximation functions. the effect of important parameters namely micro polar parameter  $\Delta$ , the suction Reynolds number  $\lambda$  and Dufour parameter  $Du$  has been studied for these functions and corresponding profiles.

Fig.1 shows the effect of Soret and Dufour effects on  $w$ . It can be seen from the graphs that increasing Soret parameter  $So$  (or decreasing Dufour parameter  $Du$ ) smaller  $|w|$ . Fig.2 represents  $w$  with chemical reaction parameter  $\gamma$ . It is found that the magnitude of  $w$  reduces in degenerating/ generating chemical reaction cases. The effect of dissipation on  $w$  is exhibited in Fig.3. It is found that smaller the dissipative effect smaller the magnitude of  $w$ . From Fig.4 represents  $w$  with micropolar parameter ( $\Delta$ ).It can be seen from the profiles that the axial velocity enhances with increase in  $\Delta$ .in the entire flow region. Fig.5 represents  $w$  with suction parameter  $\lambda$ . It is found that  $|w|$  enhances with increase in  $\lambda$ .An increase in the micropolar parameter ( $\Delta$ ) increases the magnitude of  $w$  in the flow region (fig.6).

The non-dimensional temperature ( $\theta$ ) is exhibited in Figs. 7-13 for different parametric values. It is found that the non-dimensional temperature is always positive for all variations. This indicates the actual temperature is always greater than  $T_0$ . Higher the thermo-diffusion effects (or smaller the diffusion thermo effects) smaller the actual temperature in the flow region (fig.7). Fig.8 represents  $\theta$  with heat source parameter  $\alpha$ .We notice from the profiles that the actual temperature enhances with increase in the strength of the heat generating source and reduces with that of heat absorbing source. Fig.9 represents  $\theta$  with chemical reaction parameter  $\gamma$ . It is found that the actual temperature enhances in the degenerating chemical reaction case and reduces in the generating case. The variation of  $\theta$  with Eckert number  $Ec$  is shown in the Fig. 10. It is found that the actual temperature reduces with  $Ec$  in the entire flow region.Thus higher the dissipation lesser the actual temperature in the flow region..Fig.11 shows the variation  $\theta$  with micropolar parameter  $\Delta$ .Higher the values of  $\Delta$ , lesser the actual temperature in the flow region. The variation of  $\theta$  with  $\lambda$  is exhibited in Fig.12 it is found that the actual temperature experiences an enhancement with increase in suction parameter. Fig.13 exhibits  $\theta$  with micropolar parameter  $\Delta$ . It can be seen from the graphs that the actual temperature enhances with increase in  $\Delta$ .

The concentration distribution ( $C$ ) is exhibited in Figs.14-19. We follow the convention that the non-dimensional concentration distribution is positive or negative according as the actual concentration is greater/lesser than  $C_0$ . Fig.14 shows the variation of  $C$  with Dufour parameter  $Du$ . In can be seen from the profiles that higher the thermo-diffusion effects (or smaller the diffusion –thermo effects) larger the actual concentration in the flow. From fig.15 we find that the actual concentration experiences a enhancement in the degenerating chemical reaction case and in generating chemical reaction case it reduces in the flow region. From fig.16 we find that higher the dissipation lesser the actual concentration in the flow region. The actual concentration reduces with increase in micropolar parameters  $\Delta$  in (fig.17) and  $\lambda$  (fig.19). The variation of  $C$  with  $\lambda$  is exhibited in Fig.18. It is found that the actual temperature experiences an enhancement with increase in suction parameter ( $\lambda$ ).

The micro rotation ( $\omega$ ) is shown in Figs. 20-25 for different parametric values. Higher the thermo-diffusion effects (or smaller the diffusion-thermo effects) larger the magnitude of the angular velocity (fig.20). From fig.21 we notice that the angular velocity  $\omega$  reduces in magnitude in both degenerating and generating chemical reaction cases. From fig.22 we find that higher the dissipation lesser the actual concentration in the flow region. An increase in micropolar parameter  $\Delta$  increases  $|\omega|$  in the region (fig.23). An increase in suction parameter  $\lambda$  enhances  $|\omega|$  in the flow region (fig.24). From fig.25 we find that  $|\omega|$  enhances with increase in micropolar parameter  $A \leq 1$  and reduces with higher  $A \geq 1.5$ .

The stress ( $\tau$ ) at the inner and outer cylinders is exhibited in Table-2 for different parametric values. The stress enhances at  $r=1$  and  $r=2$  with increase in strength of heat degenerating source while it reduces with that of heat absorbing source. An increase in micro rotation parameter  $A$  results in an enhancement in  $|\tau|$  at  $r=2$  and depreciates at  $r=1$ . With respect to viscosity ratio parameter  $\Delta$  we find that  $|\tau|$  enhances at  $r=1$  and reduces at  $r=2$  with increase in  $\Delta$ . With reference to Eckert number  $Ec$  it can be seen that  $|\tau|$  enhances with increase in  $Ec$  at  $r=1$  & 2. The variation of  $\tau$  with chemical reaction parameter  $\gamma$  shows that  $|\tau|$  reduces in the degenerating chemical reaction case and enhance in the generating chemical reaction case. Higher the thermo-diffusivity (or smaller the diffusion-thermo effects) larger the stress at both the cylinders. An increase in suction parameter ( $\lambda$ ) larger the stress at the cylinders.

The rate of heat transfer (Nusselt number) ( $Nu$ ) at  $r=1$  and  $r=2$  is shown in Table-2 for different parametric values. An increase in suction parameter  $\lambda$  enhances  $|Nu|$  at both cylinders. The variation of  $Nu$  with micro rotation parameter  $A$  and viscosity ratio parameter  $\Delta$  shows that  $|Nu|$  enhances with  $A$  and  $\Delta$  at  $r=1$  and while at  $r=2$ , it enhances at  $r=1$  and reduces at  $r=2$ . The variation of  $Nu$  with  $Ec$  shows that higher the dissipative heat smaller  $|Nu|$ . Also  $|Nu|$  experiences an enhances at  $r=1$  & 2 with  $\gamma > 0$  and reduces with  $\gamma < 0$ . Higher the thermo-diffusion effect (or smaller the diffusion-thermo effects) larger  $Nu$  at  $r=1$  and smaller at  $r=2$ . Also  $Nu$  reduces with increase in  $Sc$  at  $r=1$  & 2.

The rate of mass transfer (Sherwood number) at  $r=1$  and 2 is exhibited in Table-2 for different parametric values. We find that the rate of mass transfer enhances with  $\lambda$  and reduces with  $Ec$  at  $r=1$  and 2. An increase in  $\Delta$  reduces  $Sh$  at  $r=1$  and enhances at  $r=2$  while it enhances with  $A$  at both cylinders. The variation of  $Sh$  with Schmidt number  $Sc$  shows that lesser the molecular permeability larger  $|Sh|$  at  $r=1$  and 2. Higher the thermo-diffusivity smaller  $|Sh|$  at both the cylinders. With respect to  $\gamma$  we find that  $|Sh|$  enhances in degenerating and reduces in generating chemical reaction cases at both the cylinders. Decreasing Dufour parameter leads to an enhancement at the cylinders.

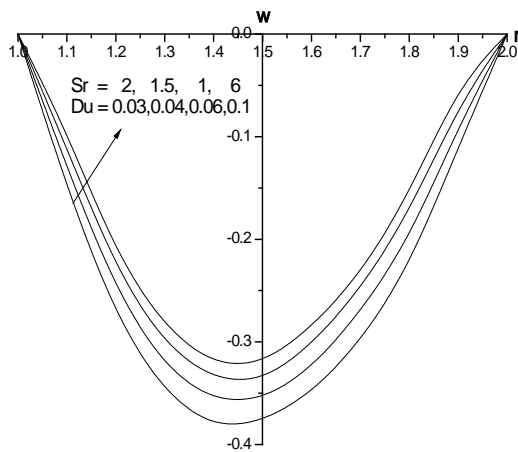


Fig. 1 : Variation of  $w$  with  $Sr$  &  $Du$   
 $\gamma=0.5, Ec=0.01, \Delta=0.5, \lambda=0.3, A=0.5$

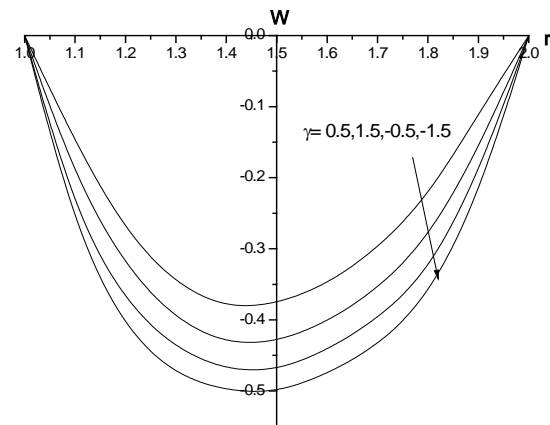


Fig. 2 : Variation of  $w$  with  $\gamma$   
 $Sr \& Du=2/0.03, Ec=0.01, \Delta=0.5, \lambda=0.3, A=0.5$

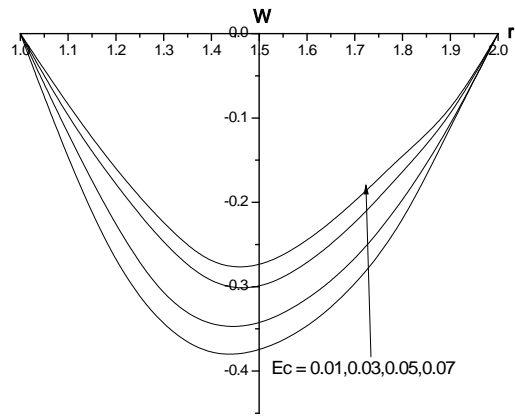


Fig. 3 : Variation of  $w$  with  $Ec$   
 $Sr \& Du = 2/0.03, \gamma = 0.5, \Delta = 0.5, \lambda = 0.3, A = 0.5$

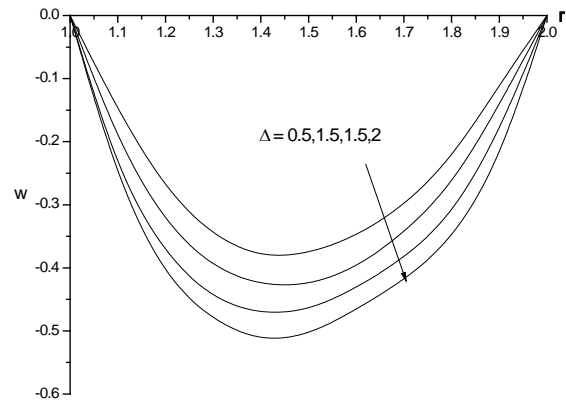


Fig. 4 : Variation of  $w$  with  $\Delta$   
 $Sr \& Du = 2/0.03, \gamma = 0.5, Ec = 0.01, \lambda = 0.3, A = 0.5$

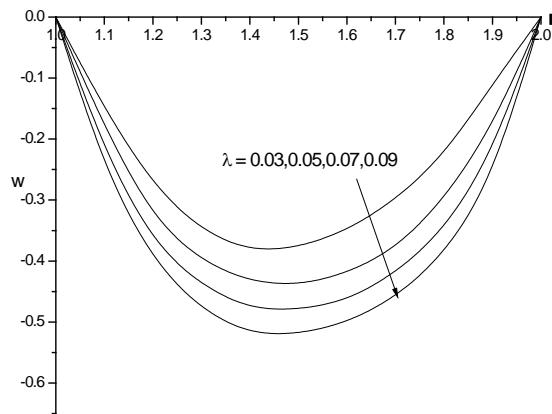


Fig. 5 : Variation of  $w$  with  $\lambda$   
 $Sr \& Du = 2/0.03, \gamma = 0.5, Ec = 0.01, \Delta = 0.5, A = 0.5$

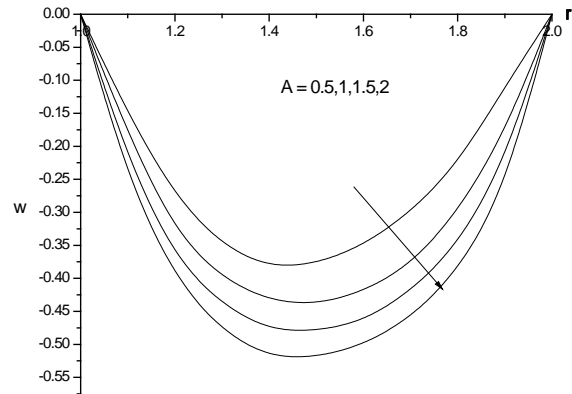


Fig. 6 : Variation of  $w$  with  $A$   
 $Sr \& Du = 2/0.03, \gamma = 0.5, Ec = 0.01, \Delta = 0.5, \lambda = 0.3$

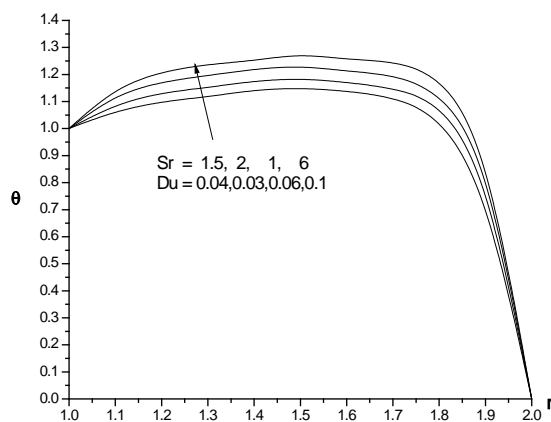


Fig. 7: Variation of  $\theta$  with  $Sr \& Du$   
 $\alpha = 2, \gamma = 0.5, Ec = 0.01, \Delta = 0.5, \lambda = 0.3, A = 0.5$

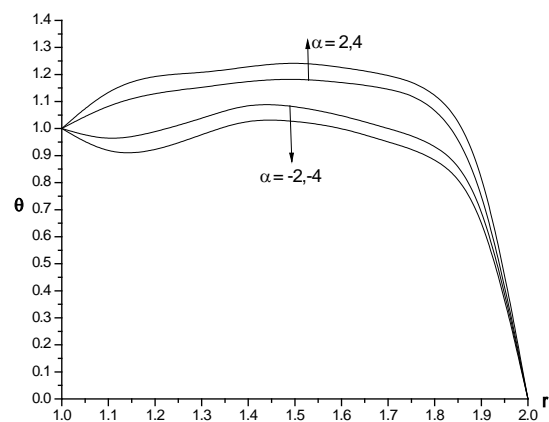


Fig. 8 : Variation of  $\theta$  with  $\alpha$   
 $Sr \& Du = 2/0.03, \gamma = 0.5, Ec = 0.01, \Delta = 0.5, \lambda = 0.3, A = 0.5$

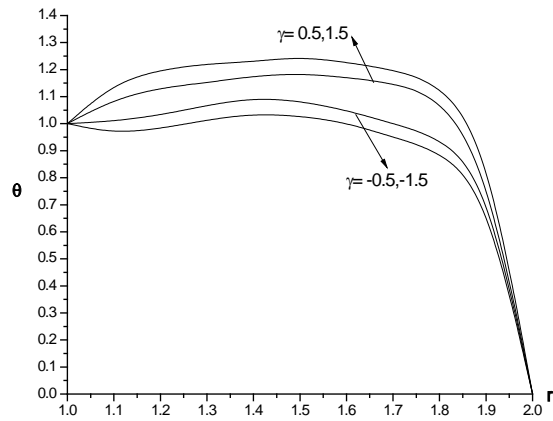


Fig. 9 : Variation of  $\theta$  with  $\gamma$   
 $Sr \& Du = 2/0.03$ ,  $\alpha = 2$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

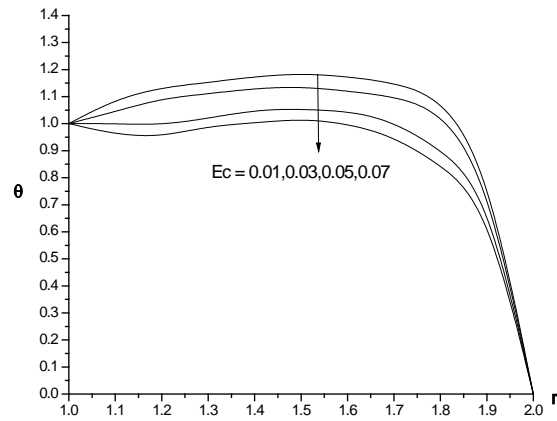


Fig. 10 : Variation of  $\theta$  with  $Ec$   
 $Sr \& Du = 2/0.03$ ,  $\alpha = 2$ ,  $\gamma = 0.5$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

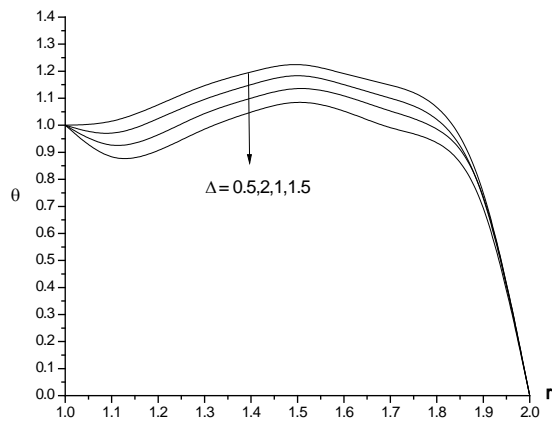


Fig. 11 : Variation of  $\theta$  with  $\Delta$   
 $Sr \& Du = 2/0.03$ ,  $\alpha = 2$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\lambda = 0.3$ ,  $A = 0.5$

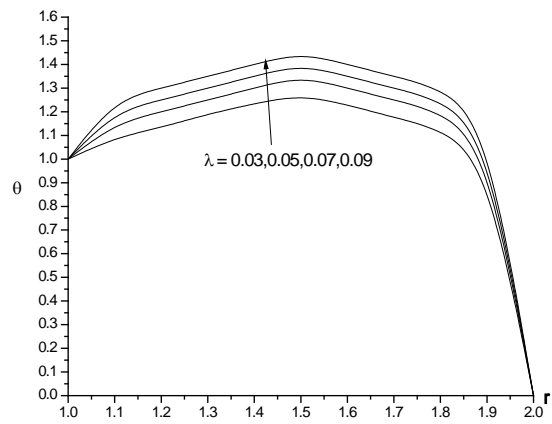


Fig. 12 : Variation of  $\theta$  with  $\lambda$   
 $Sr \& Du = 2/0.03$ ,  $\alpha = 2$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $A = 0.5$

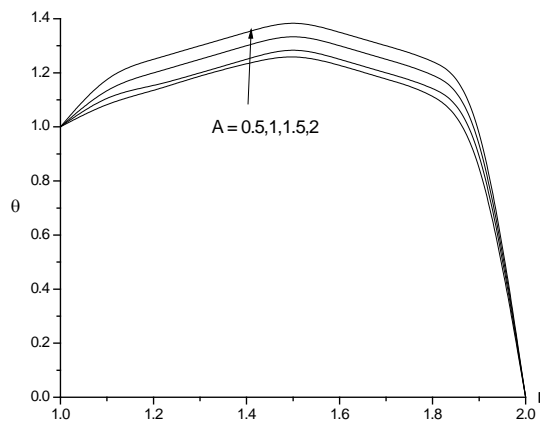


Fig. 13 : Variation of  $\theta$  with  $A$   
 $Sr \& Du = 2/0.03$ ,  $\alpha = 2$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$

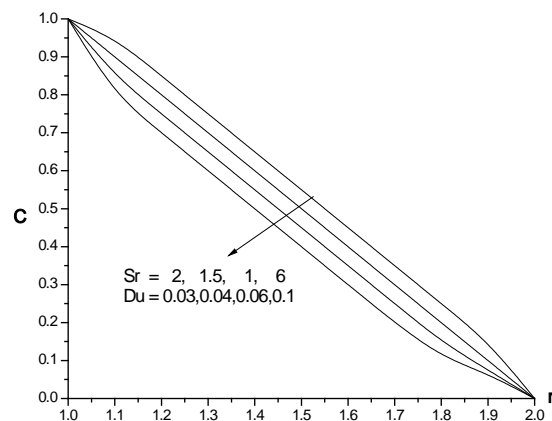


Fig. 14 : Variation of  $C$  with  $Sr$  &  $Du$   
 $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

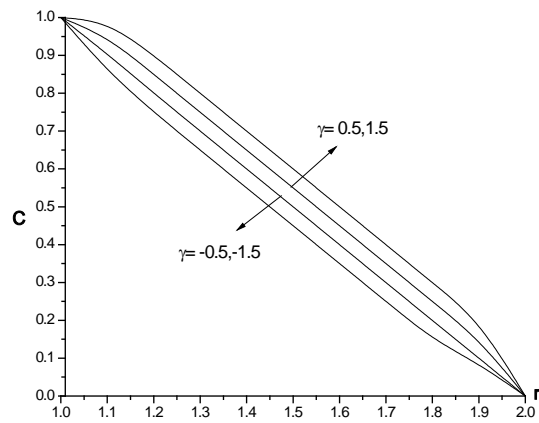


Fig. 15 : Variation of C with  $\gamma$   
 $Sr \& Du = 2/0.03$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

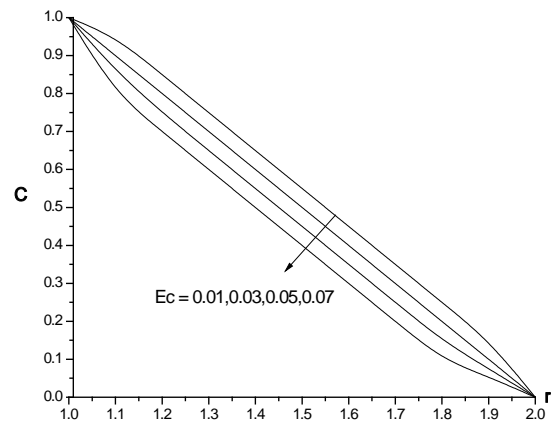


Fig. 16 : Variation of C with  $Ec$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

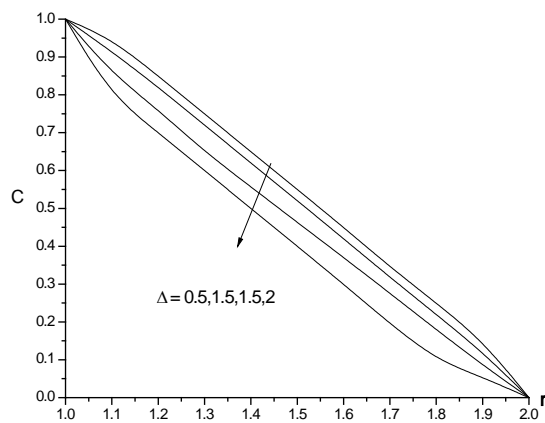


Fig. 17 : Variation of C with  $\Delta$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\lambda = 0.3$ ,  $A = 0.5$

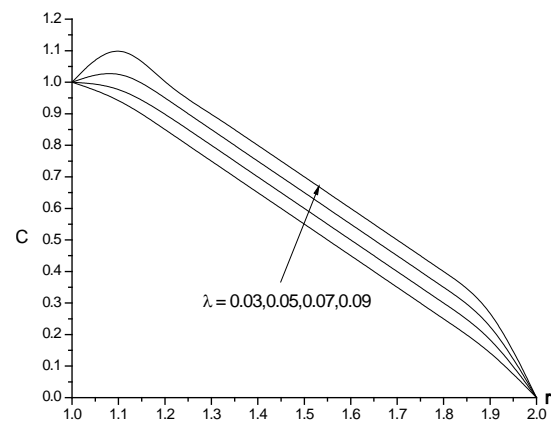


Fig. 18 : Variation of C with  $\lambda$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $A = 0.5$

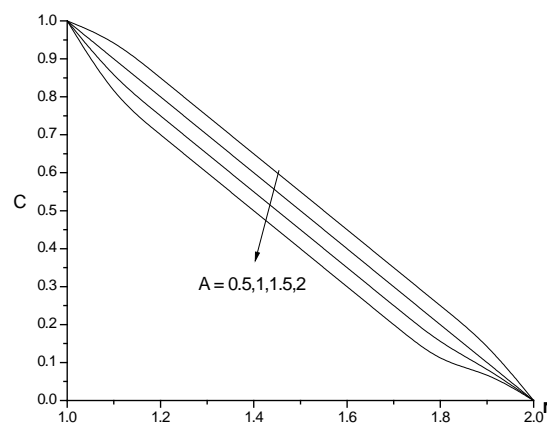


Fig. 19 : Variation of C with  $A$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$

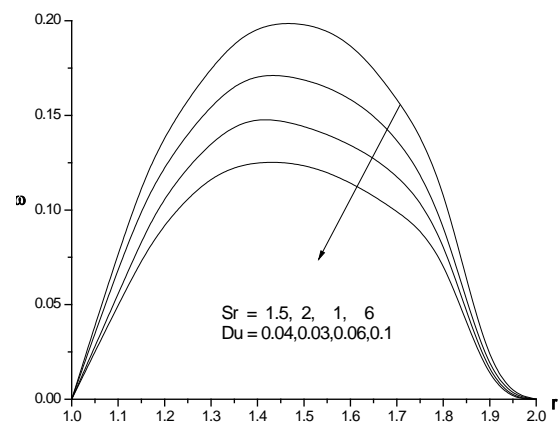


Fig. 20 : Variation of  $\omega$  with  $Sr \& Du$   
 $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

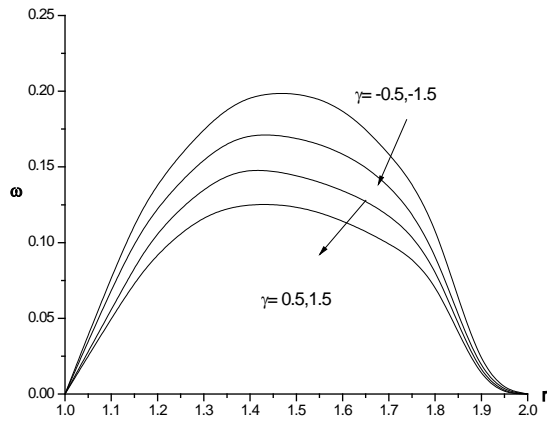


Fig. 21 : Variation of  $\omega$  with  $\gamma$   
 $Sr \& Du = 2/0.03$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

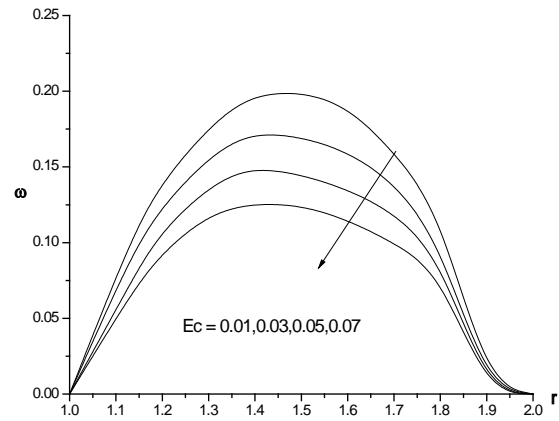


Fig. 22 : Variation of  $\omega$  with  $Ec$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$ ,  $A = 0.5$

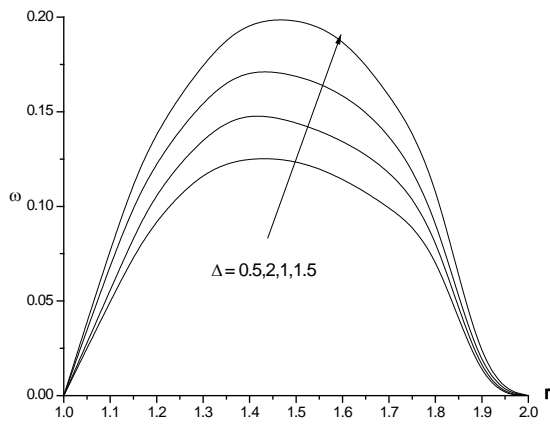


Fig. 23 : Variation of  $\omega$  with  $\Delta$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\lambda = 0.3$ ,  $A = 0.5$

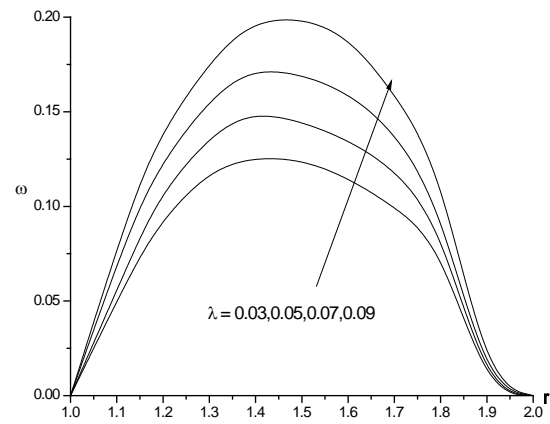


Fig. 24 : Variation of  $\omega$  with  $\lambda$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $A = 0.5$

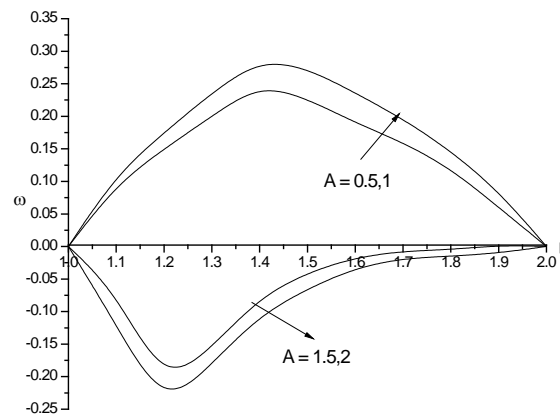


Fig. 25 : Variation of  $\omega$  with  $A$   
 $Sr \& Du = 2/0.03$ ,  $\gamma = 0.5$ ,  $Ec = 0.01$ ,  $\Delta = 0.5$ ,  $\lambda = 0.3$

**Table – 2: Skin friction( $\tau$ ), Nusselt Number( $Nu$ ), Sherwood Number ( $Sh$ )**

Sr/Du	Ec	$\gamma$	$\Delta$	$\lambda$	A	$\tau(1)$	$\tau(2)$	Nu(1)	Nu(2)	Sh(1)	Sh(2)
1.5/0.04	0.01	0.5	0.5	0.03	0.5	-0.230781	0.195265	27.5746	-39.4183	24.3414	-2.76555
1.0/0.06	0.01	0.5	0.5	0.03	0.5	-0.207915	0.177720	27.5876	-39.4816	24.1194	-2.51809
0.6/0.1	0.01	0.5	0.5	0.03	0.5	-0.190827	0.163859	27.5833	-39.5266	23.9508	-2.32974
2.0/0.03	0.03	0.5	0.5	0.03	0.5	-0.224567	0.206754	28.6789	-39.6754	24.1245	-2.45677
2.0/0.03	0.05	0.5	0.5	0.03	0.5	-0.214357	0.196785	28.7865	-39.9876	24.0345	-2.24567
2.0/0.03	0.01	1.5	0.5	0.03	0.5	-0.222607	0.190357	27.5762	-39.4372	24.7299	-3.27399
2.0/0.03	0.01	-0.5	0.5	0.03	0.5	-0.356926	0.320449	27.5289	-38.9341	23.6319	-1.37049
2.0/0.03	0.01	-1.5	0.5	0.03	0.5	-0.370676	0.339257	27.5215	-38.8622	23.5043	-1.06722
2.0/0.03	0.01	0.5	1.0	0.03	0.5	-0.285995	0.199772	27.5617	-39.3091	24.5364	-2.99128
2.0/0.03	0.01	0.5	1.5	0.03	0.5	-0.292323	0.136248	27.5678	-39.4048	24.5352	-3.00435
2.0/0.03	0.01	0.5	0.5	0.05	0.5	-0.260393	0.219239	27.6277	-39.7609	24.5576	-3.00211
2.0/0.03	0.01	0.5	0.5	0.07	0.5	-0.268106	0.225663	27.6806	-40.1356	24.5709	-3.01302
2.0/0.03	0.01	0.5	0.5	0.03	1.0	-0.269625	0.222903	27.6043	-39.5454	24.5473	-2.99987
2.0/0.03	0.01	0.5	0.5	0.03	1.5	-0.262882	0.249704	27.6413	-39.3273	24.7648	-3.16548

## 6. CONCLUSIONS

- Lesser the permeability of porous medium larger  $w$ ,  $C$ ,  $\omega$  and  $\theta$  in the flow region.
- The velocity, temperature and concentration enhance, the microrotation reduces in the degenerating chemical reaction case and enhances in the generating case, the velocity enhances, temperature, concentration and microrotation reduce in the flow region.
- An increasing suction parameter  $\lambda$  enhances  $w$ ,  $\theta$ ,  $C$  and the angular velocity  $\omega$ .
- An increasing Eckert number  $Ec$  reduces the velocity, temperature, concentration and microrotation.
- Higher the molecular diffusivity larger the velocity and concentration and smaller the temperature and microrotation in the flow region.
- An increase in heat generating source parameter ( $\alpha > 0$ ) enhances the temperature the temperature enhances in the flow region.
- Higher the thermo-diffusion effect(or lesser the diffusion-thermo) smaller the velocity, temperature and enhances the concentration and microrotation in the flow region.
- Higher the Lorentz force smaller the stress, and larger the rate of heat and mass transfer on the cylinders.
- An increasing the micropolar parameter  $\lambda$  enhances  $|Nu|$ ,  $|Sh|$  and  $|Cw|$  at both the cylinders.
- Higher the dissipative heat smaller  $|\tau|$ ,  $|Sh|$ , and larger  $|Nu|$  at  $r=1$  and  $2$

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